Rev Ind Organ (2013) 42:161–190
DOI 10.1007/s11151-013-9376-x

Spectrum Auction Design

Peter Cramton

Published online: 1 February 2013

Abstract Spectrum auctions are used by governments to assign and price licenses for wireless communications. The standard approach is the simultaneous ascending auction, in which many related lots are auctioned simultaneously in a sequence of rounds. I analyze the strengths and weaknesses of the approach with examples from US spectrum auctions. I then present a variation-the combinatorial clock auction-which has been adopted by the UK and many other countries, which addresses many of the problems of the simultaneous ascending auction while building on its strengths. The combinatorial clock auction is a simple dynamic auction in which bidders bid on packages of lots. Most importantly, the auction allows alternative technologies that require the spectrum to be organized in different ways to compete in a technology-neutral auction. In addition, the pricing rule and information policy are carefully tailored to mitigate gaming behavior. An activity rule based on revealed preference promotes price and assignment discovery throughout the clock stage of the auction. Truthful bidding is encouraged, which simplifies bidding and improves efficiency. Experimental tests and early auctions confirm the advantages of the approach.

Keywords Auctions · Spectrum auctions · Market design · Package auction · Clock auction · Combinatorial auction

JEL Classification D44 · C78 · L96

1 Introduction

Fred Kahn recognized the important role of market design in improving how markets work. He believed that prices should be set in an open competitive process, rather than

P. Cramton (✉)
Department of Economics, University of Maryland, College Park, MD, USA
e-mail: pcramton@gmail.com

administratively. I had the pleasure of working with Fred on a project to evaluate the pricing rule in California's electricity market. We examined whether the electricity market should use uniform pricing or pay-as-bid pricing (Kahn et al. 2001). In this tribute to Fred Kahn, I also focus on auction design, but in the communications industry.

Spectrum auctions have been used by governments to assign and price spectrum for about 20 years. Over those years, the simultaneous ascending auction, first introduced in the US in 1994, has been the predominant method of auctioning spectrum. The auctions have proved far superior to the prior methods of beauty contests and lotteries (Cramton 1997; Milgrom 2004).

Despite the generally positive experience with the simultaneous ascending auction, several design issues have surfaced. Some were addressed with minor rule changes. For example, bidders' use of trailing digits to signal other bidders and support tacit collusion was eliminated by limiting bids to integer multiples of the minimum increment (Cramton and Schwartz 2002). However, many other design problems remain. In this paper, I identify these problems, and describe a new approach—the combinatorial clock auction—which is based primarily on the clock-proxy auction (Ausubel et al. 2006), which addresses the main limitations of the simultaneous ascending auction.

My focus here is on spectrum auction design, rather than spectrum policy more generally. Certainly, communications regulators face many other critical challenges, such as how best to free up new spectrum for auction (Cramton et al. 1998), or whether an auction is needed at all (Federal Communications Commission 2002). For some allocations, it is better to set aside the spectrum for common property use, as is done with unlicensed spectrum. In particular, for applications that do not create additional scarcity, the commons model is better than the auction model. There are many examples of this: garage door openers, car locks, and other device controllers, but the most important is Wi-Fi. These application require little bandwidth or power, and thus, do not make the spectrum scarce. Scarcity problems are mitigated by operator separation. In contrast, mobile phones require much greater power and bandwidth, creating spectrum scarcity, and hence an auction is needed to assign the scarce resource among the competing carriers.

Spectrum auctions to date have been long-term auctions in which the winner is granted a license for 10–25 years, with a strong expectation of renewal following expiration. One might think instead that a spot market for spectrum, much like a spot market for electricity, would be a more flexible and efficient instrument. Someday that will be true. But today's hardware, especially the handset, is not sufficiently flexible to accommodate a real-time spot market. Moreover, carriers must make large specific investments in their networks. These long-term investments are better supported with a long-term license for spectrum, which is a critical input. Over the next 20 years increasingly flexible hardware will be introduced. Eventually it will make sense to organize the spectrum market much like the electricity market. The basic element will be a real-time spot market that establishes the price of bandwidth at a particular time and location. But for now, long-term spectrum auctions are both necessary and desirable.

One of the greatest challenges for the regulator is keeping up with the rapid technological development of wireless communications. Indeed, one of the main reasons for switching from beauty contests, to lotteries, to auctions was that beauty contests and lotteries were too slow. Wireless communications plays an essential role in modern economies, both in developed and developing countries. Slowing the pace of wireless innovation and development has large costs to economic growth. For this reason, regulators must do whatever they can to promote a competitive wireless industry. Allocating sufficient spectrum in a timely manner is paramount.

The combinatorial clock auction described here helps facilitate the spectrum allocation process by enabling the auction to determine how the spectrum is organized, which is called the band plan. Prior methods required that the regulator determine a fixed band plan before the auction began. As a result, before each auction there is a long regulatory process, much like the beauty contests of before, but with the companies' lobbying for particular band plans, rather than for direct spectrum awards. This is the most time-consuming and error-prone element of the spectrum management process. Thus, the new approach promises not only to improve spectrum assignments, but also to improve the band plans within which the assignments fit, and to do so with less delay.

From an auction theory viewpoint, spectrum auctions are both challenging and interesting. The government is auctioning many items that are heterogeneous but similar. Often there are competing technologies as well as companies to provide a wide range of communication services. As a result, the setting has a complex structure of substitutes and complements. This is among the most difficult auction settings that are seen in practice.

The goal for the government should be efficiency, not revenue maximization. The government should focus on ensuring that those who can put the spectrum to its highest use get it. Focusing simply on revenue maximization is short-sighted. Many steps such as technical and service flexibility, and license aggregation and disaggregation, improve efficiency and thereby improve revenues. But short-run revenue maximization by creating monopolies, which would create the highest profits before spectrum fees, and therefore would sustain the largest fees, should be resisted. Indeed, competition, which ultimately will lead to greater innovation and better and cheaper services, will likely generate *greater* government revenues from a long-run perspective. The government can best accomplish this objective with an efficient auction that puts the spectrum to its best use.

The regulator may find it necessary to introduce spectrum caps or other preferences that favor new entrants so as to level the playing field between incumbents and new entrants (Cramton et al. 2011). Incumbents include in their private value the benefit of foreclosing competition, thus driving a wedge between social value and private value. In theory the regulator can correct this externality by favoring the new entrant, but in practice this has proven to be difficult. The FCC's experience with preferences for certain bidders-set-asides, bidding credits, and installment payments-has been disappointing, at least with respect to mobile broadband communication, which is where most of the value lies.

In contrast, a good example of successful intervention was Canada's use of set asides in its 2008 Advanced Wireless Services or AWS auction. As a result, multiple deep-pocketed new entrants came to the auction and bid up the price of not only the set-aside blocks, but also the non-set-aside blocks. The result was a much more competitive auction (with much higher revenues) and the introduction prospectively of some potentially strong new service providers. The approach effectively broke up regional market-splitting by the dominant incumbents. Another successful intervention was the FCC's use of a spectrum cap in early broadband PCS auctions. The cap limited the quantity of spectrum that any one carrier could hold in a geographic area, which addressed the potential market failure of limited competition in the market for wireless services.

Despite these successes in Canada and the US, the FCC's long and sometimes troubled history with bidder preferences is an important case study for other countries that are considering preferences for various parties. Installment payments proved especially problematic, as it led to speculative bidding, bankruptcy, and lengthy delay in the use of the spectrum.

In addition, the regulator must resist the temptation to force more "winners" than the market can efficiently support. Sometimes regulators fragment the spectrum and prohibit aggregation in the auction in an effort to create as many winners as possible. The India 3G spectrum auction may be one example. Aggregation up to a suitable competitive constraint is preferred.

1.1 Three Main Points

There are three main points that I wish to emphasize:

1.1.1 Enhance Substitution

First, in terms of the auction design, it is important to enhance the substitution across the items that are being sold. Enhanced substitution is accomplished through both the product design—what is auctioned—and the auction format. Often in the spectrum setting, the product design can be just as important as the auction design.

1.1.2 Encourage Price Discovery

Second, encouraging price discovery is extremely important. We need a dynamic process, because unlike some situations, in the case of spectrum auctions, there is much uncertainty about what things are worth. The bidders need to do a considerable amount of homework to develop a crude valuation model, and they need the benefit of some collective market insights, which can be revealed in a dynamic auction process, in order to improve their decision-making. The nice thing about a dynamic auction is that through this price process the bidders gradually have their sights focused on the most relevant part of the price space. Focusing bidder decisions on what is relevant is in my mind the biggest source of benefit from the dynamic process. This benefit is generally ignored by economists, because economists assume that the bidders fully understand their valuation models, when in practice bidders almost never have a completely specified valuation model. Yes, they do a lot of homework, but there is still much uncertainty about what spectrum lots are worth, and how they should be valuing the spectrum. The experience of the 3G spectrum auctions in Europe is a good example. The bids were based more on stock prices in a bubble situation, rather than on solid analysis about values.

1.1.3 Induce Truthful Bidding

The third feature that I wish to emphasize is the importance of inducing truthful bidding. This is accomplished in the auction design through an effective pricing rule and an activity rule. The two rules work together to encourage bidders truthfully to express preferences throughout the entire auction. This truthful expression of preferences is what leads to excellent price discovery and ultimately an efficient auction outcome.

