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Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2020

IMPROVEMENTS TO AUCTION THEORY AND INVENTIONS OF NEW AUCTION FORMATS

The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel

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1 Introduction

The practice of selling valuable items to the highest bidder, or procuring valuable services from the lowest bidder, goes as far back in history as we have written records. The Greek historian Herodotus documented auctions in ancient Babylon already 2500 years ago.[1] In the Roman Empire, creditors regularly used auctions to sell off assets confiscated from delinquent debtors. In more modern times, Stockholms Auktionsverk, the oldest surviving auction house in the world was founded by the Swedish Baron Claes Rålamb in 1674. In addition to confiscated assets, Stockholms Auktionsverk auctioned a wide range of goods on behalf of willing sellers—for example, Sweden's late 17th century king, Karl XI, offered a batch of hunting weapons for sale. Similar auction houses existed all around Europe. In 1744, Samuel Baker and George Leigh sold a set of valuable books for a grand total of £826 at their newly established auction company. That London-based company was to become Sotheby's, presently the world's largest fine-arts auction house.

Setting their notorious history aside, auctions are certainly of far greater importance today than at any time in the past. Commodities such as fish, fresh flowers, and rough diamonds are sold in auctions, as they have been for centuries. Financial securities—such as government bonds—are frequently sold in modern versions of ancient auction designs. Governments have also come to rely on auctions for selling rights to timber, minerals, petroleum, and radio frequencies, as well as for procuring a wide range of goods and services from private firms. In the last decade, internet auctions have become ubiquitous. Platforms such as eBay rely on auctions to facilitate business-to-business, business-to-consumer, and consumer-to-consumer transactions; search engines like Google and Yahoo! employ auctions to sell keyword positions and advertisements.

Just as the increasing use of auctions has spurred auction research, auction research has spurred the increasing use of auctions. The resulting improvements to auction theory and inventions of new auction formats have been truly collective efforts. But two scholars stand out: Paul R. Milgrom and Robert B. Wilson, both of Stanford University, and this year's Laureates in Economic Sciences. Their research has deepened and broadened the field's analytical foundations. It answers questions of fundamental theoretical importance—how bidders are likely to behave for a variety of auction formats and under different informational conditions—as well as questions of great practical importance, such as how regulators and governments should design auctions in order to maximize social value. Moreover, insights from the Laureates' work have directly shaped the design of important real-world markets. These insights do not just enable

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[1] The Babylonian auctions were illustrated by the British artist Edwin Long in a famous 19th century painting, which was sold to Thomas Holloway in a 1882 auction for £6,615, the highest price thereto recorded for a contemporary painting. (The painting now hangs in Royal Holloway, University of London.)

sellers to raise higher revenues or buyers to procure at lower costs. They also facilitate sales to the most appropriate buyer or procurement from the most appropriate seller. In this way, auction theory can help regulators and governments around the world to put objects and activities into the hands of those agents who are best able to manage them.

Auction theory thus provides another example of serendipity in science: every now and then, important discoveries made in curiosity-driven basic research lead to unanticipated practical applications. In this case, the Laureates themselves produced both the basic research and the most important applications.

1.1 Improvements to Auction Theory

Auctions differ in two main respects: format and information. When it comes to format, what are the rules for how prices are announced, how participants place bids, how prices are updated, how the auction closes, and how "winners" are selected? For example, the English auction stipulates that open-outcry bids be given in ascending order until nobody submits a higher bid, with the object allocated to the highest bidder at that price. And with regard to information, what do the participants know about the value of the auctioned object? In most cases, each bidder has some information that is not observable to others. The information may comprise both the bidder's idiosyncratic valuation of the object (a private-value component) and her signals about the object's properties that affect its value to other bidders (common-value components).

Since participants generally have different goals and act strategically, the behavior of one bidder cannot be understood in isolation from that of other bidders. Yet auctions are not zero-sum games. On the contrary, the whole point of an auction is to create as much value as possible by assigning an object to the buyer who can make the best use of it. Hence, it was impossible to begin a rigorous analysis of auctions until non-cooperative game theory had been generalized beyond the special case of zero-sum games. That step was taken in 1950 by John F. Nash, Jr., the 1994 Laureate in Economic Sciences, in his doctoral dissertation in mathematics (Nash, 1950). The study of realistic auction settings, in which each bidder has some private information, had to wait for further extensions of Nash's theoretical framework by 1996 Laureate William S. Vickrey and 1994 Laureate John C. Harsanyi.

Vickrey (1961, 1962) provided a full-fledged analysis of different existing auction formats in his independent *private-values* model. That is, each bidder's valuation of the object provides no information about other bidders' valuations. While analytically useful, this information setting is rare in practice. In particular, for all objects whose resale value is important, the independence assumption is inappropriate.

Harsanyi (1967, 1968a,b) provided the equilibrium notion that would allow Wilson to study auctions without relying on the independence assumption. Specifically, Wilson (1969, 1977) characterized equilibrium bidding and prices in another special case, namely when the (*ex-post*) value of the object for sale is identical among all bidders, but bidders have different private information (*ex-ante*) about what that value will turn out to be. In the case of objects such as mineral rights and financial assets, this *common-values* model approximates bidder objectives much better than the independent private-values model.

However, most real-world auctions involve private-value as well as common-value components. Milgrom, who had been Wilson's doctoral student in the 1970s, made decisive discoveries about these more realistic situations. In a series of contributions from the early 1980s (especially Milgrom, 1981a,b; Milgrom and Weber, 1982) he advanced auction theory by studying a general case with both private and common values under plausible, yet mathematically tractable, conditions for bidders' information and the distribution of values across bidders. Milgrom's exhaustive analysis modified, synthesized, and generalized many earlier results. Among other things, his findings showed when a seller can expect to raise higher revenues by sharing expert appraisals—such as authenticity certificates or inspection protocols—with potential bidders. His extended analysis also made it possible to rank different auction formats in terms of their expected revenue.

1.2 Inventions of New Auction Formats

By the beginning of the 1990s, auction theorists had characterized equilibrium bidding in single-object auctions under most, if not all, relevant auction formats and information settings. Many predictions from the theory of equilibrium bidding had also been supported by empirical work using both observational and experimental data. At this time, the main research focus shifted from single-object auctions to multi-object auctions. This shift was largely due to a desire to use markets for trading a wide range of objects—like spectrum-frequency bands, electricity, and batches of "troubled debt"—that had previously been allocated in other ways. The number and size of the transactions of these objects tend to be huge, and even small efficiency gains in percentage terms can be worth billions of dollars to companies, customers, and taxpayers.

