

A Review of the L-band Auction

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In May 2008, Ofcom's L-band auction concluded. This was Ofcom's second combinatorial clock auction. The auction used an innovative format intended to encourage an efficient assignment of the 17 lots. Eight bidders competed for the lots. In sharp contrast to the first combinatorial clock auction, the 10-40 GHz auction, in which each of the ten bidders won spectrum, in the L-band auction there was a single winner—Qualcomm won all the lots. This note briefly reviews the auction.

After examining the auction data, I find compelling evidence that the auction was fully efficient. The bidders had both an opportunity and an incentive to express their true preferences. The only potential source for inefficiency is the threshold problem. It is conceivable, in principle, that the seven losing bidders could have been hesitant to bid full values, because each anticipated that it would win its desired package at a lower price by bidding less. I think this possibility is extremely unlikely. The clock stage did not give the losing bidders any information to suggest either that they would win or that bidding less than full value would reduce the price paid. Rather, the information was decidedly negative in respect of both the scope for winning and scope for reducing prices, which gives the bidder the strongest incentive to bid full value to increase the probability of winning. Thus, I expect that the losing bids did reflect true preferences, in which case the outcome was fully efficient.

The bids of several bidders were non-monotonic—a larger amount was bid for a package that was a strict subset of another package. In some cases this appeared to be the result of constraints from the activity rule, suggesting that some bidders failed to bid on the largest profitable package during the clock rounds. These binding activity rule constraints may have reduced auction revenues, but I doubt efficiency was compromised.

I begin with a description of the L-band setting. Then I present the final outcome. I analyze the clock stage, and the bidding behavior of each bidder in both the clock stage and the supplementary round.

1 Summary of environment and auction format

The L-band is a band of radio spectrum which likely will be used for mobile multimedia broadcasting. The band consists of 16 nationwide 1.7 MHz lots (LA to LP), and one 12.5 MHz lot (LQ). Each lot differs in coverage as a result of border restrictions as well as other factors. Some technologies require the spectrum be organized in blocks of three contiguous lots. Other technologies require that all the lots be contiguous. And still others do not require contiguous lots. Finally, some technologies work better with a lower density of high power antennas, and others work better with a higher density of low power antennas. Low power and high power use are incompatible on adjacent lots and nearly adjacent lots, requiring two lots of separation as a guard-band.

¹ This note was funded by Ofcom. The views expressed are my own.

Given these substantial differences across lots, the 17 lots were sold as specific lots, rather than grouped together and treated as nearly identical lots. Moreover, given the substantial differences in technologies with respect to power usage and the need for contiguous lots, a package auction was essential.

The combinatorial clock auction enables the auction to determine the successful technology, rather than the regulator. Since at every point in the auction bidders are bidding on mutually exclusive packages of lots, there is no exposure problem. A bidder never runs the risk of winning just some of what it needs. Also, bids are binding commitments, and any of the bidder's bids throughout the entire auction may be part of the winning set. This provides a strong incentive for a bidder to bid in a way that is consistent with its preferences. The combinatorial clock auction allows package bids without introducing the complexity that is often associated with combinatorial auctions. Rather the auction begins with a simple and familiar price discovery process, followed by a final round of bidding. Only at the end of the supplementary round does the auctioneer need to solve an optimization problem. The optimization takes just seconds for a problem the size of the L-band auction.

The combinatorial clock auction has an activity rule to encourage price discovery. Without an activity rule, a bidder may wait until the supplementary round before submitting serious bids, and thereby undermine price discovery. The auction used the eligibility point rule—as prices increase the package size can stay the same or decrease; it cannot increase.

Under the eligibility point rule, a bidder's best strategy in the clock stage is not the natural one of bidding on its most profitable package given the round prices. Rather, the best strategy is to bid on its largest package that is still profitable. Only with this strategy can the bidder be sure that it will be able to bid its full value for desirable packages in the supplementary round. If the bidder instead bid on its most profitable package in each round of the clock stage, the bidder likely would face severe constraints in its supplementary bids for packages larger than its final clock package. Sometimes these constraints can result in nonmonotonic bids in which the bidder submits a higher bid for a package that is a strict subset of a larger package.

2 Final outcome

The final outcome is shown in Table 1. Qualcomm won all 17 lots with a bid of 20,000 (all monetary amounts throughout are in thousands of pounds), just slightly above its final clock bid of 18,631. In the case of a single winner, it is a simple matter to determine the winner's payment. It is just the value from the value-maximizing assignment without the winner. This is the opportunity cost of Qualcomm's winning all the lots, and is shown in the second panel. The best assignment without Qualcomm awards the lots to ePortal, Vectone, and WorldSpace—the final three bidders to drop out in the clock stage. This yields a combined value of 8,334, which is Qualcomm's price.