A variety of different pricing rules are used in practice. The two most common are pay-as-bid pricing, where the bidder pays what it bid if it is a winner, and for a homogenous product, uniform pricing, where the bidder pays the market-clearing price. In the particular applications I am discussing here, there generally are not clearing prices, because of strong complementarities and heterogeneous items. As a result, a new kind of pricing rule is needed. The pricing rule that I will describe in detail later is a generalization of Vickrey's second-price rule.

I now give a brief overview of the combinatorial clock auction. The approach may appear complex. Some amount of complexity is required given the complex economic problem. Simpler versions, such as a simultaneous clock auction are possible in settings where all bidders intend to use the same technology. This may well be the case in developing countries that are conducting spectrum auctions for a particular use after the technology battles have been resolved from the experience in developed countries.

1.2 An Overview of the Combinatorial Clock Auction

The combinatorial clock auction is especially useful in situations where the regulator does not know which technology will make the best use of the spectrum. In such cases, the auction itself can determine the ultimate band plan that specifies how the spectrum is organized. Such an auction is said to be technology neutral, since it allows the competing technologies to determine the winning technologies, as well as carriers. A good example is an auction that accommodates both paired and unpaired technologies, such as LTE and WiMAX, respectively. A combinatorial auction is essential in this case, since the two uses require that the spectrum be organized in fundamentally different ways. The combinatorial clock auction is an especially simple, yet powerful, auction that lets competitive bids determine the ultimate band plan.

The combinatorial clock auction has features to address each of my three main points.

First, the product design simplifies the products whenever possible. For example, if bidders primarily care about the quantity of spectrum that they win in a geographic area, the auction should involve generic spectrum (if possible), and the bidders bid for a quantity of spectrum in each area. This simplifies the auction, enhances substitution, and improves competition. The specific assignment of spectrum lots is determined in the last stage of the auction, once the critical decisions have been made (who won how much in each area). This approach also allows a technology neutral auction, which lets the spectrum be organized in different ways for the different technologies. Each bidder indicates the quantity of spectrum and the type of use in its bids. In this case, the first stage of the auction determines not only who won how much in each area, but also the overall quantity of spectrum that is allocated for a particular use in the area.

Second, to encourage price discovery, the auction begins with a "clock" stage (i.e., each auction in the simultaneous auction process has a "clock" that shows the most recent bid price). Prices ascend for each product with excess demand until there is no excess demand for any product. This simple and familiar price discovery process works extremely well when bidders have incentives for truthful bidding. In the important case of substitutes, the clock stage determines an efficient assignment together with supporting competitive equilibrium prices. Moreover, complements are handled with no increase in the complexity of the clock process. Each bid in the clock stage is a package bid, so bidders can bid without fear of winning only some of what they need.

Bidders may find that they are unable to express preferences for all of the desirable packages in the clock stage, so following the clock stage is a supplementary round. Bidders can increase their bids on packages on which they bid in the clock stage and submit new bids on other packages. All of the clock stage bids and the supplementary round bids then are run through an optimizer to determine the value-maximizing assignment of the spectrum. This is the generic assignment.

Third, to induce truthful bidding, the auction uses Vickrey-nearest-core pricing. The efficient assignment is priced to minimize the bidders' total payments subject to competitive constraints (no group of bidders has offered the seller more). In practice, this often implies Vickrey pricing, ensuring truthful bidding. However, because of complements, there may be one or more competitive constraints that cause the payments to be greater than Vickrey payments for some bidders. In this event, the smallest deviations from Vickrey prices are used.

To induce truthful bidding throughout the clock stage, an activity rule based on revealed preference is used. This rule encourages bidders to bid in the straightforward manner of selecting the most profitable package in each round. Deviations from bidding on the most profitable package throughout the clock stage may impose a constraint on subsequent bids, either later in the clock stage or in the supplementary round.

Once the generic assignments are determined and priced, the specific assignment stage is run. Each winner submits top-up bids for each specific assignment that is better than the winner's worst specific assignment. The bids indicate the incremental value for each feasible alternative. Then an optimization program is run to determine the efficient specific assignment. Again the prices for the specific assignments are Vickrey-nearest-core prices. This concludes the auction.

This paper builds on well-developed literatures in auction theory and practice–especially combinatorial auctions and spectrum auctions. Much of the literature on combinatorial auctions is summarized in Cramton et al. (2006). The work of Ausubel et al. (2006), Ausubel and Milgrom (2006a,b), Day and Raghavan (2007), Day and Milgrom (2008), Day and Cramton (2012), Milgrom (2007, 2010), Parkes (2006) and Porter et al. (2003) is especially relevant. On spectrum auctions see Coase (1959) for the original proposal, Ausubel et al. (1997) on synergies, McMillan (1994), Cramton (1995, 1997, 2006), Klemperer (2004) and Milgrom (2004) on the performance of the simultaneous ascending auctions, and Brusco and Lopomo (2002) and Cramton and Schwartz (2002) on collusion. Kagel et al. (2010) experimentally compare the simultaneous ascending auction with a particular ascending combinatorial auction, which differs significantly from the one presented here.

I begin by describing some of the problems of the simultaneous ascending auction. Then I present the combinatorial clock auction, which retains the benefits, while addressing the weaknesses, of the simultaneous ascending auction. I emphasize two essential elements of the combinatorial clock auction: the pricing rule and the activity rule. Along the way, I summarize both experimental and field results with the combinatorial clock auction.

The combinatorial clock auction is of great practical interest. The design has been adopted for major spectrum auctions in many countries over three continents.

2 Simultaneous Ascending Auction

The workhorse for spectrum auctions since 1994 has been the simultaneous ascending auction, which is a simple generalization of the English auction to multiple items in which all items are auctioned simultaneously. Thus, unlike Sotheby's or Christie's auctions in which the items are auctioned in sequence, here all the items are auctioned at the same time.

The process is as follows: Each item or lot has a price that is associated with it. Over a sequence of rounds, bidders are asked to raise the bid on any of the lots that they find attractive, and the auctioneer identifies the provisional winner for each lot at the end of every round. The process continues until nobody is willing to bid any higher. This process was originally proposed by Preston McAfee, Paul Milgrom, and Robert Wilson for the FCC spectrum auctions. Since its introduction in July 1994, the design has undergone numerous enhancements, but the basic design has remained intact in its application worldwide for the vast majority of spectrum auctions.

An important element of the basic design is an activity rule to address the problem of bid sniping: waiting until the last minute to bid seriously (which reduces the amount of information that is generally available to other bidders and that could help them bid efficiently). The rule adopted by the FCC and used in all simultaneous ascending auctions to date is a quantity-based rule: The rule requires a bidder that wants to be a big bidder at the end of the auction must be a big bidder throughout the auction. Each bidder must maintain a level of activity, based on the quantity of spectrum for which the bidder is bidding, in order to continue with that level of eligibility later on. Thus, a bidder cannot play a snake-in-the-grass strategy where the bidder holds back and waits, and then pounces late in the auction, thereby winning without making its true intent known until the last instant.

As mentioned, the simultaneous ascending auction has been used for a long time. The FCC has conducted about 80 simultaneous ascending auctions, since it was introduced in July of 1994. The FCC has gotten good at conducting the auctions, and the design has worked reasonably well. Nonetheless, it is perhaps surprising how quickly inertia set in. The FCC was initially highly innovative in its initial choice of design, but since then the FCC has just made minor incremental improvements in response to obvious and sometimes severe problems with the original simultaneous ascending auction design.

Why has the design held up so well? The simultaneous ascending auction is an effective and simple price discovery process. It allows arbitrage across substitutes. It lets bidders piece together desirable packages of items. And, because of the dynamic process, it reduces the winner's curse by revealing common value information during the auction (Kagel and Levin 1986; Kagel et al. 1996).

But the design does, and has been observed to have, many weaknesses.

- As a result of the pricing rule, there is a strong incentive for large bidders to engage in demand reduction—to reduce the quantity demanded before the bidder's marginal value is reached in order to win at lower prices.
- Especially if there is weak competition, bidders have an incentive to engage in tacit collusion. The bidders employ various signaling strategies, where they attempt to work out deals through the language of the bids. The goal of the strategies is to allocate the items among the bidders at low prices.
- As a result of the activity rule, there are parking strategies. A bidder maintains eligibility by parking its eligibility in particular spots that the bidder is not interested in and then moves to its true interest later.
- The simultaneous ascending auction is typically done without package bids. The bidders are bidding on individual lots, and there is the possibility that a bidder will win some of the lots that it needs for its business plan, but not all. This exposure to winning less than what the bidder needs has adverse consequences on efficiency. Essentially, the bidder has to guess. Either the bidder bids for what it wants, or not. When there are complementarities, this is a tough decision for the bidder to make. The bidder may make the wrong decision and win something it actually does not want or fail to win something it does want.
- The lack of package bids also makes the simultaneous ascending auction vulnerable to hold up, which is basically a speculator stepping in and taking advantage of a bidder (Pagnozzi 2010). The speculator can make it clear to large bidders that it would be expensive to push him out of the way. As a result, the large bidders let the speculator win some desirable lots at low prices, and then the speculator turns around and sells them to the big players after the auction is over. That is the holdup strategy. It is easy to do and effective. Preventing resale would reduce this problem, but resale is desirable in a rapidly changing, dynamic industry.
- There is limited substitution across licenses, which is something I am going to emphasize. The reader might think that it would be easy to arbitrage across the lots, but in fact that is not the case. This is especially true in a large country like the United States, where the FCC splits up the frequency bands in different ways, geographically, and the bidders can only bid on individual lots, rather than packages.

