Advice on the auction model for the 3500 MHz band award

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DotEcon Ltd
17 Welbeck Street
London W1G 9XJ
www.dotecon.com

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Executive Summary

In this report we provide our recommendation to the Dutch Ministry of Economic Affairs (the Ministry) on the auction model and rules to be used for the award of spectrum in the 3500 MHz band. We base our recommendation on the auction objectives discussed with the Ministry, the structure of valuations as set out in a report suggesting the optimal lot size for this band (the Dialogic report), the spectrum caps set by ACM and any relevant insights from the recent multi-band auction.

Salient facts

There is a total of 300 MHz available, with no expected material value differences across different frequencies within the band.

The auction objectives on which we base our recommendations are:

- to ensure an efficient distribution of the available spectrum (i.e. in a way that maximises the value generated from spectrum use);
- to provide realistic possibilities to all interested parties (i.e. a level playing field);
- to generate realistic revenues (i.e. revenues reflect the value of the public resource); and
- to keep the auction as simple and transparent as possible.

The Dialogic report:

- recommends a lot size of 10 MHz in order to maximise flexibility when distributing spectrum between bidders;
- suggests the use of minimum requirements if there were any concerns that bidders may face aggregation risks with respect to a minimum bandwidth for a viable business case;
- concludes that economies of scale are exhausted at 50-60 MHz, and after this point the value of incremental spectrum is decreasing, although small entrants may require only 25-50 MHz for a viable business case; and
- highlights the importance for winners receiving non-fragmented spectrum to use it efficiently.

Bidders will be subject to a spectrum cap of 120 MHz in this band.

The recent multiband auction suggests that where the available spectrum can be split symmetrically, MNOs may be willing to settle with no or little competition for such an outcome.

Implications for auction design

Given that the spectrum is available in a single unfragmented band, with no material differences within the band, it is appropriate to separate the assignment of bandwidth amongst bidders (which can be determined in a bidding process where bidders bid for frequency-generic lots) from the assignment of specific frequencies (which can be determined in a follow-up process that ensures that all winners receive contiguous frequencies).

With a total of 300 MHz available there is a focal point for the three established MNOs to share the available spectrum, each acquiring 100 MHz. As this is also maximum bandwidth that can be deployed using a single NR carrier might thus be an optimal bandwidth from a technical point of view, an equal split in the absence of new entry might be an efficient distribution and should therefore not necessarily raise concerns with respect to the efficiency objective. However, it could raise concerns with respect to generating realistic revenues if this means that there is little competition in the auction.

However, the focal point of an equal split may also facilitate strategic bidding to block entry, allowing the MNOs to share the cost of outbidding entrants. In this case revenues might be higher (reflecting the value that the entrant places on spectrum), but there could be an efficiency concern. The value that an entrant might obtain from acquiring spectrum could be higher than the value of incremental spectrum to MNOs, but MNOs may still find it profitable to block entry to avoid greater competition downstream. To address this concern, we recommend using an information policy that does not allow bidders to infer the number of bidders competing for bandwidth too easily.

The structure of valuations, with synergies up to a bandwidth of 50-60 MHz but strongly decreasing marginal valuations thereafter, has important implications for auction design:

- concerns about bidders obtaining a minimum usable bandwidth and aggregation risks are only relevant in the event of participation from potential entrants (as if only the three MNOs participate the cap of 120 MHz guarantees that each is able to acquire at least 60 MHz, exhausting synergies), and thus must take into account the needs of both MNOs and potential entrants;
- using linear prices in combination with small blocks will limit the extent to which the process can generate realistic revenue without leaving spectrum inefficiently unsold if only the three MNOs participate; reserve prices would need

to be set at a value much below the average value per MHz in order not to choke off demand from MNOs for the last few blocks, leaving a large gap between the value of the spectrum and the revenues generated; and

- with many small lots and linear prices, there is an incentive to settle early to avoid price increments; these incentives are greater the lower reserve prices relative to the average spectrum value and the greater the number of lots a bidder expects to acquire and reinforce the effect of a focal point for MNOs to share spectrum in the absence of potential entrants. Moreover, this may also lead to entrants inefficiently winning spectrum in the event of participation from potential entrants, as MNOs may inefficiently accommodate entry to avoid prices from increasing.

If many small, identical lots are used (as would be the case if all the spectrum available is offered in 10 Hz blocks), then:

- mitigating the risk of unsold lots will require low reserve prices, owing to the valuation structure where marginal valuations are rapidly decreasing after around 60 MHz;
- incentives for bidders to settle early in the auction are strong; and
- clock-based bid collection mechanisms are needed (rather than collecting bids on individual lots) to avoid procedural inefficiencies.

Recommended auction model

We recommend assigning bandwidth using two stages:

- the first stage determines the assignment of three 60 MHz blocks, using a sealed-bid process where bidders may bid for at most one block and winners pay the lowest winning bid;
- the second stage determines the assignment of the remaining spectrum, offered in 10 MHz blocks using a clock auction, with provisions for bidders to place exit bids linked to their demand reductions, in which bidders who have not bid for a 60 MHz block in the first stage would be subject to a limit of 50 MHz in this second stage.

The assignment of specific frequencies to winners of bandwidth would be determined using a second price combinatorial sealed bid round, as in previous awards.

Assigning bandwidth in two stages provides better incentives for bidders to compete for additional spectrum at the margin, which addresses concerns that weak competition could lead to an inefficient assignment (especially in the event of participation from an entrant). At the same time, the two-stage approach does not reduce flexibility for bidders to adjust their demand at the margin, still supporting adjustments with granularity of 10 MHz. Small-scale entrants (seeking no more than 50 MHz) can bid in the second stage, whilst large-scale entrants can also compete for a lot in the first stage to obtain a minimum bandwidth.

The two-stage approach will also support the objective of generating realistic revenue as differentiated reserve prices can be set. It would be possible to set reserve prices for the initial 60 MHz blocks that are reflective of the value of spectrum without choking off demand at the margin as the reserve price on smaller lots can be chosen conservatively.

We recommend restricting bidders to bid for at most one lot in the first stage, as this reduces the risk of gaming and potentially highly asymmetrical outcomes. We also recommend that the number of lots in the first stage be set at three, as offering fewer or more lots could equally lead to gaming and/or unwanted asymmetries. Finally, we propose that bidders who do not participate in the first stage are subject to a limit of 50 MHz in the second stage. This is to remove incentives for bidders to defer bidding to the second stage, thereby undermining the two-stage approach.

A simple sealed bid process seems appropriate for the first stage. An open multi-round process would provide little benefits given that bidders can bid for at most one lot but could facilitate strategic bidding to block entry. Setting the price of these lots at the level of the lowest winning bid also mitigates incentives to place high bids simply to block entry and may lead to revenues being more reflective of valuations. At the same time, the proposed pricing rule avoids the potentially large differences in prices paid by winners that could emerge if each winner were required to pay its own bid.

For the second stage there are clear benefits from using an open multi-round process. We recommend using a clock auction with provisions for bidders to make exit bids linked to their demand reductions, as:

- a clock-based mechanism is procedurally efficient in view of the relatively large number of identical lots on offer;
- concerns about minimum bandwidth requirement are already addressed to a large extent by offering larger blocks in the first stage as well as the non-committing nature of clock bids; and

- other clock-based mechanisms like the clock-SMRA hybrid or clock auctions with restrictions to reduce demand might expose bidders to aggregation risks.

The provision for exit bids should mitigate the risk that lots might go inefficiently unsold, but without exposing bidders to aggregation risks or financial overcommitment.

With respect to the information policy, we recommend limited transparency to discourage strategic bidding to prevent entry. We would propose not to disclose the level of aggregate demand or do so only after it falls below a certain threshold as under the caps, aggregate demand information can provide a clear indication of participation from potential entrants.

For the assignment of specific frequencies, we recommend using a sealed bid second price combinatorial auction, as in previous awards.

1 Introduction

Scope of our advice

We have been asked by the Dutch Ministry of Economic Affairs (the Ministry) to recommend an auction model for the award of spectrum in the 3500 MHz band, comprising a total of 300 MHz of TDD spectrum.

In this report, we set out our recommended model for the auction and specific rules for this award.

Our recommendations are guided by the auction objectives that have been communicated to us in the consultancy brief[1] and further discussed with the Ministry in several calls, taking account of technical considerations and the likely structure of valuations as set out in a report prepared for the Ministry by *Dialogic innovatie & interactie* (Dialogic)[2] and the spectrum caps set by ACM for this award. We have also been asked to consider any implications that might be drawn from the recently completed multi-band auction for the forthcoming award process and to take account of the spectrum caps that result from the application of the spectrum capping regulations.[3]

Structure of the report

The remainder of this document is structured as follows:

- In Section 2 we summaries our understanding of the salient facts that need to be taken into account when establishing recommendations for an auction model;
- In Section 3 we spell out the implications of these facts for auction design.
- Section 4 then contains our recommendations and the reasoning underpinning them.

A full set of auction rules is provided in the Annex.

[1] Offerteaanvraag veilingtool 3,5 GHz frequenties; email from 23 June 2020

[2] Dialogic, Onderzoek kavelgrootte veiling 3,5 GHz-vergunningen, prepared for Ministerie van Economische Zaken en Klimaat, 2020, 28 August 2020.

[3] Regeling van de Staatssecretaris van Economische Zaken en Klimaat van 6 maart 2020, nr. WJZ/20063207, tot vaststelling van de maximale hoeveelheid frequentieruimte voor mobiele communicatie (Capregeling frequenties mobiele communicatie 2020)

2 Salient facts

2.1 Available spectrum

A total of 300 MHz of spectrum is available, ranging from 3450 to 3750 MHz. We understand that there is no difference in the value of different parts of this range, although the lowest and the highest blocks may be subject to some additional restrictions to protect the existing local use in in 3400 – 3450 MHz and 3750 - 3800 MHz. However, any such restrictions will be localised and temporary, as the current local usage rights will expire in September 2026 and any permits granted in future will require the users to protect MNOs. Therefore, the impact on the valuation of the lower and upper parts of the available spectrum is likely to be limited.

2.2 Objectives

We understand that the Ministry wishes to maximise the use of the 3500 MHz band for national mobile communications. As the available spectrum resources are scarce, they will need to be offered through an auction process with.