Table 1. Final outcome and determination of base price

Bidder	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	Usage	Bid	Opportunity cost	Base price
Qualcomm	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	H	20,000	8,334	8,334

Bidder	Without Qualcomm																Usage	Bid									
ePortal																		1	1	1	1	1	1	1	L	3,382	
Vectone			1	1	1																				H	2,338	
WorldSpace																									1	H	2,614
ePortal																		1	1	1	1	1	1	1	L	3,382	
Vectone			1	1	1																				H	2,338	
WorldSpace																									1	L	2,614

This outcome is in sharp contrast to the first combinatorial clock auction, the 10-40 GHz auction, in which there were ten bidders and ten winners. The outcome here, with one winner, raises the possibility that perhaps the losing bidders faced a threshold problem, and failed to top the Qualcomm bid, not because of low values, but because of low bids. Under this hypothesis, the losing bidders failed to bid full values, because each expected to win with the lower bid and believed that a higher bid would result in a higher price and lower profits. I find this hypothesis implausible. The bidders information during the clock stage did not suggest winning was likely, nor did it suggest that a higher bid would translate into a higher payment and lower profits. Rather the information during the clock stage suggested that winning was unlikely. This gives the bidders strong incentives to increase their bids in order to improve the chance of winning. Even if there was some bid shading by the losing bidders, the bid shading would have to be implausibly large—bidding just 42% of value—for the Qualcomm win to be inefficient.

3 Clock stage

I begin by looking at the bid data available to all bidders during the clock stage. The auction had 33 clock rounds. Table 2 shows a number of bidding statistics throughout the clock stage. The first is the total bidding activity, summing the package sizes across all bidders. The eligibility ratio (total eligibility over total supply) is a useful measure of the degree of competition. An initial eligibility ratio of 4.2 suggests a competitive auction—demand exceeds supply by a factor of 4.2. Auctions with initial eligibility ratios above 3 are typically viewed as highly competitive; initial eligibility ratios below 2 suggest weak competition. The next two columns show the average price per eligibility point and the standard deviation of price. Average prices increased steadily throughout the clock stage, although at a decreasing rate, as a smaller fraction of the lots increased in price with each round. It is interesting that the standard deviation of price was so small. On average across all rounds, the standard deviation was 10% of price. This is much less than the average bid increment of 17%. At least on the margin, the bidders viewed the lots as close substitutes and arbitrated across lots accordingly. Even at its highest (round 22), the standard deviation was only 20% of price. When the clock stage finished, the standard deviation was only 5% of price.

Table 2. Activity, prices, and price changes in the clock rounds

Rnd	Activity	Eligibility ratio	Price per point	Std dev of price	Fraction increase	Average increase
1	70	4.2	50		100%	
2	66	3.7	60	1	100%	19%
3	61	3.5	71	2	100%	19%
4	60	3.2	84	5	100%	18%
5	60	3.2	99	6	100%	18%
6	54	3.2	118	9	100%	19%
7	43	2.8	139	13	94%	18%
8	43	2.3	154	12	94%	11%
9	43	2.3	171	13	100%	11%
10	43	2.3	191	16	100%	12%
11	43	2.3	214	20	82%	12%
12	43	2.3	239	31	82%	12%
13	41	2.3	265	38	82%	11%
14	40	2.2	304	42	65%	15%
15	39	2.1	350	54	82%	15%
16	37	2.1	391	52	76%	12%
17	33	1.9	460	59	53%	18%
18	32	1.7	524	71	53%	14%
19	32	1.7	576	66	65%	10%
20	25	1.7	621	77	35%	8%
21	25	1.3	653	116	35%	5%
22	25	1.3	682	137	18%	4%
23	25	1.3	697	121	35%	2%
24	25	1.3	723	105	18%	4%
25	25	1.3	750	97	18%	4%
26	25	1.3	778	75	35%	4%
27	25	1.3	821	65	18%	6%
28	25	1.3	855	82	18%	4%
29	25	1.3	891	91	18%	4%
30	22	1.3	928	94	18%	4%
31	22	1.2	940	79	18%	1%
32	22	1.2	960	59	18%	2%
33	19	1.2	981	45		2%

Table 3 shows the demands in each round for low and high power. Demands above 1 are shown with darker shades of orange. By round 20, all low power demand had vanished. The bidders at this point could be fairly confident that high power demands would win all lots, eliminating the need for guard bands. The auction ended with supply and demand balance across all lots.