As a result of all these factors, the bidding strategies are quite complicated.

2.1 The US AWS and 700 MHz Auctions

The difficulties in arbitraging across substitutes are best illustrated in the two most recent major auctions in the United States: Advanced Wireless Services (AWS) and 700 MHz.

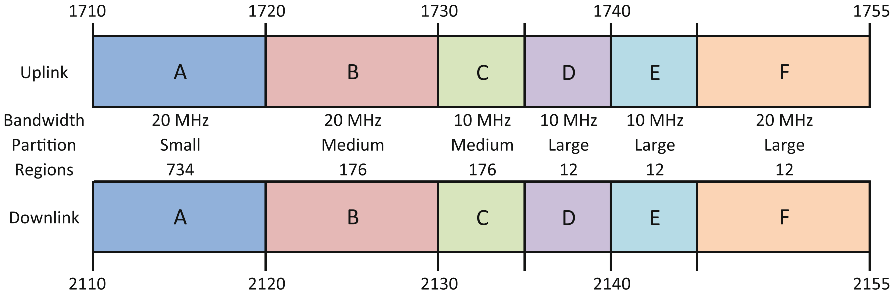

Fig. 1 The US AWS band plan: something for everyone

The AWS auction sold 90 MHz of spectrum in 161 rounds in 2006, and raised $14 billion. As in all of its auctions, the FCC began the process by settling on a specific band plan (the product design, as shown in Fig. 1), which effectively determined how the available bandwidth in each location was going to be split up into lots. Each lot is a particular frequency band covering a particular geographic area. In the case of the AWS auction, the FCC decided that six frequency blocks of paired spectrum (A–F) were to be auctioned. Three blocks were 20 MHz and three were 10 MHz. Because the US is so large, each frequency block was also partitioned geographically. And because the FCC was attempting to accommodate all types of bidders, the FCC partitioned the blocks in three different ways: for blocks D–F the country was split into 12 large regions; for blocks B and C the country was split into 176 medium-size regions; and for block A the country was split into 734 small regions. Remarkably, the different partitions do not form a hierarchy in the sense that a bidder cannot construct one of the medium-sized lots by aggregating a number of small lots. This inability to aggregate small into medium clearly limits substitution across blocks.

The underlying substitution problem was caused both by the product design—the use of specific blocks that followed three different geographic schemes—and the auction format. Figure 2 illustrates the severe problems that bidders had substituting across blocks in the AWS auction. It shows the price per 10 MHz of spectrum for each of the blocks at the end of critical days in the auction. Recall that there are six blocks, so there are six bars (A through F) at the end of each day. The 20 MHz bars are twice as wide as the 10 MHz bars, so the area of the bar corresponds to revenues at the time indicated. Finally, different shades of gray represent different bidders, so the reader can see who the provisional winners are at the various times in the auction. The two largest bidders are T-Mobile (diagonal stripes) and Verizon (horizontal stripes).

If there was perfect arbitrage across blocks, then the length of the bars would be the same at each time in the auction, which would indicate equal prices across blocks. Over time, the prices would move higher, but the prices would tend to move together across the blocks, as bidders would arbitrage to the cheaper lots per MHz of spectrum.

What happened in the AWS auction is extremely far from that, as is illustrated by the end of day five. At this point, the F block has already reached its final price. The A block is less than one twentieth the price of the F block. If the A block is roughly equivalent to the F block, why wouldn't Verizon, say, switch to the much cheaper A block, instead of placing bids twenty times higher on the F block? The reason has to

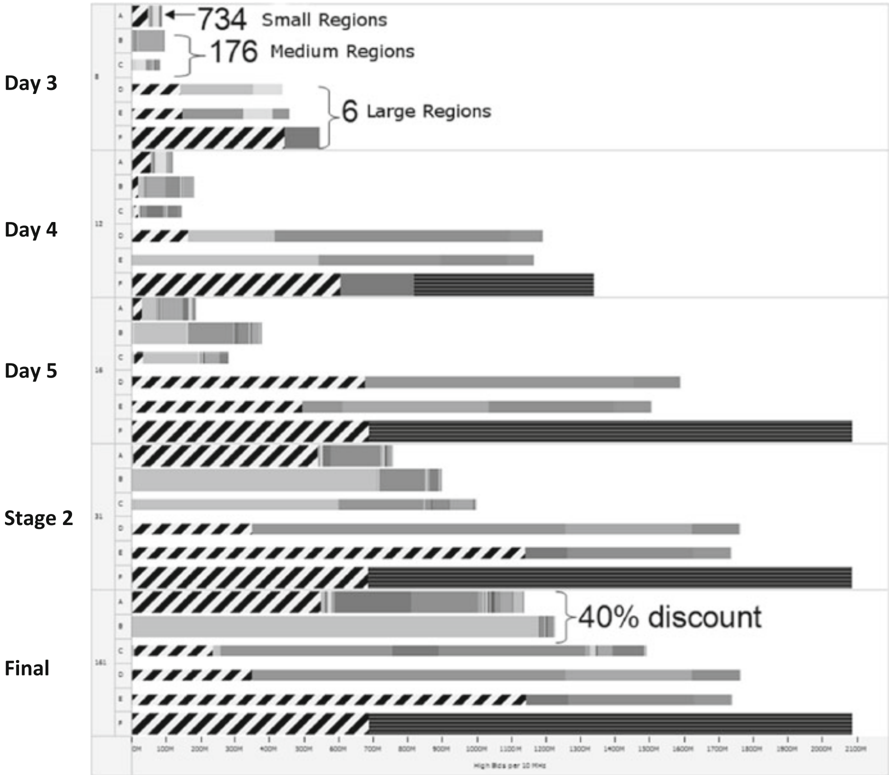

Fig. 2 The absence of arbitrage across substitutes in the US AWS auction

do with substitution difficulties. When Verizon is bumped off a large F block license, it is easy for Verizon to substitute down to the A block, submitting say the 100 or so bids on the A lots that roughly cover the corresponding F lot. The problem is that once Verizon has shifted down it would be nearly impossible to shift back up to F. The reason is that in subsequent rounds Verizon would only be bumped from some of the corresponding A block lots. Verizon would have to withdraw from many A lots in order to return to F, exposing itself to large withdrawal penalties. In addition on block A, Verizon would be vulnerable to various hold-up strategies, where speculators could pick important holes in a synergistic aggregation of lots.

Since substituting down from large (F, E, D) to small (C, B, A) lots is easier than substituting up, the auction essentially proceeded in a sequential fashion. First, the bidders competed for the large-lot blocks (F, E, D), then they competed for the medium-lot blocks (C and B), and finally the competition fell to the small-lot block (A). This explains the sequential, rather than simultaneous price process across blocks. See Bulow et al. (2009) for more on this auction.

The next major auction in the US was the 700 MHz auction in 2008. The band plan for the paired spectrum is shown above. The FCC did the same thing in this auction. Specific blocks were auctioned, using three different partitions of the US. Again the

Table 1 Band plan and final prices ($/MHz-pop) for paired spectrum in 700 MHz auction

Block	A	B	C
Bandwidth (MHz)	12	12	22
Type	Paired	Paired	Paired
Partition	176	734	12
Price	$1.16	$2.68	$0.76

different partitions did not form a hierarchy. The final prices per MHz-pop (bandwidth times population) range from $0.76 for the C block to $2.68 for the B block, as shown in Table 1. These final prices differ by over a factor of three. We see again that the substitution across blocks is far from perfect. Interestingly, this time it is the small-lot block B that sold for a high price, and the large-lot block C that sold for a low price—which is just the opposite of what happened in the AWS auction.

Although the C-block had an open access provision, which required that the carrier not discriminate against either devices or applications, the terms of open access were sufficiently watered down that I doubt it had much of an impact on the C-block price. In my view, the price difference was because competing bidders thought that competing on the C-block against Verizon (or perhaps AT&T and Verizon) was sufficiently hopeless that it would be better to focus on the A and B blocks. See Cramton et al. (2007) for more on the competitive issues in this auction.

The conclusion from the 20 years of history of spectrum auctions that have used the simultaneous ascending auction is that it works reasonably well in simple situations with a single geographic scheme. However in more complex settings, the approach leads to complex bidding strategies that complicate the auction and may undermine the efficient assignment of spectrum.

3 A Better Way: The Combinatorial Clock Auction

Fortunately, there is a better way. All that is needed is a number of complementary enhancements that ultimately simplify the bidding process, improve its efficiency, and greatly expand its power.

First, much of the game playing, such as tacit collusion and other bid signaling, can be eliminated with a shift to anonymous bids. In a combinatorial clock auction the round-by-round revelation of information is limited to aggregate measures of competition. Limiting round reports to prices and excess demand for each product gives the bidders the information needed to form expectations about likely prices and to resolve common value uncertainty, yet such reports do not allow the signaling strategies that support tacit collusion. Moreover, the streamlined report simplifies bidder decision-making and keeps the bidders focused on what is most relevant: the relationship between prices and aggregate demand.