Reaping those gains was a challenge that called for the invention of new auction formats. Auctions of a large number of interrelated objects raise issues that had not come up in (or had been overlooked in) the analysis of "standard" auction formats. For example, radio-spectrum frequencies should be sold in a multi-object auction with simultaneous sale, due to the technical and geographical complementarities across frequencies in different channels. The same is true for electricity auctions, due to economies of scale and scope in electricity distribution. But complementarities between objects, externalities across participants, and other features that often are present in auctions with interrelated objects may, for example, tempt bidders to reduce their demand and collude. This, in turn, may prevent objects from ending up with the bidders that value them the most.

Designing new auction rules to mitigate these problems typically requires a range of different inputs from auction theorists, regulators, and private companies. In this sense, practical auction design resembles engineering: a properly trained specialist can devise a suitable auction format for a specific economic setting, much like a sophisticated engineer can design a bridge for a specific geographic location.[2]

By identifying and analyzing some of the salient issues in auctions with interrelated objects, Milgrom and Wilson invented a number of valuable new auction formats and designs. The most famous example is the *Simultaneous Multiple Round Auction*, which they developed, in part with Preston McAfee, for the 1994 U.S. Federal Communications Commission's radio-spectrum auction. This auction format has since become a workhorse method of selling spectrum licenses around the world. Additional formats designed by the Laureates include *Share Auctions* (Wilson, 1979), *Combinatorial Clock Auctions* (Ausubel, Cramton and Milgrom, 2006), and *Incentive Auctions* (Milgrom et al., 2012).

The improvements to auction theory have not only led to the invention of new auction formats. They have also helped unify the analysis of different trading institutions, allowing us to see the close relationship between auctions, on the one hand, and trading via markets with posted prices or via other bargaining procedures, on the other. Concepts from auction theory also illuminate other economic interactions that, at first glance, seem quite different from auctions—such as takeover battles and wars of attrition (Klemperer, 2003). The 2007 Prize in Economic Sciences awarded to Roger B. Myerson, for contributions to mechanism design, and the 2014 Prize awarded to Jean Tirole, for contributions to the theory of regulation and competition policy, both rewarded research that used auction theory as a stepping stone.

1.3 Organization of This Review

Section 2 lays out a formal framework for the study of single-object auctions. Within this framework, starting with Vickrey's findings in the special case of private values, Wilson's work is explained for the special case of common values, followed by Milgrom's findings in the general case of both private and common values. A brief discussion follows on how the improvements to the auction theory developed by the Laureates and their coauthors have prompted empirical researchers to confront its predictions with observational and experimental data.

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[2] Auction theory is one of the two main strands of the flourishing field of *market design*. The other strand is matching theory, rewarded by the 2012 Prize in Economic Sciences to Alvin E. Roth and Lloyd S. Shapley.

Section 3 deals with multi-object auctions. A key dilemma described here is the challenges facing a regulator who seeks to balance a concern for auction revenues against a concern for efficient allocations. Some of the most important auction formats that have been invented in the past few decades to handle these challenges were created by Milgrom and Wilson, and they are described in the section.

Section 4 briefly mentions spillovers from auction research to other fields, as well as the Laureates' significant contributions to closely related fields. Section 5 offers a brief conclusion.

2 Single-Object Auctions

Describing some of auction theory's basic results is made easier with the introduction of a flexible setting that can be specialized in different directions via alternative assumptions about information conditions and trading rules for the auction at hand. Subsection 2.1 describes this general framework for single-object auctions, while Subsections 2.2–2.4 consider the theoretical findings in different special cases. The findings generate a number of empirically testable predictions. As discussed in Subsection 2.5, these predictions have been investigated using econometric as well as experimental methods.

2.1 A General Framework

Consider a seller who wants to auction off a single, indivisible object. We treat the number of (prospective) bidders as fixed, and denote this number by B. The valuation of the object by bidder $b \in \{1, \ldots, B\}$ is given by the real-valued function $v_b = V_b(\theta, \beta)$. Here, $\theta = (\theta_1, \ldots, \theta_B)$ represents a vector of private signals observed by the individual bidders. These may concern bidder b's *individual* appreciation of the auctioned object, such as the joy of gazing at a painting, or b's individual cost of using that object, such as the private cost of exploiting a mineral right. By contrast, $\beta = (\beta_1, \ldots, \beta_S)$ is a vector of real-valued state variables, which influence the value of the object to *all* bidders, such as metric estimates of minerals in the ground or factors that determine the price of the mineral. Some of these variables may only be known to the seller. In the case of the minerals, the seller may already have performed seismic and other geological tests and kept the results confidential.[3]

As will be clear in Subsections 2.2–2.4, alternative auction models make different informational assumptions: the dimension of the state variables, the precise information (signal) available to each auction participant, the distributions of θ and β, and the conditional dependence between

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[3] Bidding behavior in some auction formats also depend on the bidders' attitudes toward risk. When nothing else is said, it is assumed, for simplicity, that all bidders are risk-neutral—i.e., their valuation of any lottery equals its expected payoff.

them. Models also make different assumptions to capture different auction formats: how prices are announced, how participants place bids, how prices are updated, how the auction closes, and how winners are selected.

Early auction theory compared bidding strategies and outcomes in four auction formats. They are (in alphabetical order):

(D) the *Dutch or Clock auction*, where the price begins at a high level set by the seller and is gradually decreased until some bidder accepts and pays that price;

(E) the *English auction*, where bids ascend from a starting price (often suggested by the seller) by open outcries, observed by everybody, until no bidder is willing to make a higher bid, at which point the highest bidder wins and pays a price equal to her final bid;

(F) the *First-price* (*sealed-bid*) *auction*, where the participants submit a single bid not observed by the others (e.g., in sealed envelopes), such that the highest bidder wins and pays what she bid;

(V) the *Second-price* (*sealed-bid*) *or Vickrey auction*, where bidders submit bids in sealed envelopes, and the highest bidder wins but pays the second-highest bid.

These common auction formats are used to sell a wide range of objects, assets, and commodities. For example, the Aalsmeer Flower Auction in the Netherlands relies on Dutch auctions to sell around 20 million flowers per day, and the Federal Reserve Bank of New York uses a form of Clock auction when selling bonds to primary dealers so as to raise funds for the U.S. Treasury. English auctions are probably the most common format in use today, at least for private single-object auctions on online platforms such as eBay. First-price auctions are common vehicles for companies and organizations to procure goods or services, and for governments to award public contracts or allocate mining leases. Second-price auctions are less common, but they too have been used for centuries, for example when selling collectibles, and are presently used by internet-search engines when selling advertising space.