Efficient distribution of spectrum

The overarching objective of producing an ***efficient distribution of spectrum,*** which we understand to mean an assignment of spectrum to users such that the value for the economy and society created through its use is maximised. This is typically achieved through an auction model that allows prospective users to compete freely for the scarce frequency resources, subject only to constraints that might be needed to protect or promote effective competition in downstream markets. Competition safeguards are needed to ensure that bidders' willingness to pay for spectrum reflects the economic value that they can generate rather than any economic profits they might earn from restricting downstream competition through denying frequency resources to competitors.

Realistic possibilities for all interested parties

Closely linked to the efficiency objective but listed separately is the requirement that the auction model should offer ***realistic possibilities for all interested parties to take part*** and win spectrum. We understand this to mean that the auction model should not favour specific bidders or outcomes. Neither incumbent operators nor potential new entrants should be in any way advantaged but should have the opportunity to bid for spectrum on a level playing field.

Realistic revenues

A further objective is that the process should ***produce realistic revenues***, which we understand to mean that the auction should ensure that an appropriate share of the value of the scarce public resource that is spectrum should be realised for the public, rather than creating windfall profits for successful bidders. Put differently, the revenues received should be an appropriate reflection of the value of the public resource.

Simplicity and transparency

The final objective expressed by the Ministry is for the auction model to be ***simple and transparent***. Making sure that the rules of the auction are easily understood and that the design makes it easy for bidders to determine how they should bid helps minimise the scope for bidding mistakes which could jeopardise efficiency and increase the risk of legal challenges.

2.3 Findings from the Dialogic report

Dialogic was asked to make recommendations on the optimal lot size, based on technical constraints and the value that users might place on different amounts of spectrum.

Recommendation on block size

Based on its analysis, Dialogic recommends that the available spectrum ***should be sold in blocks of 10 MHz*** in order to maximise the flexibility for bidders to assemble their preferred portfolios and to support the widest range of outcomes. Where small blocks create a potential risk for bidders of acquiring less spectrum than is required for a reasonable (let alone viable) business case, appropriate options for specifying minimum requirements should be used rather than offering spectrum in larger blocks (including the possibility of packaging spectrum into differently sized blocks, which might limit the granularity of outcomes).

Dialogic also looked at using blocks of unequal size, but considered that this would add little value in terms of flexibility of outcomes over a subdivision into only blocks of 10 MHz from the point of view of preventing fragmentation and facilitating differentiation between operators.[4] We understand that Dialogic did not consider the implications of using different block sizes in relation to the objectives set out above, in particular with regard to the prospect of raising realistic revenues.

Valuation structure and minimum requirements

In terms of efficient use, there is a tipping point at about 50-60 MHz where economics of scale are exhausted and incremental value of additional spectrum is declining (though smaller amounts may be useable, and entrants may require only 25-50 MHz for a viable business case).

Given the maximum width of a NR carrier, using spectrum beyond 100 MHz requires the deployment of two carriers, which is associated with some efficiency losses. This makes the incremental value of spectrum in excess of 100 MHz very small, and perhaps the main source of value of spectrum beyond the amount that can be deployed with a single carrier is strategic (i.e. driven by the desire to deny spectrum to other bidders).

Benefits from contiguity

To permit bidders to deploy the spectrum they acquire efficiently, the specific frequencies assigned to winners should be ***contiguous***.

2.4 Spectrum caps

Binding constraint from the band-specific 40% limit

We understand that the spectrum capping regulations provide for the ACM recommendation of a cap of 40% of the available spectrum within the 3500 MHz band will be implemented. This corresponds to 120 MHz and creates a binding constraint on all bidders.

The global spectrum cap (40% of all available spectrum) will not be binding: after the multiband auction, the operator with the largest amount of mobile spectrum is T-Mobile, holding a total of 250 MHz across the relevant bands. With a total of 960 MHz of spectrum being available after the 3500 MHz auction, the 40% limit amounts to 384 MHz, which would allow even T-Mobile to acquire 134 MHz (in practice: 130 MHz) of spectrum in the forthcoming auction; Vodafone could acquire up to

[4] Specifically, Dialogic considered that in the case of unequal block sizes, a split into 3x80 + 3x20 MHz, 3x70 + 3x30 MHz or 3x60 + 3x20 MHz would be the most obvious, though using smaller lots of 10 MHz instead of 20 or 30 MHz would also be possible. This would then allow the strongest bidders to acquire at least 80, 70 and 60 MHz and possibly 100 MHz respectively whilst also supporting winnings of 20, 30 and 40 MHz. Various intermediate outcomes would also be possible

169 MHz, and KPN up to 189 MHz (in practice 160 and 180 MHz respectively).

The 40% limit on sub-1GHz spectrum is obviously without relevance for this award.

2.5 Key insights from the multiband auction

We have been asked to consider whether there are any insights from the recently completed multiband auction that could be relevant for the recommendation of an auction model. In this regard, we can limit our attention to the first stage of the multiband auction, which determined the distribution of spectrum across the different winners.

This stage ran over 90 rounds, with competition focusing on the 1400 MHz band. The following diagrams showing the evolution of demand and prices throughout the process.

Figure 1: Prices in the multiband auction

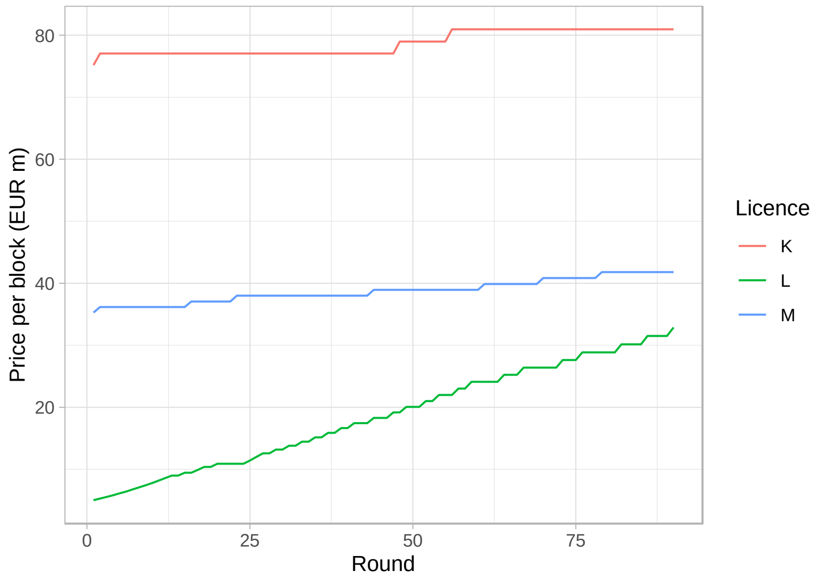
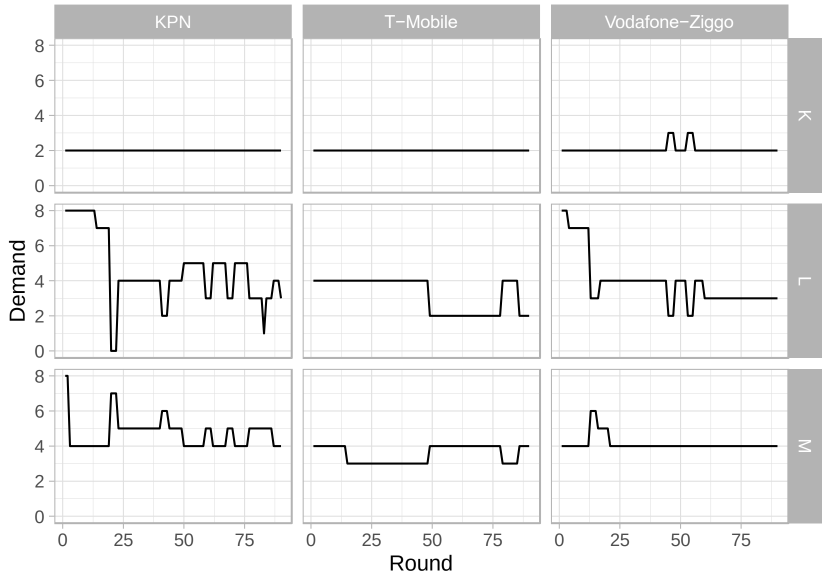


Figure 2: Demand from individual bidders in the multiband auction



These diagrams show that:

- there was no excess demand for K licences (700 MHz) in the first round, and prices rose only as a result of occasional forays of one bidder into this band as competition for L licences carried on;
- there was no excess demand for M licences (2100 MHz) from the second round onwards and prices here also rose only as a result of occasional switching between the L licences and M licences in the course of the auction.

Possibility of equal sharing can create a focal point

The fact that there was no excess demand for the 700 MHz band and that competing demand for the 2100 MHz could have resolved within a few rounds may be explained to a large extent by the fact that both bands offered the opportunity for sharing out the available spectrum (six and twelve blocks respectively) symmetrically amongst the three bidders. This provided a focal point on which bidders could settle very quickly.

By contrast, such an option did not exist in the 1400 MHz band. With eight blocks on offer, some bidders would inevitably win less than others. Without a focal point in this band, excess demand resolved only after some rounds with switches of demand into other bands, which might have indicated a genuine willingness to substitute between the bands or strategic attempts to get competitors to accommodate in the 1400 MHz band.

3 Implications for auction design

3.1 Available spectrum

Absence of substantial value differences permits use of separate steps for the assignment of bandwidth and specific frequencies, which guarantees contiguity

The available spectrum lies in a single band and we understand that value differences across different parts of the band are negligible. Even the additional usage restrictions that could affect the lower and upper end of the band are limited both geographically and temporally.

This means that we can separate the award into two steps, first determining how much bandwidth each bidder will win (bandwidth assignment phase) and then which specific frequencies each of the winners of bandwidth will be assigned (frequency assignment phase). We can easily limit the options for assigning specific frequencies to those where the assignment for each bidder is contiguous. This ensures that spectrum is used efficiently (as set out in the Dialogic report). Bidders will be able to focus on their bandwidth needs when bidding for spectrum without concerns about ending up with a fragmented assignment.[5]

Spectrum can be split equally between the three incumbents, providing a natural focal point

The available spectrum can neatly be divided into three equal parts, permitting outcomes in which each of the three established operators acquires 100 MHz. This could provide a natural focal point on which bidders might easily settle, similar to what we have observed in the 700 and 2100 MHz bands in the multi-band auction.

As equal split might be efficient in the absence of new entry no concern about efficiency, but possibly about revenue

However, we understand that 100 MHz is the maximum bandwidth that can be deployed using a single NR carrier. Using more spectrum would require the operator to deploy two carriers. As each bidder can only acquire a small amount of spectrum above 100 MHz under the cap (set at 120 MHz), this is unlikely to be a technically or economically attractive option. This means that, in the absence of new entry, an equal split of the 300 MHz available amongst the three incumbents might also be the efficient outcome.