In most instances, the spectrum lots that cover the same region in adjacent frequencies are nearly perfect substitutes. The bidder primarily cares about the quantity of spectrum in MHz that it has in the region, rather than the exact frequency location. Moreover, to minimize interference problems and maximize data speeds bidders prefer contiguous spectrum within any region. In this setting, it makes sense in the initial stage to auction generic spectrum. The initial stage determines the quantity of contiguous spectrum won in each region. The spectrum is treated as if it were a homogenous good within each region. This is an enormous simplification of what is being sold. The idea is to treat each MHz of spectrum within a geographic region and a particular frequency band as perfect substitutes. The auction first resolves the main question of how much spectrum in each region each winner gets and at what price, before the auction turns to the more subtle and less important question of the exact frequencies.

Of course, there are some auctions where the differences across frequencies are too great to allow this simplified treatment—for example, because of major interference differences by frequency, as the result of incumbents with a right to stay in the particular band. In such cases, the specific spectrum lots can be auctioned from the start; but in most cases, it is desirable to auction generic spectrum first and then determine the specific assignment in a second stage.

The specific assignment stage is simplified, since it only involves winners of the generic stage. The number of specific assignments typically is limited to the number of ways that the winners can be ordered. Thus, if there are m winners there are m! different specific assignments. For example, an auction with four winners in a particular region would have $4! = 4 \times 3 \times 2 = 24$ different possible specific assignments. If we assume separability across regions, each of the four bidders would only need to express preferences among at most 24 different specific assignments. This number is reduced further if we assume that the bidder only cares about its own specific assignment and not the location of the other winners, as is commonly the case. Then for example with four winners of equal size, each winner would only need to express three preferences: the incremental value from the bidder's first, second, and third-best specific assignment compared with its fourth-best.

The use of generic lots, wherever possible, simplifies the auction, enhances substitution, and improves price discovery. Despite these advantages the FCC has chosen in each of its roughly 80 auctions to sell specific lots. This is a common mistake in auction design. Interestingly, even in countries that recognized the advantages of selling generic lots, such as the German 3G auction, the generic lots were auctioned using a method for specific lots; that is, in the German 3G auction, even though the lots were perfect substitutes, the bidders bid on specific lots.

The first innovation is an improved product design, based on generic spectrum in each region, which accommodates multiple types of use.

Once generic lots are adopted the next innovation becomes easier to see: the adoption of simple and powerful techniques that are well-suited to auctioning many divisible goods.

The second innovation is the use of a simultaneous clock auction. This is a simplification of the simultaneous ascending auction. Each product has its own "clock," indicates its current price. Because of generic lots, each product may consist of multiple lots. In each round, the bidder is asked to indicate for each product the quantity of lots desired at the current price. At the end of the round, the auctioneer adds up the individual bids and reports the demand for each product. The price is then increased on any product with excess demand. This process is repeated until there is no excess demand for any product.

The two critical differences between the clock auction and the simultaneous ascending auction are: (1) the bidder only answers demand queries, stating the quantities desired at the announced prices; and (2) there is no need to determine provisionally winning bidders at the end of every round.

The third innovation is more subtle, but extremely powerful. One can interpret the demand vector reported by each bidder in each round as a package bid. The bidder is saying, “At these prices, I want this package of lots.” Taking this interpretation seriously yields a combinatorial auction (or package auction) without the need for any optimization. This allows bidders to express complementarities within a simple price discovery process.

Lawrence Ausubel and I have been conducting exactly this sort of package auction since 2001 for electricity and gas products in France, Germany, Belgium, Denmark, Spain, Hungary, and the United States (Ausubel and Cramton 2004). Thus far, we have conducted over 70 high-stakes auctions with this format for assets worth over $10 billion. We also used the approach in a spectrum auction in Trinidad and Tobago in 2005. The approach has been highly successful.

The clock auction may end with some products in excess supply, as a result of complementarities among lots. In addition, since the clock process follows a single price path and only includes a limited number of price points, it is desirable to allow the bidder to specify additional bids in a supplementary round following the clock stage. The purpose is to let the bidder express preferences for additional packages that were missed by the clock process. In addition, the bidder can improve its bids on packages that were already bid on in the clock stage.

Once the clock bids and the supplementary bids are collected, an optimization is run to determine the value-maximizing generic assignment and prices. This two-step process of a clock auction followed by supplementary bids, which I call a combinatorial clock auction, was proposed by Lawrence Ausubel, Paul Milgrom, and me for spectrum auctions at an FCC auction conference in 2003 (Ausubel et al. 2006). We proposed the same approach for spectrum auctions in the UK in 2006, as well as for airport takeoff-and-landing rights in 2003. Meanwhile, Porter et al. (2003) demonstrate in the experimental lab the high efficiency of a closely related approach.

Two critical elements of a successful combinatorial clock auction are the pricing rule and the activity rule. I will discuss both at length. These two important rules work together to ensure that the bids are an accurate expression of bidder preferences throughout the entire auction. The high efficiency of the combinatorial clock auction derives mainly from incentives for nearly truthful bidding. A pricing rule that is based on second pricing encourages truthful bidding; and the activity rule based on revealed preference ensures that these incentives for truthful bidding are felt throughout the clock stage.

4 UK Spectrum Auctions

The need for a technology-neutral auction is commonplace in today’s world of rapidly developing communications technologies and applications. Although the regulator can typically identify the viable candidate technologies based on early development, the regulator cannot decide how available spectrum should be split among the technologies without a market test. Examples are numerous, and several will be discussed here.

Ofcom, which is the independent regulator and competition authority for the UK communications industries, was the first to recognize and act on this need for a technology-neutral auction. In spring 2006, Lawrence Ausubel and I proposed to Ofcom a version of the combinatorial clock auction. Since June 2006, I have been working with Ofcom in developing, testing, and implementing the design for a number of its auctions. Two such auctions—the 10–40 GHz auction and the L-band auction—have occurred already. Both went well, and provided a useful field test for the economically much larger 800 MHz and 2.6 GHz auctions. Several countries in addition to the UK have since adopted the design for 4G auctions involving one or many spectrum bands.

Ofcom has three main goals for the auction design: The auction should be technology neutral, which allows alternative viable technologies to compete for the spectrum on an equal basis. The auction should accommodate flexible spectrum usage rights, which permits the user to decide how the spectrum would be used, subject to minimizing interference externalities with neighbors. And the auction should promote an efficient assignment of the spectrum, which puts the spectrum to its best use.

Simplicity and transparency are important secondary objectives. On simplicity, Ofcom recognized that satisfying the main objectives posed serious challenges, which could not be addressed with an auction design that is too simple. Moreover, simplicity has to be assessed in recognition of the complexity of bidder participation. For example, the simultaneous ascending auction has simple rules, but incredibly complicated bidding strategies. In contrast, the combinatorial clock auction has more complex rules, but the rules have been carefully constructed to make participation especially easy. For the most part, the bidder can focus simply on determining its true preferences for packages that it can realistically expect to win. In a combinatorial clock auction it is the auctioneer that needs to do the complex optimization, whereas the bidders can focus on their values for realistic packages.

Revenue maximization was explicitly excluded as an objective. Nonetheless, an efficient auction necessarily will generate substantial revenues. Indeed, my advice to countries is to focus on efficiency. A focus on revenues is short-sighted. In my view, the government is better off finding as much spectrum as possible and then auctioning it so as to put the spectrum to its best use. This approach creates a competitive and innovative market for communications, which has substantial positive spillovers to the rest of the economy. Under this approach, long-term revenues likely will far exceed those that would come from the maximization of short-term auction revenues.

I now explain the details of two essential rules in the combinatorial clock auction: the pricing rule and the activity rule. The rules may appear complex, but the complexity actually simplifies the bidding strategies, which makes it easier for bidders to participate in the auction.

5 The Pricing Rule: Vickrey-Nearest-Core Pricing

Prices are determined at two points in the auction: after the clock stage, including the supplementary bids, to determine the base prices for the winners in the value-maximizing generic assignment; and after the assignment stage to determine the additional payments for specific assignments.

The pricing rule plays a major role in fostering incentives for truthful bidding. Pay-as-bid pricing in a clock auction or a simultaneous ascending auction creates incentives for demand reduction (Ausubel and Cramton 2002). Large bidders shade their bids, in recognition of their impact on price. This bid shading both complicates bidding strategies and also leads to inefficiency.

In contrast, Vickrey pricing provides ideal incentives for truthful bidding. Each winner pays the social opportunity cost of its winnings, and therefore receives 100 percent of the incremental value created by its bids. This aligns the maximization of social value with the maximization of individual value for every bidder. Thus, with private values, it is a dominant strategy to bid truthfully. See Ausubel (2004, 2006) for an analysis in a clock auction.