Auctions are always supervised by an auctioneer, whose task is to enforce the auction rules. Historically, the auctioneer was always a physical person. Nowadays, computer software can replace human auctioneers in order to facilitate greater speed and transaction frequency. In multi-item auctions (see Section 3), computers also serve to enable more complex transactions than would be feasible with all-human auctioneers. Similarly, bidders have historically placed bids themselves. Nowadays, computerized agents—commonly referred to as proxy-bidding agents—often place bids on behalf of human bidders, based on pre-specified information about willingness to pay and standardized bid increments or decrements.

2.2 Private Values

In his seminal paper, Vickrey (1961) considered alternative single-object auctions in a setting with purely private values. In terms of the notation we just introduced, Vickrey's model deals with the special case without any common-value components in the bidders' information, $S = 0$. Moreover, he assumed that the valuations coincide with the private signals: $v_b = \theta_b$. That is, each bidder b knows her own private valuation of the object, θ_b, but θ_b is neither known to the seller, nor to the other bidders, $j \neq b$.[4] Specifically, Vickrey assumed that the valuations θ_b were statistically independent random variables—his model is therefore also known as the independent private-values model. Finally, he assumed that the bidders were risk-neutral and *ex-ante* symmetric: each valuation θ_b is drawn from the same distribution.

Main Findings and Insights

Vickrey (1961) first investigated the "strategic equivalence" between the four main auction formats (D)–(V). He found that the First-price auction and the Dutch auction are strategically equivalent: the price at which a bidder optimally plans to drop out in a Dutch auction corresponds to her optimal bid in a First-price auction. However, these auction formats have no dominant strategies, as optimal bidding depends on the strategies of other bidders. Lowering one's bid lowers the price if one wins, but also lowers the probability of winning. Optimal bids balance these two considerations.

Vickrey (1961) also showed that each bidder b in the Second-price auction—in contrast to the First-price auction and the Dutch auction—has a dominant strategy, namely to bid exactly what the object is worth to her, θ_b. Doing so, she wins and earns a positive amount whenever her valuation is above the highest competing bid, and she does not win whenever her valuation is below the highest competing bid. Reducing the bid below θ_b merely runs an unnecessary risk of forgoing a gain, and increasing the bid above θ_b merely runs an unnecessary risk of making a loss. Furthermore, the English auction and the Second-price auction are strategically equivalent: the price at which a bidder optimally plans to drop out in an English auction corresponds to her valuation of the object for sale. Consequently, in both auctions, the bidder with the highest valuation wins and pays a price equal to the second highest value.

Using game-theory arguments, Vickrey (1961) showed that all of the auctions formats (D)–(V) are efficient in his model—in the sense that the auctioned object will end up in the hands of the bidder who values it the most—and that they all yield the same expected revenue for the seller. The latter finding is known as the *revenue-equivalence theorem*.

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[4] For the case when bidders have information about each other that is not available to the seller, see Cremer and McLean (1988).

Extensions and Further Research

Vickrey (1961, 1962) discussed two important extensions of the private-values model. One was multi-object auctions with multiple identical objects for sale. These types of auctions will be discussed in Section 3. Another extension was *ex-ante* asymmetries. Vickrey showed, by example, that when bidders are asymmetric, the four auction formats cannot generally be ranked in terms of expected revenue. For some distributions, a First-price auction is revenue-superior to a Second-price auction, but for other distributions the ranking is reversed. Vickrey also found the First-price auction to be inefficient with asymmetric bidders, as opposed to the Second-price and English auctions, which are always efficient.

In a 1961 paper, Vickrey proved the revenue equivalence theorem for a uniform distribution of values, and then extended it to a more general class of distributions (Vickrey, 1961, 1962). In a paper published two decades later (Myerson, 1981), 2007 Laureate Roger B. Myerson showed that revenue equivalence extends to any efficient auction with risk-neutral and *ex-ante* symmetric bidders, when the bidder with the lowest possible valuation makes no expected payment.[5] This type of analysis has been extended to various information settings and auction models, as illustrated in the remainder of this section as well as in Section 3.

2.3 Common Values

The private-values assumption assigns each bidder's valuation only to her own particular and independent circumstances, such as her taste or private use of the object for sale. But most often, values are interdependent rather than private. In the presence of interdependencies, a bidder may want to revise her own estimate if she learns about another bidder's estimate of the value. Consider an oil-lease auction where a number of companies bid for the right to drill for oil in a previously unexplored area. How much oil can be extracted is a common piece of uncertainty to these bidders. Each of them may privately hire an expert to perform seismic and other geological tests. The results of these tests allow each bidder to imperfectly estimate the value of the oil lease. However, if a bidder would learn another bidder's test results, she would likely revise her initial estimate.

Wilson (1969) took the first and crucial steps towards building a theory of auctions with interdependent values. This model is sometimes referred to as the *mineral-rights model*, further analyzed nearly a decade later (Wilson, 1977). In terms of the general framework introduced in

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[5] A further result that emerges from Myerson (1981) and Riley and Samuelson (1981) concerns *the optimal auction problem*: which mechanism maximizes the seller's expected revenue? In some important cases, each of the four auction formats mentioned above is an optimal auction if combined with a carefully selected reserve price (minimum bid). See Holt (1980) and Harris and Raviv (1981) for related contributions.

Subsection 2.1, this model represents the special case in which $S = 1$ and $v_b = \beta_1 = \beta$ for all bidders b, but each bidder is uncertain about this common value. Because of that uncertainty, each bidder conditions her bid on some private information that is correlated with β. Since bidders have different pieces of private information, their estimates of β will differ. Formally, each bidder b observes a signal θ_b, with conditional probability density function $h_b(\theta_b \mid \beta)$. A higher value β leads to stochastically higher θ_b, so the signals are positively correlated, although they are independent conditional on β. This is known as the *conditional-independence assumption*.

Main Findings and Insights

Wilson (1969) provided the first equilibrium analysis of optimal bidding under common values when he derived Bayesian Nash equilibria in his mineral-rights model. A central finding is that the winner is likely to have overestimated the true value. To see this, consider again an oil-lease auction. Bidder b's signal θ_b now corresponds to her estimate of how much oil can be extracted. A high θ_b thus reflects optimism by b about the lease value. With such optimism, she will want to submit a high bid. Indeed, since the bidders' estimates have independent errors, the most optimistic bidder will win the auction. But as the θ_b are independent, the winner is likely to have overestimated the true value.