To the extent that this is the case, bidders settling on this outcome would not raise any efficiency concerns. However, to

[5] As spectrum will be initially offered in the form of frequency-generic lots, this also supports auction formats that rely on a clock mechanism for collecting bids (auctioneer specifies price – bidders nominate demand), which is preferable where there are many lots for reasons of procedural efficiency.

the extent to which MNOs settling early for such an outcome could suppress competition in the auction, which may then end at or close to reserve prices, this might be of concern in terms of generating realistic revenues. Given this, we recommend setting reserve prices at a level that reflects the value of the spectrum.

Potential efficiency concerns if focal point facilitates bidding to keep out efficient new entry may need to be addressed through information policy

There could be efficiency concerns, however, because an equal split of spectrum implies that the burden of keeping out an entrant can also be shared equally between the incumbents. Thus, the presence of such a focal point could facilitate strategic bidding to deny spectrum to a new entrant. This would drive up prices and increase revenues but would be inefficient if the value of a small amount of spectrum for a new entrant exceeds the value loss suffered by incumbents from a small reduction of their spectrum endowment below 100 MHz.

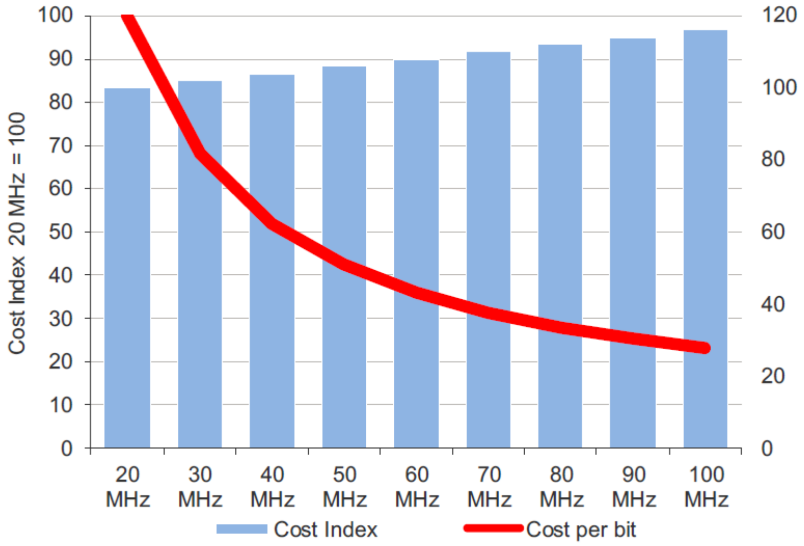
Such concerns would need to be addressed through an appropriate information policy aimed at not allowing bidders to infer whether a new entrant is (still) competing in the auction.

3.2 Valuation structure

Scale economies are exhausted at around 50-60 MHz and the value of incremental spectrum is dropping sharply beyond this point

We understand that incumbent operators enjoy substantial scale economies up to a bandwidth of around 60 MHz but that the value of additional spectrum then begins to drop off fairly quickly. Dialogic talks about a ‘tipping point’ at 50-60 MHz and references a study prepared by Coleago that shows a sharply declining cost per bit up to a spectrum amount of 50-60 MHz (though pointing out that neither the methodology nor the underlying data are known).

Figure 3: Cost per bit depending on amount of 3.5 GHz spectrum



Source: Coleago Consulting, The business case for 5G: lowering the cost per bit. 4th Annual European 5G Conference, 29-30 January 2020, taken from Figure 6 of the Dialogic report

This general valuation pattern matches the valuations that we have seen in other contexts.

The initially large incremental value of spectrum, dropping off sharply after a certain critical amount of around 50-60 MHz can be deployed, has implications for minimum requirements but also for reserve prices, which in turn has implications for bidding behaviour.

3.2.1 Minimum requirements

In the absence of entry, caps eliminate concerns about minimum requirement

In the absence of new entry, the spectrum cap of 120 MHz per operator ensures that with three bidders each bidder is guaranteed to win at least 60 MHz. As this appears to be the point at which scale economies are largely exhausted and incremental value of additional blocks falls below average value, there is no need to be concerned about protecting bidders from aggregation risks. Being guaranteed to win 60 MHz, bidders should not be exposed to the risk of winning an unusably small amount of spectrum or overpaying for the bandwidth they win.

However, bidders could face aggregation risks if there are more than three bidders

However, where there are more than three bidders the cap does not provide such a guarantee. In this case, if spectrum is simply offered in small blocks without any other measures, then there is a risk that some bidders could be left with an unusable amount of spectrum, or (if bidders are not obliged to take up small amounts of spectrum) that some spectrum could remain inefficiently unsold. This means that any measures that are adopted for protecting bidders from acquiring less than a minimum amount of spectrum must address concerns about aggregation risks in the case of new entry.

In order to provide realistic possibilities to all interested parties, such measures should also afford entrants to be protected against aggregation risks, i.e. the measures should reflect the spectrum needs of new entrants. Dialogic has identified viable entry business cases with around 30-50 MHz of spectrum. This means that if measures such as minimum requirements were to be used, they would need to allow bidders to set their own level as there is no minimum requirement that would apply to all types of bidders.

3.2.2 Linear prices and realistic revenue

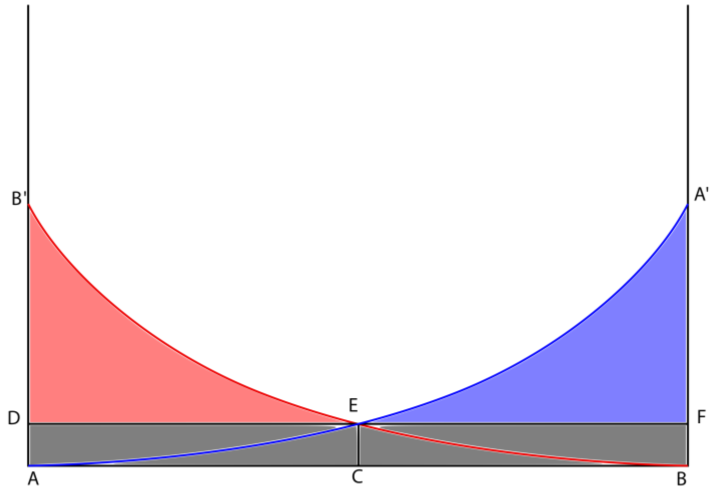
Prices in a competitive process are typically set in a way that ensures that winners of spectrum will pay at least the amount that losing bidders offered to pay. In the case where all bidders win some spectrum, but some lots are disputed between bidders, this means that the price that winners pay for disputed lots will be at least the price that competitors had offered to pay. This determines a lower bound on the amount that winners ought to pay, but depending on the structure of valuations, this may be only a small portion of the value of the resource, leaving bidders with substantial surplus.

This is the case, for instance, when prices are set to reflect opportunity costs, and the value of disputed lots is small relative to the value of all the spectrum won by each bidder.

When this happens, it is not possible to set a linear price (i.e. a unique or very similar price per MHz) that ensures both that spectrum does not remain inefficiently unsold and that revenues are reflective of the value of spectrum. We illustrate this with a stylised example in Box 1.

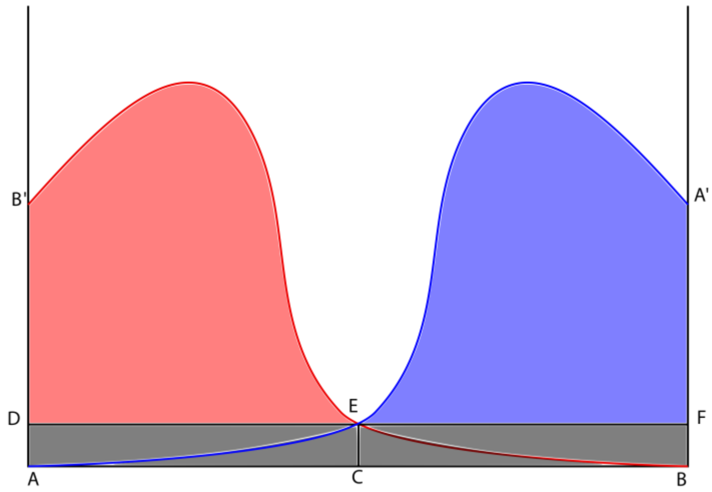
Box 1: Revenue and surplus – a stylised example

Consider the simple case where a total amount of spectrum equal to the length of the line AB in the following diagram should be distributed efficiently amongst two bidders. Let the red curve B'B depict the value that Bidder 1 places on incremental spectrum starting from A and moving to the right. Similarly, let the blue curve A'A depict the value that Bidder 2 places on incremental spectrum, starting from B and moving to the left. The efficient distribution is found where the incremental spectrum values are equal, which would imply that Bidder 1 should receive C units and Bidder 2 should receive the remainder (B – C). Giving more spectrum to Bidder 1 and less spectrum to Bidder 2 would create less additional value for Bidder 1 than the value lost to Bidder 2 (the blue curve is above the red curve), and vice versa.



If we were to run a simple clock auction starting at a price of zero, assuming that bidders truthfully reveal their demand at the various prices, total demand will equal total supply at a price of D, at which bidding stops. Bidder 1 pays ACED and Bidder 2 pays CBFE, so total revenue corresponds to the areas shaded in grey. The total value of the spectrum obtained by Bidder 1 is the entire area under the red curve up to C, so the bidder enjoys a surplus (value less payment) of B'DE – the area shaded in red. Similarly, Bidder 2 enjoys a surplus of EFA' – the area shaded in blue.

The extent to which the auction will realise an appropriate share of the total spectrum value in the form of revenues depends on the valuation structure. If the value of incremental spectrum is initially very high and then falls sharply as we approach what is the efficient distribution, the revenues generated would only be a small proportion of the total spectrum value and a large share would remain with bidders in the form of surplus, as a comparison with the following diagram clearly shows.



Reserve price for a 10 MHz block may need to be low in order to avoid the risk of unsold spectrum

If we were to use linear prices and ensure that spectrum does not remain inefficiently unsold in the absence of entry, then reserve prices should not exceed the marginal value of the last spectrum block acquired by the marginal MNO. For a 10 MHz block and assuming an equal distribution of spectrum across three incumbents, this would be the lowest value across all bidders attributed to increasing spectrum holding from 90 to 100 MHz.[6] Compared with the average per-block value, this may be rather low. It will certainly be substantially below the average block value of endowments up to 60 MHz.