Table 4 shows the total demand and prices throughout the clock stage. Looking at the prices we see a strong tendency for bids to shift to less expensive lots. By the end of the auction, all the lots had converged to roughly the same price per eligibility point. From this it appears that many bidders view the lots as close substitutes. As a result, a variation of the auction format using generic lots as in the 2.6 GHz auction might have worked well. I believe that such a variation would have resulted in the same auction outcome in this case.

The clock rounds appeared to be helpful for both price discovery and assignment discovery. The final clock assignment is identical to the final auction outcome. Moreover, the price paid was determined by the bids of the last three remaining bidders. Most of these bids were from the clock stage.

Table 5. Bidding behavior in the clock stage, rounds 1-7

Rnd, Bidder	Eligibility	Activity	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	
1	79	70			50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	150	
Qualcomm	19	19	950	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
WorldSpace	3	3	150	H																	1	
Vectone	19	19	950	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
ePortal	19	13	650	L		1	1	1	1	1	1	1	1	1	1	1	1	1				
Arqiva	10	7	350	H									1	1	1	1	1	1				
JRC	2	2	100	L		1															1	
Adolphus	4	4	200	H	1	1	1	1														
MLL	3	3	150	H	1	1	1															
2	70	66			60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	55	180
Qualcomm	19	19	1,135	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	180	H																		1
Vectone	19	19	1,135	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ePortal	13	13	775	L				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	7	7	420	H									1	1	1	1	1	1	1			
JRC	2	2	115	L	1																	1
Adolphus	4	3	180	H																		1
MLL	3			H																		
3	66	61			72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216
Qualcomm	19	19	1,350	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	216	H																		1
Vectone	19	19	1,350	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ePortal	13	13	924	L		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	7	7	504	H									1	1	1	1	1	1	1			
JRC	2			L																		
Adolphus	3			H																		
4	61	60			79	76	76	86	86	86	86	86	86	86	86	86	86	86	86	86	73	259
Qualcomm	19	19	1,595	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	245	H																		1
Vectone	19	19	1,595	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ePortal	13	13	1,091	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	7	6	516	H									1	1	1	1	1	1	1			
5	60	60			91	91	91	103	103	103	103	103	103	103	103	103	103	103	103	103	84	285
Qualcomm	19	19	1,878	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	285	H																		1
Vectone	19	19	1,878	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ePortal	13	13	1,303	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	6	6	582	H	1	1	1															
6	60	54			109	109	109	124	124	124	124	124	124	124	124	124	124	124	124	124	92	328
Qualcomm	19	19	2,235	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	328	H																		1
Vectone	19	16	1,863	L	1	1	1				1	1	1	1	1	1	1	1	1	1	1	1
ePortal	13	13	1,567	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	6	3	372	H																		
7	54	43			131	131	131	136	136	136	149	149	149	149	149	149	149	149	149	149	101	394
Qualcomm	19	19	2,637	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WorldSpace	3	3	363	H	1	1																1
Vectone	16	8	1,037	H	1	1	1														1	1
ePortal	13	13	1,844	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Arqiva	3			H																		

I now examine the actual bids by each bidder in the clock stage. This information was not available to the bidders during the clock stage. Tables 5-7 shows the bidding behavior for all bidders in each round of the clock stage. Activity reductions are shown in pink; the bidder's name is highlighted in pink when the bidder drops out. The first row of each panel gives the round number, total eligibility, total activity, and current clock prices. Demand was robust across all lots in each of the early rounds. By round 3, three of the eight bidders had dropped out. By round 7, we were down to four bidders.