To see why, suppose there are only two *ex-ante* symmetric bidders, $B = 2$. Before submitting her bid, bidder $b \in \{1, 2\}$ observes the private signal $\theta_b = \beta + \varepsilon_b$, where β is the true value and ε_b an idiosyncratic, mean-zero noise term. When bidder b submits her bid for the oil lease, her best estimate of its value is thus $E[\beta|\theta_b] = \theta_b$. As equilibrium bids are increasing functions of these estimates, the bidder with the highest idiosyncratic signal ε_b wins the auction. This means that winning gives (qualitative) information about the other bidder's signal. If bidder 1 wins, she can safely conclude that $\theta_2 < \theta_1$. If she wins, her expected value is therefore $E[\beta|\theta_1,\ \theta_2 < \theta_1] < \theta_1$. Winning the auction can thus be "bad news" about the value of the oil lease—and it will be bad news unless bidder 1 drew a negative idiosyncratic signal (i.e., $\varepsilon_2 < \varepsilon_1 < 0$). This phenomenon has been coined the *winner's curse*.[6]

Wilson provided the first rigorous theoretical investigation of the winner's curse, and his analysis revealed exactly how the bidders in a First-price auction should optimally "shade" their bids to avoid overbidding. This requires more sophisticated strategic thinking than with private values.

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[6] Empirical evidence about the presence of the winner's curse was first presented by Capen et al. (1971) in the context of auctions for offshore oil-drilling leases. Capen et al. (1971, p.641) wrote: " ... common sense tells us, that a lease winner tends to be the bidder who most overestimates reserves potential, it follows that the 'successful' bidders may not have been so successful after all." However, this was only the very beginning of a rich empirical literature, which we discuss in Subsection 2.5.

Even if the pure common-values model conveys a number of interesting insights, it does not have much to say about the efficiency of allocations under different auction rules. Since all bidders are alike except for their idiosyncratic *ex-ante* signals, they have the same *ex-post* valuation of the object: $v_b = \beta$. This means that any auction rule is (trivially) efficient in the sense that it always allocates the object to a bidder who values it the most. As we shall see in the next subsection, this is no longer true when bidders are influenced by private-value as well as common-value components.

Extensions and Further Research

Wilson (1967) demonstrated how information asymmetries can shape equilibrium bidding in common-value auctions. He analyzed an auction where one bidder knows the common value of the auctioned object, while another bidder only has a noisy estimate. In a sealed-bid auction, the uninformed bidder must randomize her bid. The informed bidder wins most of the time and can expect to make the largest profit, but the uniformed bidder can still profitably bid for the object.

Ortega-Reichert (1968) generalized some of the results presented by Wilson (1969), and Rothkopf (1969) independently discussed the problem of common values, although he did not consider Bayesian Nash equilibrium bidding.[7] Various forms of the common-values model have been further studied by many authors, including Wilson (1977), Milgrom (1979, 1981b), Engelbrecht-Wiggans et al. (1983) and Maskin and Riley (2000). Among other things, this subsequent work found that several results are sensitive to bidder asymmetries. However, Wilson (1967, 1969, 1977) provided the necessary stepping stones for Milgrom's analysis of auctions in the presence of private-value as well as common-value components. That work is discussed next.

2.4 Private and Common Values

Independent private values (Vickrey, 1961) and pure common values (Wilson, 1969, 1977) are limiting special cases of more general information settings. Many real-world single-object auctions can be expected to involve values with private as well as common elements. Milgrom (1981b) laid the groundwork for the analysis of such hybrid auctions. In particular, he demonstrated the crucial role of the *monotone likelihood ratio property* assumption when studying strategic interactions under incomplete information. This property—which he rigorously analyzed previously (Milgrom, 1981a)—says that higher (numerically valued) signals represent

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[7] The equilibrium concept, Bayesian Nash equilibrium, was developed in generality by the 1994 Laureate John C. Harsanyi in a series of articles at the end of the 1960s (Harsanyi, 1967, 1968a,b). Note also that Wilson (1969) was available as a working paper in 1966.

relatively "better news" than lower signals: a higher realized private signal thus makes a higher state value more likely. In particular, Milgrom's monotone likelihood ratio property together with Wilson's conditional-independence assumption imply that bidders use monotonic bidding strategies—that is, a bidder's optimal bid is monotonically increasing in her signal. The property also guarantees that fulfilling a relevant first-order condition is sufficient to establish optimality of such a strategy.

Armed with these preliminaries from Milgrom (1981b), Milgrom and Weber (1982) could study an informational setting that went substantially beyond all the earlier studies: they covered essentially the whole domain laid out in Subsection 2.1. In their model, each bidder draws a privately observed signal θ_b—e.g., some information about her own cost of extracting oil—as well as a signal about state variables β that influence the common value of the auctioned item—e.g., the amount of oil available or the expected oil price over the lease horizon. Some of the state variables are potentially only known to the seller.

The values are written as $v_b = V_b(\theta, \beta)$. Milgrom and Weber (1982) postulated that the full vectors of signals θ and states β enter each bidder's valuation function V_b in a nondecreasing way—that is, variables are coded such that any higher entry weakly raises each bidder's payoff. To analyze this model, they maintained the simplifying assumptions that bidders are risk-neutral, as well as *ex-ante* symmetric in how their preferences depend on the signals. What makes the Milgrom-Weber model tick is a key assumption that the private signals and variables known only to the seller are *affiliated*. Affiliation is a form of positive correlation, which is closely related to the monotone likelihood ratio property.

In the oil-lease example, suppose a certain company obtains a more favorable private signal of the oil tract's value. Roughly speaking, affiliation means that this higher signal makes it (weakly) more likely not only that the oil tract indeed has higher value, but also that other oil companies have higher estimates. To illustrate the idea, suppose there are only two bidders, $b \in \{1, 2\}$, whose signals θ_1 and θ_2 have two possible realizations, higher and lower, that are labeled H and L, such that $\theta_1^H > \theta_1^L$ and $\theta_2^H > \theta_2^L$. If f denotes the joint probability density function of these signals, affiliation implies that $f(\theta_1^H, \theta_2^L)f(\theta_1^L, \theta_2^H) \leq f(\theta_1^L, \theta_2^L)f(\theta_1^H, \theta_2^H)$. In other words, it is more likely that the two signals have the same (high or low) value rather than opposite values.[8] Existing data appear to support the monotone likelihood ratio property and affiliation assumptions.[9]

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[8] Consider a joint distribution of a set of variables given by probability density function f (as is the case in Milgrom and Weber, 1982). Then the precise definition of affiliation is that $f(x \vee x')f(x \wedge x') \geq f(x)f(x')$ for all vectors x and x', where $x \vee x'$ are the (component-wise) maxima of x and x' and $x \wedge x'$ are the minima of x and x'.