3.2.3 Incentives to settle early

With linear prices in a pay-as-bid auction there are strong incentives for bidders to reduce demand to avoid price increments

In multi-unit pay-as-bid auctions with linear prices there are incentives to reduce demand early rather than compete for incremental spectrum up to its marginal valuation in order to keep overall prices down. We illustrate this with a simple stylised example in Box 2.

[6] If bidders were dissimilar or if bidders were strongly motivated to bid for spectrum in excess of 100 MHz to deny spectrum to their competitors, this assumption might need to be adjusted. Specifically, if there were one bidder whose valuation for the 11th or 12th block exceeds a weaker competitor's valuation of the 10th or 9th block, then the reserve price must not exceed the weaker bidder's valuation of the 9th or 8th block respectively.

Box 2: Reducing demand to keep prices low – a stylised example

Suppose that a bidder might be interested in a number of lots with the following valuations:

Blocks	Total value (EUR m)	Incremental value (EUR m)
1	50	50
2	90	40
3	120	30
4	130	10
5	138	8
6	143	5

Suppose that reserve prices are set conservatively at EUR 3 m. As the marginal value of the 6th block is still above reserve, the bidder would still find it profitable to acquire six blocks. However, suppose that the bidder could end the auction by reducing demand to four blocks. (i.e. demand of four blocks will 'fit' with the demand from other bidders). Doing so would leave the bidder with a surplus of EUR 118 m (EUR 130 m – EUR 12 m). Conversely, winning a fifth block only generates greater surplus if the price per block stays below EUR 4 m.[7] Given this, the bidder is only likely to bid for more than four blocks if it expects to win five blocks for a price of less than EUR 4 m, which is substantially lower than the incremental value of the fifth block.

Note that the incentive to settle early is greater the lower the reserve price and the larger the number of blocks the bidder would wish to acquire, as these two factors determine the initial surplus that the bidder can 'lock in' by reducing demand, which sets the bar for the surplus the bidder must expect to be able to achieve from competing for incremental spectrum. If all spectrum is offered in small blocks (of which incumbents will wish to acquire a sizeable number) and reserve prices must be set at a relatively low level to avoid unsold spectrum, this provides a strong incentive for settling early.

In the absence of entry, spectrum is likely to sell at, or close to reserve, so using linear prices in combination with a low reserve might not generate realistic revenues

In the absence of new entry, this reinforces the effect of a focal point and settling for an equal split becomes even more attractive that if starting prices were higher. As noted above, this should not raise efficiency concerns as the equal sharing outcome may well be efficient. However, as prices would not rise much above reserve, reserve prices will determine revenue. If reserve prices must be set at a level that is low relative to the

[7] The bidder would be better off with five blocks at price p than with four blocks at a price of 3 as long $138 - 5p > 118$, which means $p < (138 - 118)/5 = 4$.

average value per block, this will obviously limit the extent to which the auction can be said to generate realistic revenues.

With new entry, there may be an efficiency concern if incumbents were to be too accommodating

If there is new entry, incumbent bidders targeting a reasonably large amount of spectrum might be too accommodating, which in turn could give rise to inefficient entry. They might reduce their demand early to permit entry on a small scale to benefit from lower prices overall, rather than compete for additional spectrum to make full use of the bandwidth of a NR carrier. In this case the process would not be able to establish whether the entrant's valuations are higher than MNOs value of additional spectrum.

Opportunity-cost based format could avoid incentives for settling early, but would still produce low revenues in the absence of entry

The incentives for settling early would not exist in an auction format that employs an opportunity-cost based pricing rule (which, as has been alleged by some bidders, could even create incentives for engaging in strategic bidding to drive up prices). However, such an auction format would be likely to meet strong objections from bidders for several reasons, including the uncertainty they face over their final financial commitment and the resultant difficulties in managing budget restrictions. In addition, such an auction format may not provide realistic revenues in the absence of new entry as opportunity costs are limited by tight spectrum caps.

3.3 Block size

Smaller blocks reinforce the effects from the valuation structure in terms of bidding incentives and revenues

The block size – and whether all blocks are of the same size – is relevant for the considerations above.

If all spectrum is offered in small blocks, the reserve prices will have to be sufficiently small to mitigate the risk that some of the lots may go inefficiently unsold, reflecting the decreasing marginal valuation of spectrum. For example, the reserve price of 10 MHz blocks would have to be less than half the reserve price that could be set for 20 MHz blocks, reflecting that under marginally decreasing valuations the value per MHz of the last 10 MHz is lower than for the last 20 MHz, and that the relevant consideration is not to price off demand from the weakest bidder for the last 10 MHz, rather than not pricing off demand for the last 20 MHz. Low reserve prices in combination with each bidder needing to acquire many blocks creates strong incentives for reducing demand early.

The constraint on reserve prices becomes weaker with larger blocks as the number of blocks a bidder wishes to acquire becomes smaller and demand reductions will have a greater impact on the total amount of bandwidth. Of course, offering all spectrum in large blocks comes with a loss of flexibility and a restriction of possible outcomes, as pointed out by Dialogic.

Small blocks are likely to require package bidding or a clock-based mechanism for collecting bids

When there are many lots available (as in this case, as if all the available spectrum is offered in blocks of 10 MHz, there will be 30 lots on offer), any auction format that evaluates bids on a per-lot basis (such as the standard SMRA) is likely to result in procedural inefficiencies as many rounds will be required to resolve small amounts of excess demand. Thus, in such cases it is more appropriate to use an auction format that uses a clock-based mechanism for collecting bids and setting price increments.

Clock-based mechanisms require using frequency-generic lots, which is also desirable for contiguity of assignments

Clock-based mechanisms only work if the spectrum can be offered in the form of frequency-generic lots, which requires that the value of a block depends mainly on the bandwidth and is not affected (much) by the specific frequencies that a bidder will be assigned. We understand this to be the case, noting that the use of frequency-generic blocks is desirable in any case for reasons of contiguity. Splitting the award into an initial assignment of bandwidth in the form of frequency-generic blocks followed by the assignment of specific frequencies to winners of bandwidth allows us to guarantee that all final assignments are contiguous.

4 Recommended auction model

A two-stage process for the assignment of bandwidth followed by a separate assignment of specific frequencies

We recommend first assigning bandwidth by offering frequency-generic lots, which would be assigned in two stages:

- the first stage would determine the assignment of three blocks of 60 MHz each, using a sealed-bid; and
- the second stage would determine the assignment of the remaining spectrum, offered in 10 MHz blocks, using a clock auction with provisions for bidders to be able to place exit bids linked to their demand reductions.

The assignment of bandwidth would be followed-by the frequency assignment stage, which would determine the assignment of specific frequencies to the winners of bandwidth, using a combinatorial second-price sealed bid approach as has been used in previous awards.

In the remainder of this section, we provide further details for each of these stages and steps and set out our reasoning for the recommendation.

4.1 Assignment of bandwidth

We recommend assigning bandwidth in two stages:

First stage: single sealed bid auction of three 60 MHz blocks, all winners pay the lowest winning bid

The first stage would consist of a single round sealed bid process to assign three blocks of 60 MHz in line with the following provisions:

- There are three blocks of 60 MHz on offer.
- Each bidder may bid on at most one block by specifying the amount that it would be prepared to pay for the block.
- The bids received are ranked in descending order, with ties being broken at random.
- The three highest bids become winning bids. If fewer than three bids are received, all of these become winning bids.
- Each winning bidder pays the amount of the lowest winning bid.

Second stage: a clock auction with exit bids where bidders who have not placed a bid in the first stage are limited to at most 50 MHz

The second stage would consist of a clock auction with provisions for exit bids in line with the following provisions:

There are at least twelve 10 MHz blocks on offer. Should there be any unsold spectrum from the first stage, this will be added in the form of additional 10 MHz blocks in this second stage, so if one block remained unsold in the first stage, this would mean that there are 18 blocks of 10 MHz each.

Bidders who have not placed a bid in the first stage cannot bid for more than five blocks. Other bidders may bid on as many blocks as they can acquire under the spectrum cap, considering any spectrum won in the first stage.

Bidding takes place over several rounds of fixed length (subject to extensions that may be triggered by bidders). In each round, the auctioneer announces a price per block (the clock price) and bidders specify the number of blocks they wish to acquire at that price, subject to the constraints above and an activity rule that prevents bidders from increasing demand over the course of the bidding process.

A bidder who reduces demand relative to the previous round may place so-called exit bids. Exit bids are linked to a bidder's specific demand reduction and indicate the price at which a bidder would wish to acquire additional lots (alongside the lots indicated in its clock bid), subject to the following constraints:

- the number of additional lots cannot exceed the demand reduction;
- the price offered must be at least the clock price in the round before the demand reduction and cannot exceed the round clock price at which the demand reduction is made.

Exit bids remain valid throughout the auction and may become winning bids. However, an exit bid can only become a winning bid if the bidder is already assigned a number of lots equal to the lots in its clock bid after the corresponding demand reduction. Bidders may submit exit bids within the limits set out or withdraw previously placed exit bids when making new clock bids.

If demand aggregated across all bids in a round exceeds supply, the auctioneer increases the clock price and calls a further round. Otherwise, the clock rounds end.

At the end of the auction, each bidder wins its clock bid and pays the corresponding clock price per lot. If demand at final clock prices is below supply, the auctioneer may accept exit bids as winning bids to assign blocks that would otherwise remain unsold, subject to the requirement that an exit bid can only be accepted if the bidder who made the exit bid has already been assigned the number lots in its clock bid after the demand reduction to which the exit bid is linked. Bidders who win with an exit bid will be assigned the corresponding number of additional lots and pay for those lots the price for indicated in the exit bid.

4.1.1 Why two stages?

Owing to the valuation structure, selling all spectrum in small blocks gives rise to concerns about strategic demand reduction and revenues

Our recommendation for a two-stage process for the assignment of bandwidth is largely driven by the valuation structure. As discussed in the previous section, the high value of the first 50 or 60 MHz combined with a relatively sharp decline in the value of incremental spectrum thereafter can give rise to concerns about weak incentives to compete for incremental spectrum. This could lead to an inefficient assignment, including the possibility of inefficient entry. These concerns are strongest where the entire spectrum is offered in the form of small blocks. Thus, by reducing the number of small blocks offered in the second stage, the incentives to settle early instead of competing for additional blocks are also reduced.