Unfortunately, as a result of complements, it may be that the Vickrey prices are too low in the sense that one or more bidders would be upset with the assignment and prices paid, claiming that they had offered the seller more. For example, suppose there are two items, A and B, and three bidders. Bidder 1 bids $4 for A, bidder 2 bids $4 for B, and bidder 3 bids $4 for A and B. The Vickrey outcome is for 1 to win A, 2 to win B, and each winner pays $0. Bidder 3 in this case has a legitimate complaint, "Why are you giving the goods to bidder 1 and 2, when I am offering $4 for the pair?" The basic problem is that with complements, the Vickrey outcome may not be in the core. Some coalition of bidders may have offered the seller more than the sum of the Vickrey prices. (The core is defined as a set of payments that support the efficient assignment in the sense that there does not exist an alternative coalition of bidders that has collectively offered the seller more.) This point has been emphasized in Ausubel and Milgrom (2002).

The solution is to increase one or more prices to assure that the prices are in the core. In order to provide the best incentives that are consistent with core pricing, the auctioneer finds the lowest payments that are in the core; that is, such that no alternative coalition of bidders has offered the seller more than the winning coalition is paying.

If we are auctioning a single item, then this is the second-price auction. Suppose the highest bidder bids $100 and the second-highest bidder bids $90. The item is awarded to the highest bidder, who pays the second-highest price of $90—which is the social opportunity cost of awarding the good to the highest bidder. Alternatively, we can think of assigning the item to maximize value, so we assign it to the highest bidder, and then we find the smallest payment that satisfies the core constraints. In this case, the second-highest bidder would be upset if the highest bidder paid less than $90, so $90 is the bidder-optimal core price. When the items are substitutes, then the bidder-optimal core point is unique and identical to the Vickrey prices.

The payment-minimizing core prices, or bidder-optimal core prices, typically are not unique when the Vickrey prices are outside the core. Thus, it will be important to have a method of selecting a unique bidder-optimal core point when there are many such points. One sensible approach that has been adopted in each of the recent Ofcom auctions for both the base prices and the assignment prices is to select the payment minimizing core prices that are closest to the Vickrey prices. This is what I call Vickrey-nearest-core pricing. Since the set of core prices is convex—a polytope formed from the intersection of half-spaces—and the Vickrey prices are always unique, there is a unique vector of core prices that is closest in Euclidean distance to the Vickrey prices. Not only are the prices unique, but since they are bidder-optimal-core prices, they also maximize the incentive for truthful bidding among all prices that satisfy core constraints (Day and Milgrom 2008).

The approach then is to take all of the bids from the clock stage and the supplementary bids, determine the value maximizing assignment, and then determine the payment-minimizing core prices that are closest to the Vickrey prices. It is my experience that bidders are quite happy with this approach: They like the idea of minimizing payments, and they recognize the importance of making sure that the prices are sufficiently high that no coalition of bidders has offered the seller more. Prices are as small as possible subject to the competitive constraints.

Calculating the winning assignments and prices involves solving a sequence of standard optimization problems. The basic problem is the winner determination problem, which is a well-understood set-packing problem. The main winner determination problem is to find the value maximizing assignment. To guarantee uniqueness, there is a sequence of lexicographic objectives, such as: (1) maximize total value; (2) minimize concentration; (3) maximize quantity sold; and (4) randomize. First the auctioneer maximizes total value. Then a constraint that the value equals this maximum value is added, and concentration is minimized. Then another constraint that concentration equals this minimum level is added, and the quantity sold is maximized. Finally, the constraint that the quantity sold equals this maximum quantity is added and an objective based on random values for each bid is maximized. This guarantees uniqueness.

Calculating the prices is a bit more involved. First, the Vickrey prices are determined by solving a sequence of winner determination problems, essentially removing one winner at a time to determine each winner's social opportunity cost of winning its package. Then the bidder-optimal core prices are determined by using a clever constraint generation method that was proposed in Day and Raghavan (2007). Having found the Vickrey prices, another optimization is solved to find the most violated core constraint. If there is none, then the process is finished, since the Vickrey prices are in the core. Otherwise, this most-violated constraint is added, and the optimization is resolved, again finding the most violated core constraint. It is added to the optimization, and again the optimization is resolved. This is continued until there is no violated core constraint, and then the process is finished.

The reason that that Day–Raghavan approach is a highly efficient method of solution is because in practice there are typically only a handful of violated core constraints; thus, the procedure stops after just a few steps. In contrast the number of core constraints grows exponentially with the number of bidders and that makes including all of the core constraints explicitly an inefficient method of solving the problem, both in time and memory.

As mentioned, the tie-breaking rule for prices is important, since typically ties will arise along the southwest face of the core polytope. Finding the prices that are closest to the Vickrey prices involves solving a simple quadratic optimization. This yields a unique set of prices. Uniqueness is important. It means that there is no discretion in identifying the outcome, either in the assignment or the prices.

An example will help illustrate all of these concepts: Suppose that there are five bidders—1, 2, 3, 4, 5—bidding for two lots: A and B. The following bids are submitted:

$$\begin{aligned}
\mathrm{b}_1\{\mathrm{A}\} &= 28\\
\mathrm{b}_2\{\mathrm{B}\} &= 20\\
\mathrm{b}_3\{\mathrm{AB}\} &= 32\\
\mathrm{b}_4\{\mathrm{A}\} &= 14\\
\mathrm{b}_5\{\mathrm{B}\} &= 12
\end{aligned}$$

Bidders 1 and 4 are interested in A, bidders 2 and 5 are interested in B, and bidder 3 is interested in the package A and B.

Determining the value maximizing assignment is easy in this example. Bidder 1 gets A and bidder 2 gets B, which generates 48 in total value. No other assignment yields as much. Vickrey prices are also easy to calculate. If we remove bidder 1, then the best assignment gives A to bidder 4 and B to bidder 2, resulting in 34, which is better than the alternative of awarding both A and B to bidder 3, which yields 32. Thus, the social opportunity cost of bidder 1's winning A is $34 - 20 = 14$ (the value lost from bidder 4 in this case). Similarly, if we remove bidder 2, then the efficient assignment is for bidder 1 to get A and bidder 5 to get B, resulting in 40. Then the social opportunity cost of bidder 2's winning B is $40 - 28 = 12$ (the value lost from bidder 5). Hence, the Vickrey outcome is for bidder 1 to pay 14 for A and for bidder 2 to pay 12 for B. Total revenues are $14 + 12 = 26$. Notice that bidder 3 has cause for complaint, since bidder 3 offered 32 for both A and B.

Now consider the core for this example. The core is represented in the payment space of the winning bidders—in this case the payments of bidders 1 and 2. Each bid defines a half-space of the payment space:

- Bidder 1's bid of 28 for A implies 1 cannot pay more than 28 for A.
- Bidder 2's bid of 20 for B implies 2 cannot pay more than 20 for B.
- Bidder 3's bid of 32 for AB implies that the sum of the payments for A and B must be at least 32.
- Bidder 4's bid of 14 for A implies that bidder 1 must pay at least 14 for A.
- Bidder 5's bid of 12 for B implies that bidder 2 must pay at least 12 for B.

The core is the intersection of these half-spaces as shown in Fig. 3.

This example is quite general. First, in contrast to some economic settings, in an auction the core is always nonempty. The reason is that the core always includes the efficient outcome; all of the constraints are southwest of the efficient point, since the efficient point maximizes total value. Second, the core is always a convex polytope, since it is the intersection of numerous half-spaces. Third, complementarities, like bidder 3's bid for AB, are the source of the constraints that are neither vertical nor horizontal. These are the constraints that can put the Vickrey prices outside the core.

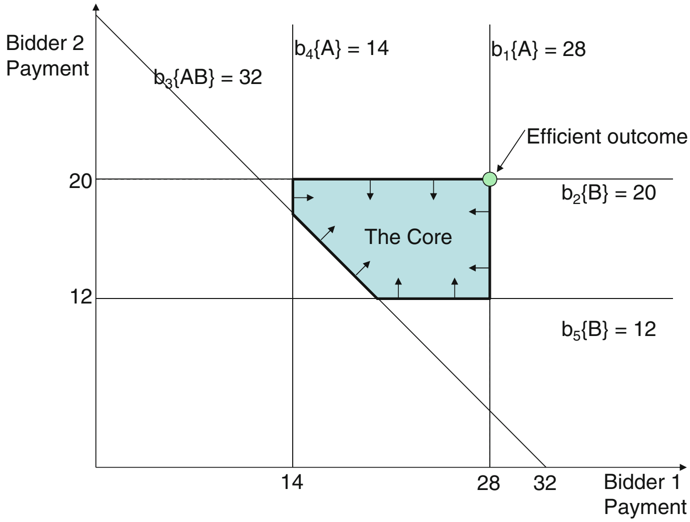

Fig. 3 The core

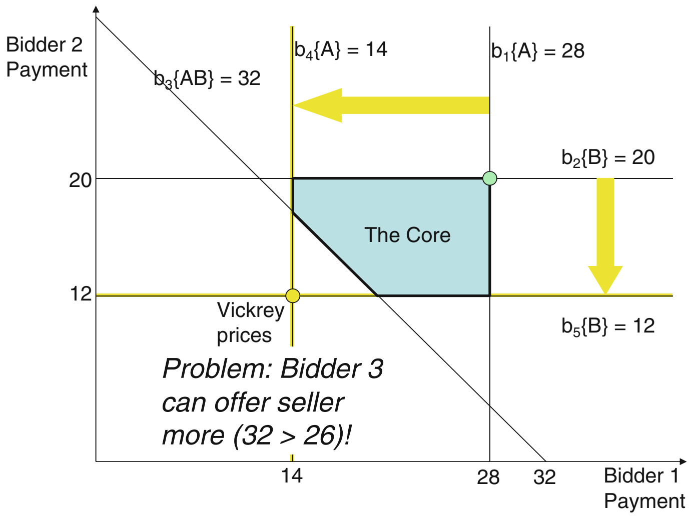

Fig. 4 Vickrey prices: how much can each winner's bid be reduced holding others fixed?