[9] Empirical tests to investigate the validity of the monotone likelihood ratio and the affiliation properties have been developed by, e.g., de Castro and Paarsch (2010), Li and Zhang (2010), and Roosen and Hennessy (2004). These studies also provide empirical support for the affiliation assumption in their respective auction *cum* information

Main Findings and Insights

Milgrom and Weber (1982) derived precise conditions for equilibrium bidding under various auction rules in their informational setting. These findings enabled them to establish two key results. First, the English auction generates weakly higher expected revenue than the Second-price auction, and the Second-price auction generates weakly higher expected revenue than the Dutch and the First-price auctions. Second, a seller can expect a higher revenue by adopting a policy of providing expert appraisals of her own private information.

Both these results relate to the so-called *linkage principle*. This principle rests on the insight that a certain auction format generates higher expected revenue than another, whenever its prices better aggregate the private information of bidders. Consider, for example, an English auction and a sealed-bid auction. In contrast to a sealed-bid auction, the English auction reveals information about the private signals during the bidding process—e.g., when bidders decide to drop out. Because of this, the price in the English auction reflects more of the private information than the price in the sealed-bid auction. The additional affiliated information revealed to the bidders during the bidding process will mitigate the winners' curse and, consequently, encourage more aggressive bidding, leading to higher expected revenue for the seller.

Consider the first main finding on revenue ranking. As previously concluded in Section 2.2, the English and the Second-price auctions are strategically equivalent in the independent private-values model. But because bidders in the English auction learn something useful about the private signals in the affiliated values model, and there is no scope for such learning in a sealed-bid auction, the common-value element destroys the strategic equivalence between the English and the Second-price auctions in the affiliated values model. The linkage principle then assures that expected revenue is weakly higher in the English auction than in the Second-price auction. By contrast, Dutch and (sealed-bid) First-price auctions continue to be strategically equivalent. In both these auction formats, the winner only knows that the other bids (and thus the private signals of others) are lower than her own, and she does not learn how much lower they are until after the auction is over. Consequently, the English auction gives weakly more expected revenue to the seller than the Second-price auction, which in turn gives weakly more expected revenue than the First-price and Dutch auctions.[10]

A similar logic supports the intuition behind the second main finding: a seller can expect a

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environments—the first with respect to costs and the last two with respect to bids.

[10] That English auctions generate more revenue than the sealed-bid First-price auctions seems to be conventional wisdom among real-world auctioneers. However, the theoretical result assumes symmetric and risk-neutral bidders; the ranking can in principle be reversed if the bidders are risk-averse (Maskin and Riley, 1984). See Holt (1980), Milgrom and Weber (1982) and Matthews (1987) for more on auctions with risk-averse bidders. The English auction has another remarkable property: it can allocate goods efficiently even with asymmetric bidders (Maskin, 1992; Krishna, 2003).

higher revenue by revealing her private information (in a nonstrategic way) to potential buyers. The situations with and without information revelation by the seller can be thought of as two different auctions. When applying the logic of the linkage principle, if the seller shares her private information, then the price in the resulting auction aggregates more private information, compared to the case in which the seller keeps the private information to herself. The added information will mitigate the winners' curse and therefore encourage more aggressive bidding, which in turn implies higher revenues. This insight provides a theoretical foundation for the common practice to provide potential bidders with expert appraisals before the auction starts—e.g., authenticity certificates in fine arts auctions, inspection reports in property auctions, and test-drilling results in gas and oil tracts.

2.5 Theory and Data

Precise predictions about equilibrium bidding and prices in specific auction formats paved the way for empirical testing. One approach in this work has been to use rich observational data emanating from public auctions. Another approach has been to use experiments to test predictions about specific auctions. The 2002 Laureate Vernon L. Smith is a forerunner of the latter approach—for instance, in his pioneering study with Stephen Rassenti and Robert Bulfin on auction mechanisms for airport-time slot allocation (Rassenti et al., 1982).

This subsection reviews some results from the plentiful empirical work on single-object auctions. Our short review focuses on the empirical research most tightly related to Milgrom and Wilson's central contributions.

Reduced-Form Observational Studies

The aim in the early empirical literature was to test the most basic theoretical predictions with observational data. For example, Johnson (1979) used a change in the auction practices of the U.S. Forest Service to test the revenue-equivalence theorem and found higher revenues in sealed-bid auctions. But the evidence was weak, and Hansen (1986) and others have failed to find significant revenue differences between auction formats. A likely reason for the inconclusive early tests is that the auctions under consideration fail to satisfy the strict information conditions that underlie the private-values model.

In the late 1980s, Kenneth Hendricks and Robert H. Porter realized that the common-values model and the affiliated-values model provide realistic and useful foundations for empirical research. In a series of papers—especially Hendricks et al. (1987), Hendricks and Porter (1988) and Porter (1995)—they studied data from drainage-lease auctions offshore of U.S. federal land.

Hendricks et al. (1987) and Hendricks and Porter (1988) argued that “neighbor” firms, which own tracts adjacent to the one leased in a particular auction, are well-informed about lease values, whereas “non-neighbor” firms are relatively uninformed and thus will suffer more from the winner’s curse. The common-values and affiliated-values models both predict that non-neighbors are less likely to participate in an auction, and less likely to win when participating. Moreover, neighbors should make positive average profits, whereas non-neighbors should make zero profits on average but negative profit conditional on neighbors not bidding. Relying on reduced-form econometric models, the researchers found convincing evidence that these specific predictions hold up in oil and gas auctions. A further observational study by Porter (1995) exploited data on amounts of extracted oil, industry costs, and the price of oil to construct *ex-post* estimates of the winners’ returns. Porter could thus directly gauge the winner’s curse and test various predictions of the Laureates’ models.

Studies such as these convinced the research community that private information is highly relevant in the study of auctions, and that the game-theory models provide credible explanations for observed bidding in auctions. Hendricks and Paarsch (1995) presented a comprehensive survey of the early empirical work on auctions.

Structural Observational Studies

More recently, empirical researchers have adopted a structural approach—i.e., the researcher assumes the theory to be true and seeks to deduce unobserved bid functions and valuation distributions from observed bidding behavior. Of course, this involves making specific assumptions about functional forms, as well as about bidder expectations and preferences. For example, the analyst may assume that bidders’ expectations satisfy Bayesian equilibrium conditions and that preferences satisfy risk neutrality. The advantage of the structural approach is that one can explore counterfactuals. For example, what would happen with a sudden shift in valuations or under a different auction format?

New structural-estimation techniques were developed by Laffont and Vuong (1993) and Paarsch (1992, 1997). These enabled Laffont et al. (1995) to use prices from an ascending auction of eggplants in France, as well as other agricultural products, to estimate bidder-value distributions. Dozens of other researchers have used structural estimation to analyze auctions—see, for example, Athey and Haile (2006) and Athey et al. (2011).