Table 6. Bidding behavior in the clock stage, rounds 8-18

Rnd	Rnd, Bidder	Eligibility	Activity	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ
8	Qualcomm	43	43	2,931	H	157	157	157	150	150	150	164	164	164	164	164	164	164	149	164	116	433
8	WorldSpace	3	3	433	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	Vectone	8	8	1,156	H	1			1	1	1										1	1
8	ePortal	13	13	2,061	L		1	1	1	1	1	1	1	1	1	1	1	1	1			
9	Qualcomm	43	43	3,244	H	173	173	173	173	173	173	180	180	180	180	180	180	180	156	164	128	498
9	WorldSpace	3	3	498	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	Vectone	8	8	1,292	H	1	1													1	1	1
9	ePortal	13	13	2,298	L	1	1	1	1	1	1	1	1	1	1	1	1	1				
10	Qualcomm	43	43	3,627	H	199	199	190	190	190	190	198	198	198	198	198	198	198	172	172	141	598
10	WorldSpace	3	3	598	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	Vectone	8	8	1,453	H	1	1	1	1	1										1	1	1
10	ePortal	13	13	2,544	L	1	1	1	1	1	1	1	1	1	1	1	1	1				
11	Qualcomm	43	43	4,058	H	239	239	218	218	218	209	218	218	218	218	218	218	218	189	189	155	658
11	WorldSpace	3	3	658	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	Vectone	8	8	1,739	H			1	1	1	1	1										1
11	ePortal	13	13	2,867	L	1	1	1	1	1	1	1	1	1	1	1	1	1				
12	Qualcomm	43	43	4,533	H	263	263	262	262	262	240	251	240	240	240	240	240	240	189	189	155	757
12	WorldSpace	3	3	757	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	Vectone	8	8	1,968	H					1	1	1	1	1								1
12	ePortal	13	13	2,988	L			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	Qualcomm	43	41	5,040	H	263	263	262	288	288	288	301	276	276	276	264	264	264	198	198	163	908
13	WorldSpace	3	3	559	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	Vectone	8	8	1,995	H											1	1	1	1	1	1	1
13	ePortal	13	11	2,983	L			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	Qualcomm	41	40	5,772	H	263	263	262	331	331	331	346	317	317	317	304	323	323	257	243	200	1044
14	WorldSpace	3	3	1,044	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	Vectone	8	8	2,390	H											1	1	1	1	1	1	1
14	ePortal	11	10	2,947	L						1	1	1	1	1	1	1	1	1	1	1	1
15	Qualcomm	40	39	6,653	H	263	263	262	331	331	331	398	365	365	365	350	420	420	334	316	260	1279
15	WorldSpace	3	3	910	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	Vectone	8	7	2,398	H	1	1	1	1													1
15	ePortal	10	10	3,274	L	1	1	1	1	1	1	1	1	1	1							
16	Qualcomm	39	37	7,423	H	302	302	301	381	356	356	458	420	420	420	350	420	420	384	363	299	1471
16	WorldSpace	3	3	1,471	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	Vectone	7	6	2,517	H																	1
16	ePortal	10	9	3,296	L	1	1	1	1	1	1	1	1	1								1
17	Qualcomm	37	33	8,732	H	362	362	361	457	427	427	550	504	504	420	350	420	420	461	436	359	1912
17	WorldSpace	3	3	1,912	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	Vectone	6	4	2,271	H																	1
17	ePortal	9	7	2,946	L	1	1	1	1	1	1	1										1
18	Qualcomm	33	32	9,964	H	434	434	433	548	512	512	660	504	504	420	350	420	420	461	436	431	2485
18	WorldSpace	3	3	1,328	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	Vectone	4	3	2,485	H																	1
18	ePortal	7	7	3,011	L									1	1	1	1	1	1	1	1	1

At round 13, ePortal begins making steady reductions. Vectone's reductions begin in round 15.