Small blocks are required to allow bidders to adjust their demand at the margin in small steps to support outcomes of sufficient granularity around a distribution of spectrum that also reflects minimum requirements, but this does not mean that all spectrum needs to be offered in the form of small blocks. Indeed, separating the award into two stages in which bidders first can acquire a critical amount of spectrum and then add further bandwidth to this endowment provides the same support for variations of the distribution of bandwidth at the margin in small steps.

The block size for the first stage (of 60 MHz) is determined by what might be considered as a conservative minimum requirement, based on the valuation structure. Incumbents and new entrants who wish to come into the market at a large scale would be able to secure a base endowment that allows them to take advantage of scale economies. The protection against aggregation risks is provided regardless of whether there is new entry as each winner will have at least 60 MHz of spectrum.

Potential small-scale entrants can satisfy their spectrum demand in the second stage but will have to compete with incumbent's demand for incremental spectrum without the distortions that could arise if all spectrum were offered in the form of small blocks. Potential large-scale entrants will have an opportunity to acquire the spectrum they need in the first stage.

Separating the award into two stages also addresses concerns about generating realistic revenues under the valuation structure discussed above, by allowing to set different prices (and reserve) for initial 60 MHz blocks and small 10 MHz blocks. Even if the auction ends with prices at or close to reserve, the two stage-approach allows for setting the reserve price of larger spectrum blocks to be more reflective of their substantial value without creating the risk of leaving spectrum unsold. The reserve price of small blocks can be set conservatively to avoid choking off demand for additional spectrum. This allows us to mitigate the risk that spectrum might go inefficiently unsold without jeopardising the objective of generating realistic revenues.

We note that it would of course be possible to include different sized blocks with different reserve prices in a single stage to permit some switching. However, we advise against this, as providing switching opportunities would then re-introduce concerns about potentially pricing off demand. This would then need to be addressed through explicit constraints on switching or through introducing further bidding requirements (see Box 3).

Box 3: Large and small blocks in a single stage

Consider the case where we include large and small blocks in a single stage and run a simple clock auction. Suppose for simplicity that there are three 60 MHz blocks with a price of 60 and twelve 10 MHz blocks with a price of 5. Suppose also that the value of incremental spectrum beyond 60 MHz is 6. Suppose that there are three bidders who target 100 MHz each initially but want to buy at least 60 MHz provided that this bandwidth can be acquired at a price of less than 200.

Initial demand for small blocks will be 30, as each bidder bids for a full 100 MHz (note that settling early for 60 MHz would still leave excess demand and settling for 40 MHz would leave all large blocks unsold). The price of small blocks will increase, and aggregate demand will fall to 18 when they exceed 6 as bidders are no longer interested in incremental bandwidth. At the point at which the price for a small block reaches 10, one or more bidders would switch to the large blocks and bidding would end, but 120 MHz of spectrum would remain unsold.

Such an outcome could be avoided if, for example, bidders were only allowed to bid for small blocks alongside one of the large blocks. This would however remove the ability of bidders to switch between large and small blocks, which would have been the main reason for including the differently sized blocks in a single stage.

Alternatively, one could require that bidders place exit bids when reducing demand so that some of the small blocks could eventually be assigned at lower prices. However, this would result in aggregation risks and could also mean that bidders find it difficult to manage their financial exposure.

We also note that there are alternative options for mitigating incentives to settle early by de-coupling the prices for inframarginal and marginal units, such as Ausubel's 'clock auction with clinching'[8] (see Box 4). However, whilst this auction format should lead to higher clock prices than the simple clock auction (as the incentive to settle early is removed), it entails selling a potentially sizeable proportion of the overall supply at reserve and might therefore not generate realistic revenues, given the specific valuation structure.

Box 4: Ausubel's clock auction with clinching

In this auction format, the incentives for strategic demand reduction in a multi-unit pay-as-bid auction are removed by allowing bidders to win blocks at the (lower) prices at which they are effectively no longer competed for, i.e. at the prices of the rounds in which the total demand of other bidders falls below available supply. To illustrate how this works, consider the following example:

Three bidders A, B and C bid for a total of ten blocks. Each bidder makes a bid for four blocks at the reserve price (say EUR 3 m per block).

Now consider bidder A. Without any demand from bidder A, total demand would eight blocks, i.e. there are two blocks for which A does not face any competition. These two blocks could be assigned at reserve. The same holds for B and C, so under the rules each bidder would obtain ('clinch') two blocks at reserve. As total demand exceeds the available supply, price goes up and a further round is called.

Suppose that bidders maintain their demand until the price reaches EUR 4 m, at which point bidder C reduces its demand to three blocks.

This means that total demand from B and C would now be only seven blocks. There would be three unsold blocks if A did not take part in the auction (i.e. one more than previously), so A would clinch a third at a price of EUR 4 m. The same holds for B. The situation for C remains unchanged.

Suppose that the price increases further to EUR 5 m, at which point B reduces its demand to three blocks, so total demand equals supply. The auction ends and:

- A receives two blocks at EUR 3 m, one block at EUR 4 m and one block at EUR 5 m;
- B receives two blocks at EUR 3 m and one block at EUR 4 m; and
- C receives two blocks at EUR 3 m and one block at EUR 5 m.

Note that there is no incentive to reduce demand early as doing so only benefits competing bidders, who can 'clinch' additional blocks at lower prices. Also note that the prices reflect opportunity cost:

- the blocks assigned to each bidder for EUR 3 m would remain unsold if the respective bidder did not take part in the auction;
- the price of the third block for A and B of EUR 4 m reflects the valuation that C puts on his fourth block; and
- the price of the fourth and third block for A and C respectively reflects the value that B puts on his fifth block.

[8] Ausubel, L (2004) "An Efficient Ascending-Bid Auction for Multiple Objects," *American Economic Review* 94(5)

> Whilst this should provide strong incentives to bid according to valuations, an obvious downside of the mechanism is that it leaves bidders with the maximum surplus that is compatible with supporting an efficient outcome. This format would therefore not meet the requirement that the process should generate realistic revenues.

In any case, the format may also raise discrimination concerns as it results in prices that favour larger bidders.[9]

4.1.2 Limitation of one lot per bidder in the first stage

Bidders limited to one block to prevent gaming leading to non-optimal asymmetries between winners

Under the spectrum cap of 120 MHz, bidders could in principle be permitted to acquire two large blocks in the first stage. However, we recommend limiting each bidder to at most one large block, as allowing bidders to acquire multiple larger blocks might affect competition in both stages and could lead to more asymmetric outcomes than would occur otherwise.

Specifically, suppose that there is no new entry and one of the three incumbent bidders had won two large blocks in the first stage. This bidder would no longer be able to take part in the second stage, where a bidder who has not yet acquired any spectrum would then compete against a bidder who has already secured 60 MHz. The bidder who needs to acquire its whole endowment in the form of smaller blocks would have a greater incentive to reduce demand early in the process to keep prices lower, as the savings from lower prices in the second stage apply to a larger number of blocks. At the same time, such a bidder would be exposed to the risk of having its prices driven up by the bidder who already has secured a large block in the first stage (albeit at a higher price per MHz) and would be less affected by an increase in the price of the small blocks in the second stage. This risk can only be avoided by settling for 60 MHz, and therefore an outcome in which two bidders win 120 MHz and the third one is left with 60 MHz may be more likely.

The prospect of potentially being able to buy spectrum at lower prices in the second stage (if the bidder is unsuccessful in the first stage) and the risk of being exposed to more aggressive

[9] In order to see this, consider the example but with modified demands where A and B bid for five blocks each and C bids for two blocks. This means that without the bid from A (and B, respectively), aggregate demand would be seven, so there are three blocks for which A (and B) do not face competition and thus either of them would clinch three blocks at reserve. By contrast, without C's bid there would be no excess supply so C does not obtain any lots at the lower price.

competition in the second stage from a bidder who has secured large blocks will affect bidding incentives in the first stage, and may lead to inefficiencies.

Allowing bidders to acquire two lots in the first stage would also be of concern in the case of new entry. In this case, if one bidder acquired two blocks in the first stage, then the remaining three bidders would be left to compete for 180 MHz in the second stage (with one of them having already secured a larger block). Such a scenario could lead to substantial and potentially undesirable asymmetries in spectrum holdings.

4.1.3 Number of lots offered in the first stage

Though it would be possible to offer fewer or more large blocks, doing so would have downsides

We have considered the alternatives of offering two and four large blocks respectively but find that there are downsides to either of those options.

- Offering only two large blocks may lead to a situation where in the second stage there is competition between two bidders who are seeking incremental spectrum and one or more bidders who need to satisfy all their bandwidth needs through small lots. This would have similarly distortive effects on competition as those discussed above for the case where a bidder might be allowed to obtain two lots in the first stage.
- With four blocks, competition would be limited even in the case of new entry, so price would very likely be determined by the reserve price set rather than related to the value of the spectrum for bidders. A more aggressive approach to setting reserve might be needed to ensure that revenues are reflective of valuations. This could be avoided by allowing bidders to acquire multiple blocks in the first stage. However, this could have similar distortive effects on competition as offering only two blocks and could promote a highly asymmetric outcome where two bidders obtain 120 MHz each in the first stage, with only 60 MHz being available for the remaining bidder(s).

4.1.4 Bid limit for bidders who do not participate in the first stage

Contingent bid limit to avoid the first stage being bypassed

With reserve prices of the 10 MHz blocks set at a lower level, and with potentially unsold lots from the first stage being broken down into smaller blocks and included in the second stage, bidders might have an incentive to refrain from bidding for larger spectrum blocks in the first stage. This would render the two-stage setup ineffective and could result in inefficient outcomes (e.g. if incumbents misjudge whether or not there is interest from entrants who might pursue a sizeable spectrum endowment).

In order to remove such incentives, we recommend that bidders in the second stage will be limited (e.g. though constraints on eligibility) to bid for at most five blocks (50 MHz), unless they have also placed (unsuccessful) bids for large blocks in the first stage. This means that a bidder could not obtain more than this amount of spectrum by bidding only for small blocks in the second stage. Bidding in the second stage would still be open for small scale entrants requiring 50 MHz or less. In the case of more than three bidders making bids for a large block in the first stage, bidders who have failed to secure a large block would not be limited by this constraint.

4.1.5 Sealed bid for the first stage

For the first stage, we suggest a sealed bid auction of three large blocks of 60 MHz each and price being determined by the lowest winning bid. Each bidder would be limited to winning at most one block.