Without complementarities, all of the constraints will be vertical and horizontal lines, and there will be a unique extreme point to the southwest: the Vickrey prices.

The graphical representation of the core is also a useful way to see the Vickrey prices. Vickrey is asking how much can each winner unilaterally reduce its bids and still remain a winner. As shown in Fig. 4, bidder 1 can reduce its bid to 14 before bidder 1 is displaced by bidder 4 as a winner. Similarly, bidder 2 can reduce its bid to 12 before being displaced by bidder 5. Thus, the Vickrey prices are 14 and 12. The

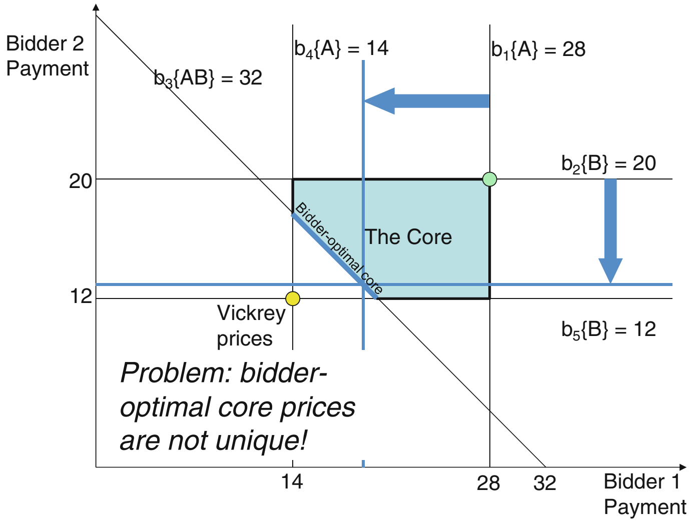

Fig. 5 Bidder-optimal core prices: jointly reduce winning bids as much as possible

problem is that these payments sum to 26, which violates the core constraint coming from bidder 3's bid of 32 for AB.

Bidder-optimal core prices can also be thought of as maximal reductions in the bids of winners, but rather than reducing the bids of each winner one at a time, we jointly reduce all the winning bids, as shown in Fig. 5, until the southwest face of the core is reached. As can be seen, this does not result in a unique core point, since the particular point on the southwest face depends on the rate at which each winner's bids are reduced. The bidder-optimal core points consist of the entire southwest face of the core. If the southwest face is a unique point, then it is the Vickrey prices; if the southwest face is not unique, then the face is a core constraint involving complementarities, and the Vickrey prices lie outside the core.

Nonetheless, there is always a unique bidder-optimal core point that is closest to the Vickrey prices. This is seen in Fig. 6, as the bidder-optimal core point that forms a 90° angle with the line that passes through the Vickrey prices. This point minimizes the Euclidean distance from the Vickrey prices.

Vickrey-nearest-core pricing was adopted in each of the UK spectrum auctions and has been adopted in several other auctions. Erdil and Klemperer (2010) argue that marginal incentives for truthful bidding may be improved by using a reference point other than the Vickrey prices for selecting among bidder-optimal core prices. In particular, they recommend a reference point that is independent of the winners' bids. See also Ausubel and Baranov (2010) for additional analysis.

Bidder-optimal core pricing has several advantages. First, it minimizes the bidders' incentive to distort bids in a Pareto sense: There is no other pricing rule that provides strictly better incentives for truthful bidding. Bidder-optimal core pricing implies Vickrey pricing, whenever Vickrey is in the core. For example, when lots are substitutes, Vickrey is in the core, and the bidders have an incentive to bid truthfully.

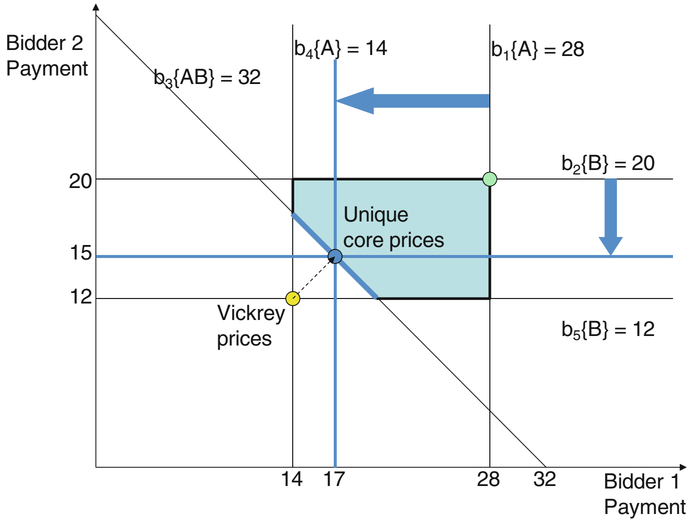

Fig. 6 Core point closest to Vickrey prices

Since the prices are in the core, it avoids the problem of Vickrey prices' being too low as a result of complements.

6 The Activity Rule: Revealed Preference

Good price discovery is essential in realizing the benefits of a dynamic auction. Good price discovery stems from providing incentives for the bidders to make truthful bids throughout the auction process. The pricing rule discussed in the prior section is an essential element, but one also has to be concerned about what is seen on eBay every day: bid sniping—jumping in at the last instance in an auction and thereby holding information back. Absent an activity rule, bidders will have an incentive to hold back to conceal information. The activity rule is intended to promote truthful bidding throughout the auction process.

Nearly all high-stake auctions, such as the FCC spectrum auctions, have an activity rule. The FCC uses a quantity-based rule. This rule has worked reasonably well in the FCC's simultaneous ascending auctions; but in a combinatorial clock auction with Vickrey-nearest-core pricing, we need a more complex rule: one that is based on revealed preference (Ausubel et al. 2006). Such a rule is effective at getting bidders to bid in a straightforward way throughout the clock stage, selecting the most profitable package given the current prices.

The traditional activity rule in both simultaneous ascending auctions and clock auctions has been a quantity-based rule: To be a large winner at the end of the auction, the bidder must be a large bidder throughout the auction. In particular, each lot corresponds to a particular quantity of spectrum, measured in either MHz-pop or in "eligibility points". The bidder starts with an initial eligibility based on the bidder's initial deposit. To maintain this level of eligibility in future rounds, the bidder needs to bid on a sufficiently large quantity of spectrum in the current round, where "sufficiently large" is stated as some percentage, typically between 80 and 100 % of the bidder's current eligibility. If the bidder bids on a smaller quantity, the bidder's eligibility is reduced in future rounds. This quantity-based rule has worked reasonably well, although as mentioned, it does create an incentive for parking eligibility on lots that a bidder is not truly interested in, especially if the eligibility points are not a good measure of relative value across lots. (The FCC's MHz-pop measure is especially poor with small lots. Spectrum in New York City is much scarcer than spectrum in Montana. As a result, spectrum values are much higher in New York City on a per MHz-pop basis. Despite this fact, which has been demonstrated in many dozens of spectrum auctions, the FCC still continues to use MHz-pop as the quantity measure in its auctions, which exacerbates parking and other problems that are associated with the activity rule.)

In many clock auctions, an activity requirement of 100 % is used, which means that the bidder cannot increase the size of the package, as measured in eligibility points, as prices rise. For the case of a single product, this means that the bidder must bid in a manner that is consistent with a downward-sloping demand curve.

In a combinatorial clock auction, one can use this quantity-based rule in the clock stage, but one also needs to specify how the rule limits bids in the supplementary round. This linkage between the clock bids and the supplementary bids is of critical importance, for otherwise the bidder could snipe: submit all of its bids in the supplementary round.

Ofcom proposed the following, which I call the *eligibility point rule*: During the clock stage the bidder cannot increase the package size. Moreover, whenever the bidder reduces the package size, the bid on all larger packages is capped by the prices at the time of the reduction. For example, if during the clock stage a bidder drops from a package of size 10–6 at prices p, then for all packages q of size 7–10, the supplementary bid cannot be more than $p \cdot q$.

The eligibility point rule, which Ofcom used in its first two combinatorial clock auctions, has the advantage of simplicity. For each package there is at most a single linear constraint on the supplementary bid. However, it has a potentially serious problem: The straightforward strategy of bidding on the most profitable package in the clock stage is a poor strategy. A bidder following such a strategy would find that its supplementary bids would be sharply constrained, well below true values. To avoid this problem, the bidder must instead bid in the clock stage to maximize package size, subject to a nonnegative profit constraint. That is, the bidder throughout the clock stage bids on the largest package that is still profitable.