Table 7. Bidding behavior in the clock stage, rounds 19-33

Rnd	Rnd, Bidder	Eligibility	Activity	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ
19	Qualcomm	32	32	10,937	H	434	434	433	548	512	512	660	504	554	462	385	462	462	553	523	517	2982
19	WorldSpace	3	3	1,301	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	Vectone	3	3	1,718	H						1	1	1									
19	ePortal	7	7	3,382	L							1	1	1	1	1	1	1				
20	Qualcomm	32	25	11,808	H	477	477	476	548	512	512	726	605	665	554	462	554	554	664	523	517	2982
20	WorldSpace	3	3	1,704	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	Vectone	3	3	1,824	H							1	1	1					1	1	1	
20	ePortal	7			L																	
21	Qualcomm	25	25	12,410	H	477	477	476	548	512	512	726	726	798	665	462	554	554	797	575	569	2982
21	WorldSpace	3	3	1,941	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	Vectone	3	3	1,572	H				1	1	1								1	1	1	
22	Qualcomm	25	25	12,955	H	477	477	476	603	563	563	726	726	798	665	462	554	554	956	690	683	2982
22	WorldSpace	3	3	1,430	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	Vectone	3	3	1,430	H	1	1	1														
23	Qualcomm	25	25	13,240	H	572	572	571	603	563	563	726	726	798	665	462	554	554	956	690	683	2982
23	WorldSpace	3	3	1,570	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	Vectone	3	3	1,715	H	1	1	1							1	1	1					
24	Qualcomm	25	25	13,738	H	686	686	685	603	563	563	726	726	798	665	508	609	609	956	690	683	2982
24	WorldSpace	3	3	1,729	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	Vectone	3	3	1,729	H				1	1	1											
25	Qualcomm	25	25	14,257	H	686	686	685	784	732	732	726	726	798	665	508	609	609	956	690	683	2982
25	WorldSpace	3	3	1,726	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	Vectone	3	3	1,726	H										1	1	1					
26	Qualcomm	25	25	14,775	H	686	686	685	784	732	732	726	726	798	665	660	792	792	956	690	683	2982
26	WorldSpace	3	3	2,033	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	Vectone	3	3	2,057	H	1	1	1							1					1	1	
27	Qualcomm	25	25	15,593	H	823	823	822	784	732	732	726	726	798	665	792	792	792	956	828	820	2982
27	WorldSpace	3	3	2,184	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	Vectone	3	3	2,184	H					1	1	1										
28	Qualcomm	25	25	16,249	H	823	823	822	784	732	952	944	944	798	665	792	792	792	956	828	820	2982
28	WorldSpace	3	3	2,249	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	Vectone	3	3	2,249	H										1	1	1					
29	Qualcomm	25	25	16,925	H	823	823	822	784	732	952	944	944	798	865	1030	1030	792	956	828	820	2982
29	WorldSpace	3	3	2,338	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	Vectone	3	3	2,338	H			1	1	1												
30	Qualcomm	25	22	17,627	H	823	823	1069	1019	952	952	944	944	798	865	1030	1030	792	956	828	820	2982
30	WorldSpace	3	3	2,410	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	Vectone	3			H									1				1			1	
31	Qualcomm	22	22	17,868	H	823	823	1069	1019	952	952	944	944	878	865	1030	1030	871	956	828	902	2982
31	WorldSpace	3	3	2,474	H	1	1							1						1		
32	Qualcomm	22	22	18,238	H	946	946	1069	1019	952	952	944	944	878	865	1030	1030	871	956	952	902	2982
32	WorldSpace	3	3	2,614	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	Qualcomm	19	19	18,631	H	946	946	1069	1019	952	952	944	944	1010	995	1030	1030	1002	956	952	902	2982
						1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

In round 20, ePortal drops out. There are no longer any bidders bidding for low power usage. In round 30, Vectone drops out. WorldSpace submits its final competing bid in round 32. The clock stage ends in round 33 with Qualcomm still bidding for all 17 lots.

3.1 Bidding behavior including supplementary bids

I now examine the bidding behavior of each of the eight bidders in the clock stage and the supplementary round. Supplementary bids are intended for bidders to increase their clock bids to

full value and add bids on any additional packages that are relevant but were not bid on during the clock rounds. Bidding behavior varied a great deal across bidders. As can be seen in Table 8, only two bidders submitted a significant number of supplementary bids. I suspect most bidders believed that the supplementary bids would not improve their chance of winning.

Table 8. Number of package bids, considering only the highest bid on the package

Bidder	Primary bids	Supplementary bids	All bids
Qualcomm		1	1
WorldSpace	12	2	14
Vectone	21		21
ePortal	1	15	16
Arqiva	4		4
JRC	1	1	2
Adolphus	2		2
MLL		14	14
Total	41	33	74

Table 9 shows Qualcomm’s bids. Qualcomm’s bidding was straightforward. It bid for all lots in every clock round as well as in its single supplementary bid. The supplementary bid was the winning bid and so is highlighted in bright green. Based on the clock bidding of others, Qualcomm must have been quite confident that it would win all lots. It did raise in the supplementary round its final clock bid slightly from 18,631 to 20,000, but this raise was unnecessary for it to win all lots, given that it was the only active bidder in the final clock round and therefore the constraints on other bidders’ supplementary bids were clear.

Qualcomm did not bid for smaller packages that might fit well with other bidders. Qualcomm likely had large complementarities from winning the entire L-band.