Experimental Studies

In laboratory experiments, an analyst can control the valuations of the participants. By comparing the bids observed in the lab with those predicted by the theory, she can precisely test her model's predictions. Early experimental work focused on testing basic predictions. For example, Coppinger et al. (1980) and Cox et al. (1982) tested the revenue-equivalence theorem in private-values settings and showed that it failed to hold. Furthermore, while the strategic equivalence between the English the Second-price auctions seems straightforward in theory, subjects in laboratory experiments more easily arrive at their dominant strategies in the context of an English auction than in the context of a Second-price auction (Kagel et al., 1987).[11]

Some experimental findings robustly support the theory. For example, Bayesian Nash equilibrium reliably predicts the direction of change in behavior from changes in the environment (see, e.g., Cox et al., 1982, 1988). Experimental work has also demonstrated that some subjects frequently fail to play equilibrium strategies. For example, the winner's curse is quite prevalent, at least for relatively inexperienced bidders (see Kagel and Levin, 1986). In itself, this finding confirms a central insight of game theory, that in general there is no compelling reason to expect behavior to converge immediately to equilibrium (see Weibull, 1995). Equilibrium theory is primarily a theory of players who have either had an opportunity to learn from experience or an opportunity to otherwise coordinate their expectations.[12]

Kagel (1995) and Kagel and Levin (2014) survey the experimental research on auctions.

3 Multi-Object Auctions

Real-world auctions have increasingly been used to sell multiple objects at the same time. One type of multi-object auction involves divisible objects such as government debt and electricity. Another type involves nonidentical multiple objects, which may be either complements or substitutes, such as radio frequencies or bus routes. In practice, the line between the identical and nonidentical object categories can be blurry. For example, electricity is homogeneous and finely divisible at its source, but the cost of delivering electricity at one location may, for example, depend on how much that is delivered at other locations.

In this section, we first describe auctions of divisible objects (Subsection 3.1). We then turn to the more complicated case of heterogeneous, interrelated objects (Subsection 3.2). There, we mainly focus the attention on spectrum auctions. These involve exceptionally large values, and governments face a challenging trade-off between raising revenue and allocating the spectrum efficiently.

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[11] Auction theorists such as Ausubel (2004) have sought to express this intuition, and recent work pinpoints differences in the degree of "obviousness" of dominant strategies (Li, 2017).

[12] Of course, auction theory will have different predictions if participants fail to act rationally when they place their bids. The 2017 Laureate Richard Thaler provided an early discussion of this possibility—in the context of the winner's curse—in the very first Anomalies article published in the *Journal of Economic Perspectives* (Thaler, 1988).

In the process of resolving this problem, this year's Laureates and their collaborators have used theory to address a range of practical issues. As a result, they have significantly reduced the barriers to trading interrelated objects. We end by describing some of the auction formats the Laureates and others have invented (Subsections 3.3 and 3.4).

3.1 Share Auctions

Wilson (1979) formulated the seminal common-values model for the auctioning of shares. First, each bidder observes a private signal of the true object value. Subsequently she specifies a price for each possible share of it—effectively, a bid is thus equivalent to a demand schedule. Wilson considered both uniform-price auctions, where the same price is charged for each share of the object, and discriminatory (pay-as-bid) auctions. He also made the first attempt to rank different auction types and formats, in terms of the revenue they raise.[13] His model is particularly suitable for studying auctions of public debt.

Main Findings and Insights

The model by Wilson (1979) conveys two main insights. First, when seeking to purchase multiple units, a bidder knows that the price she offers for marginal units may affect the price that she pays for inframarginal units. Compared to single-object formats, a bidder thus has an additional motive to "shade" the bids below her trues value. Second, due to the rich strategy space, bidders can sometimes coordinate on several different equilibria, some of which yield very low profit for the seller.

Extensions and Further Research

Building on Wilson's insights about share auctions, Klemperer and Meyer (1989) developed a general model of competition through supply functions, where the bidders are sellers rather than buyers. At the time, in the late 1980s, procurement auctions with supply-function bids became increasingly important as several countries privatized their energy markets. Like Wilson's model, Klemperer and Meyer's model also had multiple equilibria, but they observed that adding particular forms of uncertainty would select a unique equilibrium. Subsequent research by von der Fehr and Harbord (1993) and Kremer and Nyborg (2004) in supply-function and demand-function settings, respectively, showed that uniqueness also follows if bid functions are required to be sufficiently discrete. Like the presence of uncertainty, such discreteness is empirically justified. In

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[13] The allocation mechanism is the same for both these two formats, since each bidder is allocated the demanded fraction of the good at the equilibrium price, but the payment differs.

the context of electricity markets, it turns out that bid caps and capacity constraints can heavily influence equilibrium outcomes (for a survey, see Holmberg and Newbery, 2010).

Wilson (2002) offered a fascinating account of his engagement with the practical design of electricity markets, which raised new theoretical issues. He also contributed to resolving these new problems by way of careful formal analysis. For example, Wilson (2008) characterized a supply-function equilibrium in an auction market, which is realistically constrained by limited capacities of links in a transportation network and by limited input/output capacities of participants.

Realistic generalizations of the share-auction model thus imply that multiplicity problems may be less severe than Wilson's original analysis suggested. His concern about multiplicity of equilibria nonetheless proved prescient, as the problem reappears with even more consequence, once we turn our attention to more general multi-object auctions (see Section 3.2).

Empirical Work

By now, auctions in electricity markets as well as bond markets have been the subject of rich empirical research. Broadly speaking, the data support the presence of bid shading, with degrees of shading reflecting participants' market shares. In electricity markets, Holmberg and Newbery (2010) conclude that markups are typically small in the wake of much spare capacity, but get dramatically larger when capacity is a binding constraint. These are also robust features of theoretical models.

In bond markets, large actors and actors with potential information advantages also seem to have some market power. But these effects are small (see Hortaçsu et al., 2018, and the references therein). It is also noteworthy that the preferred bond-market model remains the original share-auctions model, generalized only to account for realistic bid-discreteness along the lines proposed by Kastl (2011). Hortaçsu and McAdams (2018) investigated treasury auctions in Turkey. Their main finding is that a (counterfactual) switch from a discriminatory auction to a uniform-price auction would not significantly increase revenue.

3.2 Auctioning Interrelated Objects

In the early 1990s, an explosion of the demand for mobile communication made the U.S. federal government decide to use an auction for allocating radio-spectrum licenses among telecommunication firms. Previously, the U.S. Federal Communications Commission (FCC) had only been allowed to rely either on administrative procedures—commonly referred to as "beauty contests"—or on lotteries. These methods had notably failed in a number of complex settings, at the expense of both taxpayers and end-users (Binmore and Klemperer, 2002; Cramton, 2013; Kasberger, 2020; Klemperer, 2002; McMillan, 1994). The obvious alternative is to adopt an auction to assign licenses. In fact, as early as in the 1950s, the 1991 Laureate Ronald H. Coase argued that the basic principle should be to allocate objects, such as broadcasting licenses, to the firms who will make the most efficient use of them, and the best way to identify these firms is to assign the objects through a competitive price mechanism (Coase, 1959).