A sealed bid is a simple process for the case where each bidder can win at most one block

Sealed bid processes are often seen to be inferior to open multi-round bidding because the latter support price discovery, allowing bidders to adjust their demand gradually as bidding converges towards a market-clearing outcome and bid back if they are unsuccessful, which gives them full control over their outcomes.

The advantages of open bidding process are clearly important in multi-band awards where the composition of the portfolio that a bidder pursues may vary in response to changes in relative prices and may be relevant also in single-band auctions where multiple lots are on offer and the resolution of excess demand requires that each bidder adjusts its own demand in response to price changes. However, they are of limited relevance in the case where each bidder can acquire at most one lot and the number of bidders is likely to be limited.

Where each bidder may acquire at most one lot, there may still be some benefits from open bidding process in terms of reducing common value uncertainty: being able to observe the points at which others drop demand may help each bidder to confirm or update its own valuation. However, with single lot demands, these benefits arise only where the number of bidders substantially exceeds the number of available lots and where the private value component of bidders' valuations is small. Only in these conditions can the behaviour of competitors provide information about the likely value of the lots on offer.

This is not the case in the first stage, and therefore a single round sealed bid is the simplest way of auctioning the large blocks.

Using a sealed bid with prices linked to winning bids mitigates the risk of strategic bidding to prevent entry, and may lead to prices being closer to valuations

Using a sealed bid instead of an open multi-round process also mitigates the risk that incumbent operators might bid strategically to block entry, which could discourage participation from entrants in the first place. With three lots available and the constraint that bidders may bid for at most one block in this stage, the need for a second round in an open bidding process would immediately reveal that there is participation from an entrant. This could lead to incumbent operators to continue to bid to deny spectrum to entrants. In addition to the intrinsic spectrum value, incumbents would be able to express the full value of avoiding the impact of new entry on downstream competition (which new entrants by definition do not have) and would therefore be more likely to win. Anticipating this, prospective new entrants might decide not to take part in the auction.[10]

With a sealed bid auction, a decision must be made about the pricing rule. One option is to make every winning bidder pay opportunity cost, which in this simple instance are determined by the highest losing bid or if all bids are winning, the reserve price. This creates the strongest incentives for bidders to reveal their valuation, as the amount they bid does not affect the price

[10] Note that in this case incumbents would not face a free-rider problem that would exist in cases where bidders can acquire multiple lots, where different incumbents may bear a larger share of the cost of blocking entry associated with acquiring the additional lots (at a higher price) that need to be won by incumbents to block entry. This is because in this case bidders may acquire at most one lot, and the only way in which incumbents can block entry is by acquiring exactly one lot each.

they pay if successful, but only the probability of winning.[11] This promotes efficiency when the auction is competitive, but in the absence of competition revenues are fully determined by the reserve price rather than related to the value of the spectrum to winners. In addition, a second price sealed bid auction would allow incumbent operators to bid the maximum amount they would be willing to pay to block entry, knowing that they will only be required to pay the minimum price that would be needed to outbid entrants – or reserve if there are no entrants.

By contrast, if each winner must pay the amount of its bid, bidders should not be expected to bid their valuation, as this would leave them with zero surplus in the case of winning. The surplus is larger the further one's bid is below valuation, but at the same time reducing the bid amount will reduce the chance of winning. Bidders will have to determine the amount they wish to bid by finding the optimal balance between the impact of lowering the bid amount (bid-shading) on their chances of winning and on the surplus they enjoy if they are successful. This requires bidders to make assumptions about their competitors, their valuations and their strategy. Also, with multi-unit auctions, different winners may end up paying very different prices.

Our proposed pricing rule sits somewhere between these extremes. If all bidders pay the amount of the lowest winning bid, the bid amount affects price only for the marginal bidder. However, this is enough to discourage very high bids aimed only at blocking entry, as such a strategy would only be successful if all incumbents make high bids, and in this case they are exposed to paying a price above valuations. The proposed pricing rule leaves a somewhat reduced incentive for bid shading compared with a strict pay-as-bid rule but would still leave scope for prices to be linked to the value of spectrum for bidders even in the absence of entry as long as bidders take into account the prospect of additional competition. It is therefore better suited to meeting the objective of raising a

[11] Indeed, with independent private values and each winner restricted to a single, indivisible item, bidding one's valuation is a dominant strategy.

realistic revenue. Also, there is no difference in the price paid across different winners.

4.1.6 Clock auction with exit bids for the second stage

For the second stage, we suggest offering (at least) twelve 10 MHz blocks in a clock auction with provisions for bidders to submit exit bids linked to their demand reductions. Bidders who have not placed a bid in the first stage would be subject to a limit of 50 MHz.

Benefits of an open multi-round process with multiple lots on offer

With multiple blocks on offer, there are clear advantages from permitting bidders to adjust their demand in response to price movements to reach a market clearing outcome. Using a sealed bid process would require that bidders submit multiple, mutually exclusive bids for different quantities at different prices. As they may win any of these bids, they must cede some control over outcomes. Bidders may wish to limit the number of possible outcomes by not placing bids on all quantities in which they could potentially be interested. As there will then be some 'missing bids' this can lead to inefficient outcomes and bidders run the risk of losing out altogether.

Clock-based mechanism for collection of bids provides a few options, but clock auction is simplest format and protects against aggregation risk

As noted above, the relatively large number of identical (frequency-generic) blocks on offer means that a clock-based mechanism for the collection of bids has advantages in terms of procedural efficiency.

We discard fully combinatorial formats such as the Combinatorial Clock Auction or the Combinatorial Multi-Round Auction as these formats are unduly complex given that we have spectrum in a single band and any remaining aggregation risks can be dealt with by other means such as permitting bidders to specify a minimum requirement or are already addressed in the simpler format.

This leaves several candidate formats, namely the SMRA-Clock hybrid format (as used in the recent Dutch multi-band auction), a clock auction with restrictions on reductions in demand to prevent unsold lots, or a simple clock auction with exit bids to mitigate the risk of unsold lots.

The first two formats expose bidders to the risk of winning an unwanted subset of the lots on which they have placed bids.

- In the Clock-SMRA-Hybrid the auctioneer designates provisional winning bids at the end of each round. These cannot be withdrawn but are committing. A bidder who has only some, but not all of its bids designated as provisional winning bids, therefore faces the risk of winning a subset of the lots it wishes to acquire if the auction ends before the bidder has been outbid. The bidder cannot exit cleanly if it can no longer afford to pursue the number of lots it would need to have a viable case or where the price per lot exceeds the average block value of the endowment the bidder can afford.

- In a clock auction where demand reductions are only accepted to the extent that they do not result in excess supply, there is similarly no opportunity for the bidder to exit cleanly. Rather, the bidder is committed to take up the lots that would remain unsold if its demand reduction were accepted in full.[12]

Although aggregation risks should to a large extent be addressed by offering large blocks in the first stage, there may be some residual concerns related to entry – either small scale entry or large-scale entry that resulted in one bidder who is interested in obtaining 60 MHz or more having lost out in the first stage. To remove these aggregation risks, under either format there would need to be a provision that releases bidders from their obligation to honour provisionally winning bids or a portion of their previous clock demand if they would as a result win fewer blocks than they have specified as their minimum requirement and prefer to leave the auction without any spectrum. Such provisions create the risk of lots remaining unsold unless there are further rules that stipulate how these released lots would be fed back into the auction process (see Box 5 for a simple example).

[12] This format was suggested as an alternative to the SMRA-Clock-Hybrid by the peer reviewer of the multiband auction design. It was not used because in our view the potential difference between submitted and processed bids creates uncertainty for bidders about their ability to switch demand between lot categories. Potential switching impediments are of course not of concern here as we are dealing with a single band.

Box 5: Aggregation risks, minimum requirements and unsold lots

Consider the case where there is new entry and bidders pursue the following targets:

- two bidders are targeting 100 MHz each;
- a third bidder is aiming for 80 MHz; and
- a fourth bidder wants to acquire 40 MHz.

This implies excess demand, so prices will go up and eventually reach the point at which the smaller bidder may wish to withdraw from the auction completely.

However, the bidder would always be a provisional winner on at least two blocks in the Clock-SMRA-Hybrid, and a submitted bid for zero blocks would not be processed in full in the clock auction with restrictions demand reduction. This exposes the bidder to the risk of winning 20 MHz, which may not be usable at all, or which have a much lower per-block value than the price at which the bidder cannot longer afford to go for four blocks.

Allowing the bidder to specify a minimum requirement of 40 MHz could address this issue, as provisional winning bids for fewer than four blocks could be withdrawn, or demand reductions would have to be accepted in full if the bidder would otherwise be winning fewer than four blocks. However, this then could create excess supply of 20 MHz, which could not be taken up by the remaining bidders (who may already have reduced demand) unless there are some provisions for re-activation of previously submitted bids or the allocation of unsold spectrum to remaining winners.

By contrast, a simple clock auction does not create any aggregation risks as bidders are free to reduce demand even if this results in excess supply when bidding ends. This translates of course immediately into a risk of unsold spectrum when demand reduction is not gradual, but bidders drop from a larger number of blocks straight to zero.

Exit bids to mitigate the risk of unsold lots

To address this concern, we recommend that bidders can make exit bids linked to their demand reductions, which specify the prices at which they would be prepared to buy the additional lots. Such bids will be constrained to be within the parameters at which the demand reduction is made, namely:

- the price offered for additional lots will need to be between the price of lots in the round when the demand reduction was made and the preceding round; and
- the number of additional lots that the bidder offers to acquire cannot exceed the demand reduction.

Where a bidder reduces demand by more than one block in a round, it can place multiple exit bids, subject to the prices specified for these exit bids not increasing in quantities (see Box 6 for an example)

Box 6: Exit bids

Suppose a bidder makes a clock bid for six blocks in round n at a clock price of 100. In round n+1, the clock price increases to 110 and the bidder reduces its demand to three blocks. The bidder can now place exit bids for one, two or three lots it would wish to acquire in addition to the three blocks at a price of 110 subject to the following constraints:

- All exit bids must be strictly lower than 110.
- No exit bid must be lower than 100.
- Exit bids for more lots must not specify a higher amount.

The following exit bids would for example be compliant with these rules:

- One block (in addition to three blocks at 110): 109
- Two blocks (in addition to three blocks at 110): 106
- Three blocks (in addition to three blocks at 110): 103

Note that these exit bids are optional, i.e. a bidder does not need to place exit bids at all or may only place one or two of the exit bids specified above.