Table 9. Qualcomm primary and supplementary bids

Rnd	Elig	Act	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LJ	LK	LL	LM	LN	LO	LP	LQ	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	
<i>Qualcomm</i>																																						
1	19	19	950	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	180
2	19	19	1,135	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	180
3	19	19	1,350	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216
4	19	19	1,595	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	79	76	76	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	73	259
5	19	19	1,878	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	91	91	91	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	84	285
6	19	19	2,235	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	109	109	109	124	124	124	124	124	124	124	124	124	124	124	124	124	124	92	328	
7	19	19	2,637	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	131	131	131	136	136	136	149	149	149	149	149	149	149	149	149	149	149	101	394	
8	19	19	2,931	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	157	157	157	150	150	150	164	164	164	164	164	164	164	164	164	164	149	116	433	
9	19	19	3,244	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	173	173	173	173	173	180	180	180	180	180	180	180	180	180	180	180	156	128	498	
10	19	19	3,627	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	199	199	199	190	190	190	198	198	198	198	198	198	198	198	198	198	172	141	598	
11	19	19	4,058	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	239	239	218	218	218	209	218	218	218	218	218	218	218	218	218	189	155	658		
12	19	19	4,533	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	263	263	262	262	262	240	251	240	240	240	240	240	240	240	240	189	155	757		
13	19	19	5,040	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	263	263	262	288	288	288	301	276	276	276	264	264	264	264	198	163	908			
14	19	19	5,772	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	263	263	262	331	331	331	346	317	317	317	304	323	323	257	243	200	1044			
15	19	19	6,653	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	263	263	262	331	331	331	398	365	365	365	350	420	420	334	316	260	1279			
16	19	19	7,423	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	302	302	301	381	356	356	458	420	420	420	350	420	420	384	363	299	1471			
17	19	19	8,732	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	362	362	361	457	427	427	550	504	504	420	350	420	420	461	436	359	1912			
18	19	19	9,964	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	434	434	433	548	512	512	660	504	504	420	350	420	420	461	436	431	2485			
19	19	19	10,937	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	434	434	433	548	512	512	660	504	554	462	385	462	462	553	523	517	2982			
20	19	19	11,808	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	477	477	476	548	512	512	726	605	665	554	462	554	554	664	523	517	2982			
21	19	19	12,410	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	477	477	476	548	512	512	726	726	798	665	462	554	554	797	575	569	2982			
22	19	19	12,955	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	477	477	476	603	563	563	726	726	798	665	462	554	554	956	690	683	2982			
23	19	19	13,240	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	572	572	571	603	563	563	726	726	798	665	462	554	554	956	690	683	2982			
24	19	19	13,738	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	686	686	685	603	563	563	726	726	798	665	508	609	609	956	690	683	2982			
25	19	19	14,257	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	686	686	685	784	732	732	726	726	798	665	508	609	609	956	690	683	2982			
26	19	19	14,775	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	686	686	685	784	732	732	726	726	798	665	660	792	792	956	690	683	2982			
27	19	19	15,593	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	823	823	822	784	732	732	726	726	798	665	792	792	956	828	820	2982				
28	19	19	16,249	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	823	823	822	784	732	952	944	944	798	665	792	792	956	828	820	2982				
29	19	19	16,925	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	823	823	822	784	732	952	944	944	798	865	1030	1030	792	956	828	820	2982			
30	19	19	17,627	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	823	823	1069	1019	952	952	944	944	798	865	1030	1030	792	956	828	820	2982			
31	19	19	17,868	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	823	823	1069	1019	952	952	944	944	878	865	1030	1030	871	956	828	902	2982			
32	19	19	18,238	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	946	946	1069	1019	952	952	944	944	878	865	1030	1030	871	956	952	902	2982			
33	19	19	18,631	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1	946	946	1069	1019	952	952	944	944	1010	995	1030	1030	1002	956	952	902	2982			
SB			20,000	H	1	1	1	1	1	1	1	1	1	1	1	1	1	1																				