Efficiency and Social Value

Following the FCC policy shift, multi-object auctions turned from an esoteric topic at the fringe of microeconomic theory to a hot research topic almost overnight. The stated policy intention was to achieve an "efficient and intensive use of the electromagnetic spectrum" (U.S. Congress, 1993, p.82), with revenue as a secondary objective only (see McMillan, 1994, p.147).[14] The main argument for using auctions reflects two types of efficiency concerns. First, assigning the objects to the most productive suppliers avoids (or saves on) costs in a secondary market for object reassignments.[15] Second, generating funds through markets rather than taxation avoids costly tax distortions. Considering different countries and times, economists have found expected social deadweight losses in the range of 0.17 to 0.56 dollars per tax dollar raised; see, e.g., Ballard et al. (1985) or Feldstein (1999) and the references therein.

However, the fact that regulators and governments can avoid tax distortions by generating high revenues does *not* imply that an auction should aim to maximize revenues. Using maximal revenue as an objective for an auction of something like a set of spectrum licenses—or as a measure of the auction's "success"[16]—may be too short-sighted and lead to a monopoly market. A higher degree of competition in the market will likely generate higher long-run government revenues and higher long-run welfare due to better and cheaper services.

Consequently, both revenue-minimizing beauty contests and revenue-maximizing auctions have been rejected by economists and policy makers on the grounds of public welfare. The "intermediate case" is not easy to address, and designing an auction to obtain efficient outcomes is a nontrivial task—for example, due to problems with strategic misrepresentation, shill bidding, common-value components, collusion, budget restrictions[17], or externalities; see, e.g., Ausubel

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[14] Later, governments in Europe would similarly seek to allocate spectrum licenses so as to maximize social welfare; see, e.g., EU Directive (2002) and Kasberger (2020).

[15] In fact, well-known results in mechanism design theory show that no bargaining protocol can correct initial misallocations. More precisely, not even costless resale can normally ensure efficient outcomes in the wake of incomplete information (Cramton et al., 1987; Jehiel and Moldovanu, 1999; Myerson and Satterthwaite, 1983).

[16] See, e.g., Hazlett and Muñoz (2009) or Hazlett et al. (2012) for a critical analysis of using revenues as success criteria.

[17] Bidders may be budget constrained and sometimes they may even have to rely on external funding. Such financial constraints affect bidding strategies and, therefore, also the outcome of the auction. In a seminal paper, Che and Gale (1998) analyzed several standard auction formats when bidders have absolute or flexible spending

and Milgrom (2002a), Hazlett and Muñoz (2009), Hazlett et al. (2012), Jehiel and Moldovanu (2003) and Klemperer (2002). Some of these problems are discussed next.

Challenges in Multi-Object Auctions

In view of the efficiency focus, a seemingly natural solution is to use the Vickrey-Clarke-Groves (VCG) auction (shorthand for the publications Vickrey, 1961; Clarke, 1971; Groves, 1973). This auction is a generalization of the Second-price auction and it is known to be efficient in the private values framework. The VCG auction allocates objects according to the maximal sum of bids and charges each bidder b the externality she imposes on the other bidders—the total value of the auction for all other bidders if b is absent minus their total value if b is present. However, even if the VCG auction is efficient in the private-values framework, the VCG auction may result in inefficient outcomes in more complex settings as illustrated in the following two examples.[18] The first example also shows how buyers may engage in shill bidding (bids are given under alternative names with the sole purpose of manipulation). The second example shows how a VCG auction is susceptible to collusion.

Example 1. *Two objects* $(A$ *and* $B)$ *are auctioned to two bidders* $($*called 1 and 2*$)$*. Bidder 1 regards objects* A *and* B *as complements and is willing to pay 2 units for the pair, but only 0 for each individual object. Bidder 2 on the other hand is willing to pay 1 unit for the pair* (A,B) *and 0.5 for each of them individually. If both bid truthfully, the VCG auction assigns bidder 1 the pair* (A,B)*—as this assignment maximizes the total value of the auction—at a total price of 1, which is the externality imposed by bidder 1 on bidder 2.*[19] *This outcome is efficient.*

But suppose that losing bidder 2 misrepresents his values and bids 2 for object A *and 0 for object* B*. In addition, bidder 2 masquerades as an additional "bidder 3" willing to pay 2 units for object* B*, and 0 for object* A*. Now, bidders 2 and 3—i.e., bidder 2 in reality—are assigned one object each at a price of 0. A clever misrepresentation and shill bidding thus convert losing bidder 2 into a winner and an efficient allocation into an inefficient one.* $\square$

Example 2. *Two objects* $(A$ *and* $B)$ *are auctioned to three bidders* $($*called 1, 2 and 3*$)$*. Bidder 1 is identical to bidder 1 in Example 1. Bidders 2 and 3 are willing to pay 0.5 units for each*

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limits.

[18] Both examples are from Ausubel and Milgrom (2002a), which in turn drew from a report to the FCC by Charles River Associates and Market Design Inc. from 1997.

[19] Formally, this externality is defined as the difference between the total value of the auction if bidder 1 is absent, so bidder 2 instead is assigned the pair (A, B) at value 1, and the total value of the auction for bidder 2 if bidder 1 is present, so bidder 2 is assigned nothing at value 0. Consequently, bidder 1 is charged $1 - 0 = 1$. Note also that bidder 2 is charged zero because the value of the auction for bidder 1 equals 2 whether bidder 2 participates or not—i.e., bidder 2 does not impose any externalities on bidder 1.

individual object but 0 for the pair (A,B). *Truthful bidding again assigns* (A,B) *to bidder 1 at a total price of 1. But if bidders 2 and 3 collude, such that bidder 2* $($*bidder 3*$)$ *bids 2 for object* A $($*object* $B)$, *and 0 for object* B $($*object* $A)$, *they will be assigned one object each and pay 0. A clever misrepresentation and sophisticated collusion thus convert losing bidders 2 and 3 into winners and an efficient allocation into an inefficient one.* □

The source of the problems illustrated in these examples is that bidder 1 regards the two objects as *complements*. If bidder 1 had instead regarded them as substitutes (say, she is willing to pay 1 for each individual object), the problems would disappear. But in practice the complementarity problem is too prevalent to ignore. In spectrum auctions, bidders generally prefer combinations of complementary licenses (see Ausubel et al., 1997). For example, phone-service providers often seek to cover large areas, and so they prefer licenses for adjacent geographical regions. The FCC worried about other challenges as well. While bidders have different objectives and financial resources, the number of prospective bidders was uncertain. Furthermore, basic computational issues arose. With many licenses for sale, bidders could not be expected to bid for all subsets of potential objects.