Exit bids will remain valid throughout the auction unless they are explicitly withdrawn. Exit bids placed in a previous round can be withdrawn in any subsequent round if the bidder no longer wishes to acquire the additional lots.

If the last round ends with aggregate demand over all clock bids being less than the available supply, exit bids will be used to assign the lots that would otherwise remain unsold. Specifically, we suggest that the auctioneer establishes the combination of exit bids that minimises the number of unsold lots, taking account only of exit bids that specify lots in addition to whatever the bidder has already been assigned. If there is more than one combination of exit bids that meets this condition, the combination with the highest value will be chosen. Any further ties will be broken at random.

Box 7: Evaluation of exit bids

Consider that there are 12 lots on offer. Over three rounds three bidders A, B and C have placed the following bids, where C denotes a clock bid and E denotes an exit bid for additional lots at the price specified.

Round	Clock price	Bidder A	Bidder B	Bidder C
1	100	C: 6	C: 6	C: 6
2	110	C: 6	C: 3 E: +3 @ 100 E: +2 @ 102 E: +1 @ 105	C: 6
3	120	C: 5	C: 1 E: +2 @ 110	C: 4 E: +1 @ 115

Aggregate clock demand at the end of round 3 is ten, leaving excess supply of two lots. These could be used to satisfy Bidder B's or Bidder C's exit bids. The assignment that minimises the number of unsold lots is to Bidder B, so the result is:

- A wins five lots at 120
- B wins one lot at 120 and two further lots at 110
- C wins four lots at 120

Assume that Bidder B had also placed an exit bid for a single additional lot

Round	Clock price	Bidder A	Bidder B	Bidder C
…	…	…	…	…
3	120	C: 5	C: 1 E: +2 @ 110 E: +1 @ 111	C: 4 E: +1 @ 115

In this case, there are two alternative options for assigning the unsold lots that both assign all lots, i.e. giving two lots to Bidder B or giving one lot to Bidder B and Bidder C respectively. The latter generates a higher value so will be chosen. The result then is:

- A wins five lots at 120
- B wins one lot at 120 and one further lot at 111
- C wins four lots at 120 and one further lot at 115

Now assume that bidders had placed the following bids:

Round	Clock price	Bidder A	Bidder B	Bidder C
1	100	C: 6	C: 6	C: 6
2	110	C: 6	C: 3 E: +3 @ 100 E: +2 @ 102 E: +1 @ 105	C: 5 E: +1 @ 109
3	120	C: 6	C: 0 E: + 3 @ 110	C: 4 E: +1 @ 115

In this case, we award an additional lot to Bidder C at a price of 115. This still leaves one lot unassigned, but as Bidder C now already receives five lots, we can consider the exit bid of 109 from round 2, which has been expressed in addition to an endowment of five lots. Thus, the results are:

- A wins six lots at 120
- B wins zero lots
- C wins four lots at 120, one further lot at 115 and one further lot at 109

If Bidder C had not placed an exit bid in round two, its last exit bid would be considered, and one lot would remain unassigned. Although Bidder B has placed an exit bid for one additional lot at 105, this cannot be considered as it has been made for one lot ***in addition to receiving three lots.*** Thus, the results are:

- A wins six lots at 120
- B wins zero lots

- C wins four lots at 120 and one further lot at 115
- One lot remains unassigned

As these examples show, this approach does not eliminate the possibility of some lots remaining unassigned. We have considered the option of requiring rather than permitting bidders to place exit bids for every possible additional quantity when they reduce demand, possibly with a presumption that bidders would acquire the additional lots up to the quantity of their previous clock bid at the previous clock price. However, this would expose bidders again to aggregation risks as they may receive an unwanted subset of lots and would require some further measures such as the ability to specify a minimum requirement (which would then again lead to unsold lots unless there were exit bids from other bidders that could soak up the excess supply). In addition, it would expose bidders to some risk of exceeding their available budget if they had to take up some lots in addition to their final clock demand.

The concern about exposure of bidders to financial overcommitment is also the reason for permitting bidders to amend or withdraw exit bids, as they might no longer be able to afford additional lots if they approach their budget limit with their clock bids.

Information policy: limited transparency to protect against strategic bidding to block entry

In relation to the information policy, there may be a need to limit transparency to curb the scope for strategic bidding aimed at preventing new entry. Under the existing caps, excess demand of more than six blocks would clearly indicate that there is some entrant demand, which might then trigger strategic bidding to keep out new entry. This would suggest not disclosing aggregate demand.

At the same time, we acknowledge the benefits from disclosing information about aggregate demand to provide bidders with some guidance about likely market clearing outcomes.

We consider that an appropriate compromise might be to release aggregate demand information when this information does not unambiguously allow bidders to detect the presence of new entry. As mentioned above, we can only observe excess demand in excess of six blocks if there is entry. However, any individual incumbent may be able unambiguously to identify the presence of an entrant with lower levels of excess demand of it is itself bidding below the maximum number of blocks permitted under the cap. For example, suppose that an incumbent who has acquired one block in the first stage reduces its demand for in the second stage to four, in order to acquire a total of 100 MHz (rather than the maximum of 120 MHz allowed for by the cap). If the incumbent observes that excess demand is above four blocks, then this must be due to the presence of an entrant. Equally, if the incumbent reduced its demand to two blocks, then it would be able to identify excess demand if it observes excess demand above two blocks, and so on.

As our main concern in terms of neutralising strategies to block entry is that a bidder might bid for more blocks than it needs simply to prevent entry, we are not concerned about the possibility that an incumbent might be able to infer the presence of an entrant by reducing its demand below four blocks in the second stage. Thus, we propose to disclose the exact value of aggregate demand once excess demand has fallen to fewer than four blocks but withhold this information otherwise.

4.2 Assignment of specific frequencies

Second price combinatorial sealed bid for the assignment of specific frequencies

For the assignment of specific frequencies to winners of bandwidth we recommend a second price combinatorial sealed bid which allows bidders to express their preferences without concerns of having to pay more than is strictly necessary to resolve potential conflicts. Specifically:

- The auctioneer will determine all potential band plans in which each bidder obtains exactly the amount of spectrum it has won in the first phase and in which all bidders are assigned contiguous spectrum. Potentially unsold spectrum would typically be retained as a contiguous block and may be placed at either end of the band but could also be split and placed at the ends if this provided additional protection of adjacent uses.
- From these band plans, the possible assignment options for each bidder are determined. Bidders with more than one assignment option are then given the opportunity to express their maximum willingness to pay for each specific assignment option instead of being assigned other frequencies.
- The auctioneer establishes the feasible combination of bids (i.e. the combination of bids that yields one of the candidate band plans) with the highest value, with ties being broken at random.

- Each bidder wins the corresponding assignment and all bidders, individually and collectively, pay the opportunity cost of receiving the respective assignment which are established by looking at the value that could have been generated from other bidders if each individual bidder or subset of bidders had not expressed any preferences over assignments.

This means that where the preferences of bidders do not conflict, each bidder will obtain its preferred assignment without having to make any payment.

The following example illustrates this process.

Box 8: Assignment bids and prices

Suppose that two bidders (A and B) have won one large block and three small blocks each, i.e. 90 MHz, and the third bidder (C) has won the remaining blocks i.e. 120 MHz.

The candidate band plans are created by looking through all possible bidder orderings (ABC, ACB, BAC, BCA, CAB, CBA) and identifying the blocks that bidders would receive in each of these. Thus:

A and B could both receive the following blocks:

- 3450 – 3540 MHz, if placed at the bottom of the band;
- 3540 – 3630 MHz if placed in the middle with the respective other bidder placed at the bottom;
- 3570 – 3660 MHz if placed in the middle with bidder C placed at the bottom; and
- 3660 – 3750 MHz if placed at the top of the band.

C could receive the following blocks:

- 3450 – 3570 MHz if placed at the bottom of the band;
- 3540 – 3660 MHz if placed in the middle of the band; and
- 3630 – 3570 MHz if placed at the top of the band.

Now suppose that we receive the following bids:

- A bids €1,000 on the bottom position and €500 on the top position.
- B bids €2,000 on the bottom position, and €1,800 on each of the two middle positions
- C bids €1,000 on the top position.

The values of the candidate band plans are thus as follows:

- ABC: €3,800
- ACB: €1,000
- BAC: €3,000
- BCA: €2,500
- CAB: €0
- CBA: €2,300

Band plan ABC generates the highest value and will therefore become the winning band plan.

In terms of pricing, we establish the best outcome that could be obtained if each of the bidders did not have any preference. If A had placed bids of zero on all options, the corresponding values would be:

- ABC: €2,800
- ACB: €0
- BAC: €3,000
- BCA: €2,000
- CAB: €0
- CBA: €1,800

The best option would now be band plan BAC, and A's opportunity cost is given by the difference between the value of this band plan (€3,000) and the bids of other bidders in the winning band plan (€2,800), i.e. €200.

- ABC: €2,000
- ACB: €1,000
- BAC: €1,000
- BCA: €500
- CAB: €0
- CBA: €500

ABC would continue to be the best option. B's opportunity cost is given by the difference between the value of this band plan (€2,000) and the bids of other bidders in the winning band plan (€2,000), i.e. B's opportunity cost is zero.

Repeating this exercise for C shows that C's opportunity cost is equally zero.

If neither A nor B had expressed any preference, the optimal band plan would have been either ABC or BAC with a value of $1,000. The joint opportunity cost of A and B are therefore zero.

Without any preference from A and C, B would have obtained its preferred position (BAC or BCA) yielding 2000, so the joint opportunity costs of A and C are 200.

The joint opportunity costs of B and C are again zero.

This means that assignment prices are as follows:

A pays €200, which corresponds to the value lost from pushing B from its most preferred assignment to the second preference. B and C each pay nothing, as their preferences do not conflict with each other and B yields to A.

Compared with other options for the assignment of specific frequencies (such as permitting bidders to pick their preferred assignment from the set of available assignments that respect contiguity in an order determined by a criterion such as their amount of bandwidth they have acquired, or established through a separate bidding process), this approach supports a proper comparison of valuations.

It is well tried and tested and has been used for the recent multi-band auctions in the Netherlands. We see no reason to use a different approach for this award.