Table 10. WorldSpace primary and supplementary bids

Bidder	Rnd	Elig	Act	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ					
WorldSpace	1	3	3	150	H																		1	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	150			
WorldSpace	2	3	3	180	H																		1	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	180	
WorldSpace	3	3	3	216	H																		1	72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216
WorldSpace	4	3	3	245	H																		1	79	76	76	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	73	259
WorldSpace	5	3	3	285	H																		1	91	91	91	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	84	285	
WorldSpace	6	3	3	328	H																		1	109	109	109	124	124	124	124	124	124	124	124	124	124	124	124	124	124	92	328		
WorldSpace	7	3	3	363	H	1	1																1	131	131	131	136	136	136	149	149	149	149	149	149	149	149	149	149	101	394			
WorldSpace	8	3	3	433	H	1	1																1	157	157	157	150	150	150	164	164	164	164	164	164	164	164	164	149	116	433			
WorldSpace	9	3	3	498	H	1	1																1	173	173	173	173	173	173	180	180	180	180	180	180	180	180	180	156	128	498			
WorldSpace	10	3	3	598	H	1	1																1	199	199	190	190	190	190	198	198	198	198	198	198	198	198	198	198	172	141	598		
WorldSpace	11	3	3	658	H	1	1																1	239	239	218	218	218	209	218	218	218	218	218	218	218	218	218	189	155	658			
WorldSpace	12	3	3	757	H	1	1																1	263	263	262	262	262	240	251	240	240	240	240	240	240	240	189	155	757				
WorldSpace	13	3	3	559	H	1	1	1															1	263	263	262	288	288	288	301	276	276	276	264	264	264	198	163	908					
WorldSpace	14	3	3	1,044	H	1	1	1															1	263	263	262	331	331	331	346	317	317	317	304	323	323	257	243	200	1044				
WorldSpace	15	3	3	910	H	1	1	1															1	263	263	262	331	331	331	398	365	365	365	350	420	420	334	316	260	1279				
WorldSpace	16	3	3	1,471	H	1	1	1															1	302	302	301	381	356	356	458	420	420	420	350	420	420	384	363	299	1471				
WorldSpace	17	3	3	1,912	H	1	1	1															1	362	362	361	457	427	427	550	504	504	420	350	420	420	461	436	359	1912				
WorldSpace	18	3	3	1,328	H	1	1	1															1	434	434	433	548	512	512	660	504	504	420	350	420	420	461	436	431	2485				
WorldSpace	19	3	3	1,301	H	1	1	1															1	434	434	433	548	512	512	660	504	554	462	385	462	462	553	523	517	2982				
WorldSpace	20	3	3	1,704	H	1	1	1															1	477	477	476	548	512	512	726	605	665	554	462	554	554	664	523	517	2982				
WorldSpace	21	3	3	1,941	H	1	1	1															1	477	477	476	548	512	512	726	726	798	665	462	554	554	797	575	569	2982				
WorldSpace	22	3	3	1,430	H	1	1	1															1	477	477	476	603	563	563	726	726	798	665	462	554	554	956	690	683	2982				
WorldSpace	23	3	3	1,570	H	1	1	1				1	1	1									1	572	572	571	603	563	563	726	726	798	665	462	554	554	956	690	683	2982				
WorldSpace	24	3	3	1,729	H	1	1	1				1	1	1									1	686	686	685	603	563	563	726	726	798	665	508	609	609	956	690	683	2982				
WorldSpace	25	3	3	1,726	H	1	1	1				1	1	1									1	686	686	685	784	732	732	726	726	798	665	508	609	609	956	690	683	2982				
WorldSpace	26	3	3	2,033	H	1	1	1				1	1	1									1	686	686	685	784	732	732	726	726	798	665	660	792	792	956	690	683	2982				
WorldSpace	27	3	3	2,184	H	1	1	1				1	1	1									1	823	823	822	784	732	732	726	726	798	665	792	792	956	828	820	2982					
WorldSpace	28	3	3	2,249	H	1	1	1				1	1	1									1	823	823	822	784	732	952	944	944	798	665	792	792	956	828	820	2982					
WorldSpace	29	3	3	2,338	H	1	1	1				1	1	1									1	823	823	822	784	732	952	944	944	798	865	1030	1030	792	956	828	820	2982				
WorldSpace	30	3	3	2,410	H	1	1	1				1	1	1									1	823	823	1069	1019	952	952	944	944	798	865	1030	1030	792	956	828	820	2982				
WorldSpace	31	3	3	2,474	H	1	1	1				1	1	1									1	823	823	1069	1019	952	952	944	944	878	865	1030	1030	871	956	828	902	2982				
WorldSpace	32	3	3	2,614	H	1	1	1				1	1	1									1	946	946	1069	1019	952	952	944	944	878	865	1030	1030	871	956	952	902	2982				
WorldSpace	SB			2,614	H																	1																						
WorldSpace	SB			2,614	L																		1																					

Table 10 shows WorldSpace bids. WorldSpace started out with little eligibility of 3 points, and remained at that level until the last round of the clock stage. WorldSpace arbitrated between lot LQ and three smaller lots. Interestingly, WorldSpace did not always bid on contiguous lots. It often bid on the three least expensive lots, or on the LQ lot if that was less expensive. This behavior was one of the main reasons why the prices across all lots equalized on a per point basis.

WorldSpace submitted only two supplementary bids. Both were for LQ at the same price, one high power and one low power. These bids, highlighted in yellow, were important in that they were among the bids that determined Qualcomm’s payment. It is difficult to understand why WorldSpace did not enter a more complete set of supplementary bids. Based on its bidding in the clock stage, it would appear to value nearly any set of three small lots at its upper limit of 2,614. Perhaps it thought that the extra bids would have little chance of success.

Table 12 shows ePortal’s bids. ePortal required contiguous spectrum and appears to have a minimum scale of 7 lots. ePortal did submit many supplementary bids, many of which violate monotonicity. However, it was not the case that the activity rule caused ePortal’s bids to violate monotonicity. ePortal’s final clock bid proved to be part of the bid set that determined Qualcomm’s payment. All of ePortal’s bids are less than about 3,000.