Lawrence Ausubel, Paul Milgrom, and I proposed an alternative activity rule that is based on revealed preference for the combinatorial clock auction (Ausubel et al. 2006). Revealed preference is the underlying motivation for all activity rules. The intent is to require the bidder to bid in a way throughout the auction that is consistent with the bidder's true preferences. Since we do not know the bidder's true preferences, the best we can hope for is for the bidder to bid in a manner that is consistent with its revealed preferences. In the simplest case of a single-product clock auction, this is equivalent to monotonicity in quantity, just like the eligibility point rule, but when we have multiple products the two rules differ in important ways.

Table 2 An example with two bidders and two identical lots

	Marginal value		Average value	
	Bidder A	Bidder B	Bidder A	Bidder B
1 Lot	16	8	16	8
2 Lots	2	2	9	5

For the combinatorial clock auction, the *revealed preference rule* is as follows (see Harsha et al. (2010) for a stronger statement): During the clock stage, a bidder can only shift to packages that have become relatively cheaper; that is, at time $t' > t$, package $q_{t'}$ has become relatively cheaper than q_t:

$$q_{t'} \cdot (p_{t'} - p_t) \leq q_t \cdot (p_{t'} - p_t). \tag{P}$$

Moreover, every supplementary bid $b(q)$ must be less profitable than the revised package bid $b(q_t)$ at t:

$$b(q) \leq b(q_t) + (q - q_t) \cdot p_t. \tag{S}$$

Each clock bid for package q_t , as improved in the supplementary round, imposes a cap on the supplementary bid for package q.

An important advantage of the revealed preference rule is that a bidder that follows the straightforward strategy of bidding on its most profitable package in the clock stage would retain the flexibility to bid its full value on all packages in the supplementary round.

To illustrate the implications of the two activity rules, consider the following example with two bidders and two identical lots (one product) in a setting of substitutes: The bidders' preferences are given in Table 2, which indicates the marginal and average value for 1 lot and 2 lots.

Since the lots are substitutes, both bidders want to bid their true values in the supplementary round. However, consider what happens in the clock stage in response to the two different rules.

With the revealed preference rule, each bidder has an incentive to bid on its most profitable package in each round. Thus, the bidding simply moves up each bidder's marginal value (demand) curve. When the clock price reaches 2, both bidders drop from a package of size 2–1, and excess demand drops to zero. The clock stage ends at the competitive equilibrium price of 2 and the efficient assignment. Indeed, there is no need for any supplementary bids in this case. Bidder A can enter supplementary bids of 16 and 18, and bidder B can enter supplementary bids of 8 and 10, but these supplementary bids will not change the outcome in any way. Each bidder wins one lot and pays 2 (the Vickrey price). The supplementary round is unnecessary. The clock stage, by revealing the bidders marginal value information, up to the point of no excess demand, has revealed all that is needed to determine and price the efficient assignment. This is a general result with substitutes.

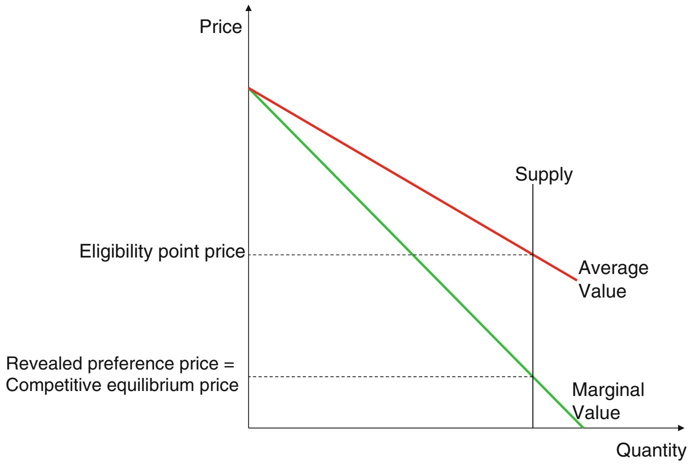

Fig. 7 Downward sloping aggregate demand implies average value > marginal value

With the eligibility point rule, bidders are forced to distort their bidding away from the straightforward strategy of profit maximization. In order to preserve the ability to bid full values in the supplementary round, the bidders instead bid on the largest package that is still profitable. This entails moving up the average value curve, since when the average value is exceeded a package is no longer profitable. Thus, when the clock price reaches 5, bidder B's average value for 2 is reached, and the bidder drops its demand to 1. Then when the clock price reaches 8, bidder B's average value for 1 is reached and bidder B drops out. At this point there is no excess demand, so the clock stage ends with bidder A demanding 2, bidder B demanding zero, and the clock price at 8. In the supplementary bid round, the bidders again submit their true preferences, and the optimization determines that each bidder should win one lot and should pay 2. The supplementary round was required to determine the efficient assignment and price the goods. Notice that the clock stage did little but mislead the bidders into thinking that bidder A would win all the items at a high price.

The reader might think that I somehow rigged this example to make the eligibility point rule look bad. This is not the case. Whenever lots are substitutes, the same features will be observed. With revealed preference, the clock stage will converge to the competitive equilibrium, revealing the efficient outcome and supporting prices; whereas with the eligibility point rule, the clock stage ends with an assignment that is excessively concentrated and prices that are too high. This result follows from the simple fact that average value exceeds marginal value, whenever aggregate demand is downward sloping, as shown in Fig. 7. Having participated in many dozens of major spectrum auctions, I can confirm that this is indeed the typical case.

What is essential for price discovery is the revelation of the marginal value information. This helps bidders make the marginal tradeoffs that are of greatest relevance in figuring out what the outcome should be. This is why I believe that the eligibility point rule is a poor choice.

To further test the two activity rules, I conducted numerous simulations that used realistic demand scenarios with significant complementarities from both technological

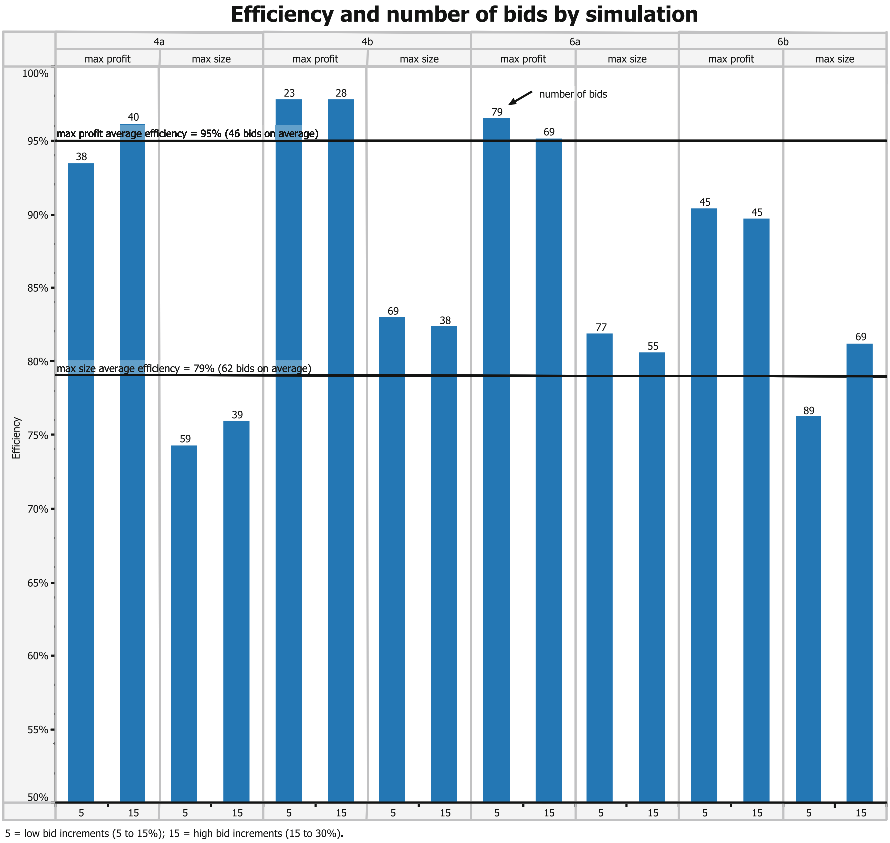

Fig. 8 Revealed preference rule yields higher efficiency and fewer bids in the clock stage

and minimum scale constraints. I assumed that the bidders bid on the most profitable package with revealed preference (max profit) and bid on the largest profitable package with the eligibility point rule (max size). The results are summarized in Fig. 8. It is clear that the revealed preference rule achieves substantially higher efficiency in many fewer rounds.

As a final test of the two activity rules, as well as other elements of the auction design, I conducted a series of full-scale tests in the experimental lab. For the tests, the Ofcom auction platform was used and indeed Ofcom staff served as the auctioneer. The subjects in the test were PhD students, who had taken an advanced course in game theory and auction theory, and had prior participation in combinatorial clock auction experiments. I chose such an experienced and expert subject pool, since in the actual spectrum auctions bidders often hire experts and devote substantial time and money to understand the strategic implications of the rules.

Each subject participated in several auctions over a two-week period. In each auction, the subject was given a bidding tool, which calculated the subject's value for each package consistent with the bidder's business plan. The scenarios as represented by the various bidding tools were chosen to be realistic. The valuation models included both substitutes and complements. Complements came from minimum scale constraints as well as technological requirements. A training session was held before the auctions to explain the details of the combinatorial clock auction, including the two different activity rules. All subjects participated in both activity rule treatments. Each subject was paid an amount that was based on her experimental profits. The average subject payment was $420.