Annex A Auction rules

A.1 Overview of the auction process

The auction proceeds as follows:

- In a first stage, there will be a single round of bidding in which bidders place bids for one of three frequency generic blocks of 60 MHz each (bandwidth assignment phase – sealed bid stage).
- In a second stage, twelve or more frequency-generic blocks of 10 MHz will be offered in a clock auction where bidders can place exit bids when they reduce demand (bandwidth assignment phase – clock stage).
- These two stages will be followed by a single sealed bid round in which winners of spectrum can place bids for the different assignments of specific frequencies they might obtain (frequency assignment phase).

A.2 Bandwidth assignment phase – sealed bid stage

A.2.1 Available spectrum and reserve price

A total of 180 MHz is available in this stage, split into three frequency-generic blocks of 60 MHz each.

The reserve price for each of these blocks is set at [EUR XXXXX].

A.2.2 Bidding process

Bidding takes place in a single round scheduled by the auctioneer. The auctioneer will communicate the scheduled start and end time to bidders at the time specified in the [auction regulations], which will be at least [one hour] prior to the scheduled start time.

During the round, each bidder will be able to submit a single bid that specifies the maximum amount the bidder is willing to pay for a block.

Bids must not be lower than the reserve price and must be expressed in whole EUR and must be submitted in the manner specified by the auctioneer.

A.2.3 Determination of winners and prices

If three or fewer bids have been received, the auctioneer will designate all bids as winning bids.

If more than three bids have been received, the auctioneer will order the bids by their amount in descending orders, with ties being broken at random, and designate the three top ranked bids as winning bids.

The price payable by each winner is the amount of the lowest of the winning bids.

A.2.4 Information provided to bidders at the end of the stage

Winning bidders will be informed about the fact that they have been successful and the price they must pay.

All bidders will be informed about:

- the start time of the first round of the clock stage of the bandwidth assignment phase; and
- the number of blocks, if any, that have not been assigned in this stage.

A.3 Bandwidth assignment phase – clock stage

A.3.1 Available spectrum and reserve price

The spectrum available in this stage comprises 120 MHz plus 60 MHz for any block that has not been assigned in the previous stage (i.e. at least 120 MHz and at most 300 MHz).

The available spectrum will be offered in blocks of 10 MHz each (i.e. at least 12 and at most 30 blocks).

The reserve price for each of these blocks is set at [EUR XXXXX].

Prior to the start of the first bidding round, the auctioneer will inform all bidders about the number of blocks available in this stage (the '***available supply***').

A.3.2 Bidding process

Bidding takes place over one or more rounds which are scheduled at the auctioneer's discretion. The auction will communicate the scheduled start and end time of a bidding round to bidders at least [5 minutes] prior to the round start. A round will end at the scheduled end time unless any bidder has triggered an extension. If an extension has been triggered, the round will end after the extension period, i.e. [30 minutes] after the scheduled end time, or as soon as the last bidder who has triggered an extension has submitted its decision, whichever is earlier.

In each bidding round, the auction will set a price per block (the '***clock price***') and each bidder will specify the number of blocks it wishes to acquire at this price (the '***clock bid***').

Bidding ends after the first round in which the sum of the number of blocks specified in the clock bids of all bidders ('***aggregate demand***') is not greater than the available supply.

If the aggregate demand exceeds the available supply, the auctioneer will schedule a further round with a higher clock price. The amount by which the clock price increases is determined by the auctioneer at its discretion.

If the number of blocks in a bidder's clock bid is lower than the number of blocks in the previous round's clock bid, the bidder may place one or more '***exit bids***'.

An exit bid specifies a number of blocks that the bidder would wish to acquire in addition to the number of blocks in its clock bid, and the price that the bidder offers to pay for these additional blocks.

For the avoidance of doubt, exit bids do not count towards aggregate demand. They remain active unless the bidder withdraws them and may become winning bids at the end of the stage.

A.3.3 Submission of bid decisions

Submission of bid decisions is possible only during the scheduled round times or an extension triggered by the bidder.

A bid decision includes:

- the bidder's clock bid;
- exit bids, if any, placed in the round;
- the withdrawal of exit bids from previous rounds (if any)

If a bidder does not submit a bid decision before the scheduled end time of the round or within the period of any extension it may have triggered, it will be deemed to have submitted a clock bid for zero blocks without any new exit bids.

For the avoidance of doubt, a bidder who has not triggered an extension will not be able to submit a clock bid after the scheduled round end even if an extension period has been triggered by another bidder.

A.3.4 Extensions

Each bidder will be given [three] single-use extension rights, each of which will grant the bidder additional time (up to [30 minutes]) to place its bid if it has not done so by the scheduled end time of the round, provided that the bidder is still able to make further clock bids and has extension rights left.

Extensions will be triggered automatically.

A.3.5 Constraints on clock bids

In the first round the number of blocks in the clock bid is limited in the following manner:

- Bidders who have submitted a bid in the previous stage can only bid on as many blocks as they can acquire without breaching the spectrum cap in this band (120 MHz), considering any spectrum they have won in the previous stage, i.e. a bidder who has bid for and won a block in the previous stage can bid on at most six blocks and a bidder who has submitted a bid in the previous stage but has not been successful can bid on at most twelve blocks.
- Bidders who have not submitted a bid in the previous stage can bid on at most five blocks.

In any subsequent round, the number of blocks in the clock bid is limited to the number of blocks in the clock bid placed in the previous round.

A.3.6 Constraints on exit bids

Let Q_n be the number of blocks in the clock bid placed by a bidder in round n and P_n be the clock price in this round. Then:

- The placement of exit bids is possible only if $Q_n < Q_{n-1}$, i.e. if the bidder has reduced its clock demand relative to the previous round.
- In this case, the bidder may place exit bids for $1 \ldots (Q_n - Q_{n-1})$ blocks, i.e. up to as many blocks as the reduction in clock demand.
- The price p^i specified for each of the exit bids $i = 1 \ldots (Q_n - Q_{n-1})$ must be strictly lower than the current clock price and not be lower than the previous round clock price, i.e. $P_{n-1} \leq p^i < P_n$.
- If multiple exit bids are placed, then the price specified for an exit bid for a larger number of blocks must not exceed the exit bid for a smaller number of blocks, i.e. $i > j \Rightarrow p^i \leq p^j$.

A.3.7 Determination of winners and prices

Each bidder will be assigned the number of blocks specified in its clock bid in the last round of bidding.

If aggregate demand in the last round of bidding is less than the available supply, the auctioneer will use the unassigned blocks to satisfy incremental demand expressed in bidder's exit bids.

Specifically, the auctioneer will establish the combination of exit bids that minimises the number of unsold blocks, always respecting the additionality of exit bids (i.e. an exit bid will only be considered if the bidder has already been assigned the number of blocks specified in the clock bid of the round in which the exit bid has been placed).

If there is more than one combination of exit bids that achieves the same minimum number of unsold blocks, the auctioneer will choose the combination with the greatest value.

If there is more than one combination of exit bids that achieves the same minimum number of unsold blocks and the same greatest value, one will be chosen at random.

Bidders will then be assigned the additional blocks at the prices specified in their exit bids.

A.3.8 Information provided to bidders

Before the start of the first round, bidders will be informed about the round schedule, available supply, the clock price for this round (which will be the same as the reserve price) and the number of extensions they have remaining.

Subsequently, after the end of each round and before the start of the next round, bidders will be informed about the round schedule, available supply, the clock price for this round, their clock demand in the previous round and any non-withdrawn exit bids. If aggregate demand in the previous round did not exceed available supply by more than three blocks, bidders will also be informed about aggregate demand in the last round.

After the last bidding round, each bidder will be informed about the number of blocks it has won and the prices it will have to pay. All bidders will be informed about the start of the next stage.

A.4 Frequency assignment phase

A.4.1 Assignment options

Each bidder will be presented with all possible frequency assignments for that bidder ('***assignment options***'). The assignment options have been determined by the auctioneer based on the requirement that each assignment option must be part of at least one feasible band plan. A feasible band plan is an assignment of frequencies to bidders in which:

- each bidder receives exactly the bandwidth that corresponds to its winnings from the previous stages;
- all assignments are contiguous; and
- any unsold spectrum will be retained as a contiguous block [at the lower/upper end of the band].

A.4.2 Bidding process

Bidding takes place in a single round scheduled by the auctioneer. The auctioneer will communicate the scheduled start and end time to bidders at the time specified in the [auction regulations], which will be at least [one hour] prior to the scheduled start time.

During the round, each bidder will be able to submit a bid for each of its assignment options that specifies the maximum amount the bidder is willing to pay for receiving this assignment rather than any other assignment.

Bid amounts must be positive and in whole thousands of Euros.

Bid amounts for any assignment option for which a bidder does not specify a bid amount default to zero.

If a bidder fails to place bids before the scheduled end of the bidding round, then the bid amounts for all its assignment options will default to zero.

A.4.3 Determination of winners and prices

At the end of the round, the auctioneer will select the combination of bids that yields a feasible band plan and has the highest value.

If there is more than one combination of assignment bids that yield a feasible band plan value with the same highest value, one will be picked at random.

Bidders will be assigned the frequencies specified in assignment options of the winning bids.

The prices that bidders will be required to pay for the frequency option they are assigned are calculated jointly, using a second-pricing approach as follows.

First, the auctioneer establishes for each subset of bidders (including individual bidders) the ‘opportunity cost’ of assigning to the bidders in the subset the options they are assigned in the winning combination of bids, by calculating the difference between:

- the greatest sum of bids from other bidders that could be achieved in any of the feasible band plans for that band; and
- the sum of winning bids from other bidders for the options they are assigned in the winning combination of bids.

Then prices are calculated jointly by applying the following conditions, which yield a unique solution:

- the sum of individual prices for each proper subset of bidders[13] cannot exceed the sum of their winning bids;
- the sum of individual prices for each proper subset of bidders[14] must be at least the opportunity cost for the subset;
- the sum of individual prices must be the smallest possible subject to prices satisfying the conditions above; and
- the sum of the squared differences between each bidder’s individual price and its opportunity cost[15] must be the smallest possible across all prices that satisfy the conditions above.

[13] Including all possible sets containing only some of the bidders and the sets containing each single bidder.

[14] As above.

[15] I.e. the assignment opportunity cost for the subset including only this bidder.

A.4.4 Information provided to bidders

Before the start of the round, bidders will be informed about the round schedule and their respective assignment options.

After the round:

- each bidder will be informed about specific frequencies it will be assigned (i.e. its winning bid) and the price payable for this assignment (the '***assignment price***'); and
- all bidders will be informed about the frequencies assigned to each winner and the total price payable by that winner, which is the sum of the prices payable in the first two stages and the assignment price, if any.