Table 13. Primary and supplementary bids of Arqiva, JRC, Adolphus, and MLL

Bidder	Rnd	Elig	Act	Bid	Usage	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO	LP	LQ	
<i>Arqiva</i>																																								
Arqiva	1	10	7	350	H									1	1	1	1	1	1	1		50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	180		
Arqiva	2	7	7	420	H									1	1	1	1	1	1	1		60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	180	
Arqiva	3	7	7	504	H									1	1	1	1	1	1	1		72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216
Arqiva	4	7	6	516	H									1	1	1	1	1	1	1		79	76	76	86	86	86	86	86	86	86	86	86	86	86	86	86	86	73	259
Arqiva	5	6	6	582	H	1	1	1														91	91	91	103	103	103	103	103	103	103	103	103	103	103	103	103	84	285	
Arqiva	6	6	3	372	H																	109	109	109	124	124	124	124	124	124	124	124	124	124	124	124	124	92	328	
Arqiva	7	3			H																	131	131	131	136	136	136	149	149	149	149	149	149	149	149	149	101	394		
<i>JRC</i>																																								
JRC	1	2	2	100	L			1	1													50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	180	
JRC	2	2	2	115	L	1																60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	55	180
JRC	3	2			L																	72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216
JRC	SB			120	L	1																																		
<i>Adolphus</i>																																								
Adolphus	1	4	4	200	H	1	1	1	1													50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	150	
Adolphus	2	4	3	180	H																	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	55	180
Adolphus	3	3			H																	72	66	66	72	72	72	72	72	72	72	72	72	72	72	72	72	66	216	
<i>MLL</i>																																								
MLL	1	3	3	150	H	1	1	1														50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	180	
MLL	2	3			H																	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	55	180	
MLL	SB			165	H	1	1	1																																
MLL	SB			165	H		1	1	1																															
MLL	SB			165	H			1	1	1																														
MLL	SB			165	H				1	1	1																													
MLL	SB			165	H					1	1	1																												
MLL	SB			165	H						1	1	1																											
MLL	SB			165	H							1	1	1																										
MLL	SB			165	H								1	1	1																									
MLL	SB			165	H									1	1	1																								
MLL	SB			165	H										1	1	1																							
MLL	SB			165	H											1	1	1																						
MLL	SB			165	H												1	1	1																					
MLL	SB			165	H													1	1	1																				
MLL	SB			165	H														1	1	1																			

The bids of the four weakest bidders are shown in Table 13. All four bidders dropped out early in the auction, and had little chance of winning. Nonetheless, MLL submitted a full contingent of bids for three contiguous small lots, apparently hoping that one of the bids would fit well with other bidders. One of Arqiva’s bids violates monotonicity.

3.2 Optimization

Two independent optimizers were used to find the optimal assignment and prices. Both optimizers found the same unique solution. The assignment and prices with 8 bidders and 74 bids were determined in a combined time of 0.14 seconds.

4 Conclusion

The L-band auction successfully demonstrated the desirable features of the combinatorial clock auction. The bidders had strong incentives to submit truthful bids during the clock stage and in the supplementary round. Assuming they did, then the outcome was fully efficient.

The auction was competitive with an initial eligibility ratio of 4.2. However, one bidder Qualcomm was much stronger than the others and appeared to have strong complementarities for all lots. As a result, Qualcomm won all the lots.

My only concern is with the activity rule. Several of the bidders failed to bid on the largest profitable package in the clock stage, consistent with the incentives of the eligibility point rule. As a result, the bids of these bidders violated monotonicity—they sometimes bid more for a smaller package that was a strict subset of a larger package. The activity rule sometimes prevented

bidders from correcting these nonmonotonic bids in the supplementary round, as a result of the bidding strategy they employed in the primary rounds, which appears to have deviated from the optimal strategy of bidding on the largest profitable package at the round prices.

Most bidders failed to submit a complete set of supplementary bids. The absence of these bids may have reduced auction revenues somewhat. Still I doubt that including these bids would have altered the final assignment. Qualcomm would still win all lots, but perhaps pay a bit more.

Finally, there may be some concern that Qualcomm winning all lots was somehow caused by the threshold problem sometimes seen in package auctions. I doubt this was the case. The losing bidders did not have an incentive to substantially shade their bids. Qualcomm almost surely won all lots because it valued the lots more than the other bidders. I am confident that the auction outcome was efficient.