The experiments confirmed that the eligibility point rule caused a major deviation from straightforward bidding in the clock stage. Bidders quickly realized the need to bid on the largest profitable package. This undermined price discovery; but, given the private value setting and simple valuation models, the poor performance of the clock stage was largely corrected by the supplementary bids and the optimization that followed. There were some instances of inefficiency when bidders deviated from bidding on the largest profitable package and then found that they were unable to bid full values in the supplementary stage.

In contrast, with the revealed preference rule, bidders almost always followed the straightforward strategy of bidding on the most profitable package. In the supplementary round, bidders typically bid full value and were not constrained by the revealed preference rule. As a result, efficiency was nearly 100 %. More recently, Bichler et al. (2011) conducted experimental tests of the combinatorial clock auction that achieved lower levels of efficiency (between 89 and 96 %), because bidders tended to submit too few bids. For the combinatorial clock auction to perform well, it is important for bidders to submit all relevant bids. The experiments that I conducted did not suffer from "too few bids" because the bidders had a bidding tool that made it easy for them to submit bids on all of the relevant packages. In my experience with real bidders, the bidders have had such tools, and indeed the development of such tools is a big task in the preparations for the auctions.

One issue that was discovered in the lab was the complexity of the revealed preference rule. The few bidders who deviated from bidding on the most profitable package in each round of the clock stage found that they were unable to bid full value in the supplementary round as a result of the revealed preference constraint. These bidders had to make adjustments to bids to satisfy the revealed preference constraints, but it was difficult for them to figure out what changes to make. The challenge for the bidder is to figure out how best to adjust numerous bids in order simultaneously to satisfy many constraints (one per round). Even the brightest PhD students found this to be a daunting task without some computational help.

One solution to the complexity problem is for the auction system to provide the bidder with some help. For example, the bidder could provide the system with its desired bids. The auction system then would indicate a summary of the bids that currently violate revealed preference constraints and suggest an alternative set of bids that satisfies all constraints and is closest (in Euclidean distance) to the desired bids. This is exactly the information that the subjects in the lab were looking for in the few instances of deviations from straightforward bidding. In the lab, the deviations were minor, and the bids would have been easily adjusted with the help of a smart auction system.

In addition to complexity, the revealed preference rule may at times be too strong. Bidders' values may change over the course of the auction—for example, as the result of common value uncertainty, or the bidder may have budget constraints. Thus, there are good reasons to simplify and somewhat weaken the revealed preference rule.

The approach adopted for the 4G auctions in several countries, such as the UK, Canada, and Australia, uses a revealed preference rule that only imposes a subset of the revealed preference constraints. Importantly all bids in the supplementary round must satisfy revealed preference with respect to the final clock round. Ausubel and Cramton (2011) provide further details.

The idea behind the rule is that it may be unnecessary to include all of the revealed preference constraints to get the bidders to adopt straightforward bidding. Since the incentive for bid sniping is not too strong, even the possibility of a revealed-preference constraint may be sufficient to induce the desired behavior. People put coins in parking meters in order to avoid the possibility of a parking ticket. We can hope that a simplified revealed preference rule will have the same effect in the combinatorial clock auction.

Specifically, all supplementary bids $b(q)$ are capped by the revealed preference constraint with respect to the final clock package q_f:

$$b(q) \leq b(q_f) + (q - q_f) \cdot p_f. \tag{S'}$$

One of the desirable features of the rule is that the final package in the clock stage plays an especially important role in limiting bids. Thus, any distortion from profit maximization in the final clock package is especially costly to the bidder. Of course, the bidder never knows which clock round will be the last, so there is always some incentive to bid consistent with profit maximization. Moreover, as excess demand falls, the probability that the current round will be the last tends to increase, strengthening the incentive for straightforward bidding throughout the clock stage.

A second desirable feature of the simplified revealed preference rule is that it makes the final clock assignment and prices much more meaningful, limiting the impact of the supplementary round and motivating aggressive bidding in the clock stage.

Proposition 1 *If the clock stage ends with no excess supply, then the final assignment is the same as the clock assignment. The supplementary round cannot alter the clock assignment.*

Proof (S') implies that the marginal value of awarding q_f to the bidder rather than q is at least the value of the lots at prices p_f:

$$b(q_f) - b(q) \geq (q - q_f) \cdot p_f.$$

It follows that any change in the final assignment cannot result in a higher total value.

Proposition 2 *If the clock stage ends with excess supply, then a winner can guarantee that it wins its clock assignment by raising its bid on its clock package by the value of the unsold lots at the final clock prices.*

Proof (S') implies that the marginal value of awarding q_f to the bidder rather than q is at least the value of the lots at prices p_f:

$$b(q_f) - b(q) \geq (q - q_f) \cdot p_f.$$

It follows that any change in the final assignment can result in a marginal value of at most $q_u \cdot p_f$, where q_u is the vector of unsold lots in the clock assignment. Thus if a winner increases its bid on q_f by the amount $q_u \cdot p_f$, the final assignment must award the bidder q_f.

The propositions demonstrate that the clock stage provides excellent price and assignment discovery whenever the final clock assignment has little or no excess supply. Clock winners know how to guarantee their clock assignment. It is not necessary to increase bids to full value. A clock winner only needs to raise its bid on the final clock package by the value of the unsold lots at the final clock prices. Potential clock losers have an incentive to bid until no profitable packages remain, since losing in the clock stage may prevent winning any package.

In the case of substitutes, the clock stage performs perfectly, if we assume a continuous clock. The pricing and activity rules provide incentives for straightforward bidding. The clock stage yields a competitive equilibrium with an efficient assignment and supporting prices. Supplementary bids are not needed to improve the assignment. The final assignment is the same as the clock assignment. The optimization simply reduces prices to reflect opportunity costs.

In the general case, the incentives for straightforward bidding are strong, but not perfect. Complements may push the Vickrey prices outside of the core, creating a threshold problem for some bidders. Nonetheless, if the clock stage ends without excess supply, then the final assignment is the clock assignment. Supplement-ary bids may affect prices, but not the assignment. If there is excess supply at the end of the clock stage, the winners can guarantee winning at least the clock assignment with a limited raise.

7 Conclusion

The combinatorial clock auction is a large advance over the simultaneous ascending auction. It eliminates the exposure problem; it eliminates most gaming behavior; it enhances substitution; and it encourages competition. The combinatorial clock auction enables a technology-neutral auction. This should be especially valuable in settings where the regulator does not know in advance how the spectrum should be organized. The auction, through the competitive bids, determines how the spectrum is organized, rather than the regulator. In an environment where the regulator has little information about what technology or use is best, letting the auction resolve such matters can greatly expand the realized value of the scarce spectrum resource.

A further advantage of the combinatorial clock auction is that it is readily customized for a variety of settings. Typically, a communications regulator will have a sequence of auctions over many years, as new spectrum gradually is made available. The combinatorial clock auction can be adapted to the unique characteristics of any particular auction. Adopting a consistent and flexible auction platform reduces transaction costs for the government and, more importantly, the bidders.

The auction design also enhances competition. The process is highly transparent and encourages price discovery. There is enhanced substitution both through the product design and the auction format. Bidder participation costs are reduced.

As in any market design problem, an important task for the regulator is to identify and mitigate potential market failures. In this setting and many others, the most important potential failure is market power. This is especially an issue in settings where there already is a highly concentrated communications market. Spectrum is an essential input for any new entrant. The approach here allows the regulator to address this potential market failure, as well as others, with a variety of instruments, such as spectrum caps, set asides, or bidding credits. The instruments must be used with care, or else they may do more harm than good.

One of the greatest harms is delaying the allocation and award of spectrum. Avoiding economic loss from delay should be a main priority of the regulator. Incumbents often will argue that spectrum awards should be put off. Such arguments may simply be a far less costly means of impeding competition than outbidding an entrant in an auction.

Fortunately, the use of a state-of-the-art auction design, such as the combinatorial clock auction and its variants, does not cause delay. These auctions can be designed and implemented, even by developing countries, in short order, provided that the country is using successful techniques that have been adopted elsewhere. The bottleneck is regulatory procedures, not auction design and implementation. Providers of auction services can readily meet deadlines of a few months, if necessary.

The combinatorial clock auction can be applied in many other industries. For example, the approach was proposed and tested for the auctioning of takeoff-and-landing slots at New York City's airports. The approach is well-suited for any setting in which there are many interrelated items, some of which are substitutes and some of which are complements.

More broadly, the approach described here is an example of using auction design to harness the power of markets. The approach leads to improved pricing of a scarce resource and improved decision making—both short term and long term. Innovation is fostered from the better pricing and assignment of the scarce resource.

Acknowledgments I thank my collaborators, Larry Ausubel, Robert Day, and Paul Milgrom for helpful discussions, as well as Nathaniel Higgins, Evan Kwerel, Thayer Morrill, Peter Pitsch, and Andrew Stocking. I thank the staff at Ofcom, especially Graham Louth, Director of Spectrum Markets, whose leadership and intellectual contribution were essential to the successful implementation of the combinatorial clock auction. I am grateful to the National Science Foundation and the Rockefeller Foundation for funding.

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