Early Designs

Early multi-object auction designs largely abstracted from these problems. One example is the static combinatorial auction proposed by Rassenti et al. (1982) for airport-time slot allocation. Another is the theoretically influential *menu auction* by Bernheim and Whinston (1986). The latter is a one-round auction format where bidders submit bids for any package of objects and the auctioneer selects an allocation of the objects to maximize total revenues, given that prices are determined by a pay-as-bid rule. Like all static auction formats, the menu auction forces each bidder to make quite uninformed guesses about other bidders' valuations and bids. In fact, even in the single-object case, sequential designs tend to be preferable in the presence of common-value components. With multiple objects, multi-round designs have the additional advantage that they allow each bidder to learn which packages are likely to be relevant and thus to economize on bidder-evaluation efforts.

The combination of so many new issues compared to single-object auctions implied that the standard auction theory discussed in Section 2 could only provide basic insights. Success would require sound intuition to fill theoretical gaps that would likely remain open for a long time. The lacuna of knowledge also called for new auction formats and rules, some of which are discussed in the remaining part of this section.

3.3 Multiple Round Auctions

This subsection discusses three new auction formats, which were invented largely as a response to the aforementioned explosion in the demand for frequencies on the radio spectrum, and the FCC's policy shift to sell batches of spectrum bands via auctions.

Simultaneous Multiple Round Auctions

For the 1994 FCC auction, the final version of the newly designed auction was the *Simultaneous Multiple Round Auction* (SMRA), sometimes referred to as the *Simultaneous Ascending Auction* (SAA). This design was based on two detailed proposals, one by Milgrom and Wilson, and the other by Preston McAfee.[20] The SMRA format allows bidders to place bids on an arbitrary number of objects over multiple rounds. The multi-round feature reduces the winner's curse because information is revealed during the auction—an application of the linkage principle discussed in Subsection 2.4.

To prevent bidders from passively waiting for others to bid, the SMRA design included so-called *activity rules*. These rules induce every bidder to make a credible attempt at placing the highest bid for at least some object in every round. Specifically, at the first round, prices are set sufficiently low that all objects are in excess demand. In each round, bidders raise their bids by an integer number of increments on any object they would like to buy. At the end of each round, a "provisional winner" is determined for each object. The process repeats itself until the excess demand is eliminated and bidding has stopped for all objects. At that point, bidding closes and the provisional winners in the last round are assigned the objects and pay their current bids.

The 1994 FCC spectrum auctions raised some $20 billion for the U.S. federal government, twice the forecasted amount. This outcome attracted considerable media attention and led other governments to set up their own auctions. The U.K. 3G spectrum auction that concluded in 2000 raised about $34 billion for the British government (Binmore and Klemperer, 2002). The SMRA auction format became the dominant design for spectrum sales worldwide, and versions of it have been used in Canada, Finland, Germany, India, Norway, Poland, Spain, Sweden, the U.K., and the U.S. These auctions have generated hundreds of billions of dollars for governments worldwide. For more details, see Bichler and Goeree (2017), Cramton (1997), Cramton (2006), Goetzendorff et al. (2018), Klemperer (2004), and Milgrom (2004).

While the SMRA has generally worked well, the format does have some well-known weaknesses (discussed in detail by Ausubel and Milgrom, 2002a; Cramton, 2013). For example, just

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[20] The foreword to Milgrom (2004) by Evan Kwerel provides an account of some of the early work on FCC auctions and some later developments. See also Cramton (1997) and McAfee and McMillan (1996).

as in share auctions, large bidders have incentives to engage in demand reduction.[21] Another issue, known as the *exposure problem*, derives from the fact that the SMRA does not allow package bids (although extensions allowing package bids were proposed by, e.g., Ausubel and Milgrom, 2002a). Therefore, a bidder looking to buy some complementary objects runs the risk of not winning all of them, while being forced to buy only some of them.

Combinatorial Clock Auctions

Several new designs were invented to deal with some of the problems with the SMRA auction, especially how best to accommodate package bidding. An important step forward was the development of the *Combinatorial Clock Auction* (CCA), first described by Milgrom, working together with Lawrence M. Ausubel and Peter C. Cramton (Ausubel and Milgrom, 2002a,b; Ausubel et al., 2006).[22]

A CCA has two main stages:[23]

1. *The Allocation Stage,* with two sub-stages, the *clock stage* and the *supplementary stage*. The clock stage, in turn, consists of multiple rounds. In each round, the auctioneer announces prices for all individual objects and bidders respond with a single bid for one package of objects. Prices increase until there is no excess demand for any object. The supplementary stage is a sealed-bid auction process in which bidders can improve their bids from the clock stage and submit additional bids for other combinations of objects. Throughout the allocation stage, all bids are all-or-nothing package bids.

2. *The Assignment Stage*. When all bids from the clock and supplementary stages are submitted, the winners and payments are determined in the "winner determination problem." The solution to this problem selects the value-maximizing combination, subject to some feasibility constraints. Prices are then calculated for each winning bidder using a Second-price rule.

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[21] Kagel and Levin (2001) experimentally investigated incentives for large bidders to engage in demand reduction. They found clear evidence in (sealed-bid) uniform price auctions, but with substantially more demand reduction in English auctions.

[22] Some early and additional aspects of the CCA auction are described in Ausubel and Milgrom (2001), Ausubel et al. (2002), Milgrom (2004), and Porter et al. (2003). Later descriptions and analyses of the CCA auction are provided by Day and Raghavan (2007), Day and Cramton (2012), Day and Milgrom (2008), and Day and Milgrom (2013).

[23] If objects can be treated as good substitutes, there is often a third stage of bidding in the CCA auction. In spectrum auctions, for example, bidders can simply indicate the desired quantity of "generic spectrum" (an amount of frequencies in a given region). Then, the third stage determines the assignment from generic spectrum to physical frequencies. Consequently, bidders only have to evaluate generic spectrum in the bidding process which simplifies bidding and economize on bidder-evaluation efforts. See, e.g., Ausubel and Baranov (2014) or Cramton (2013) for detailed discussions.

The CCA differs from the SMRA in two main respects. The CCA is a combinatorial auction that allows bidders to place package bids. Moreover, it does not identify provisional winners at the end of each round. As a consequence, only aggregate information about highest standing bids and excess demand is revealed after each round. But these are the only bits of information needed by the bidders to form expectations about prices and to resolve common-value uncertainty.

As argued by Cramton (2013), the CCA has advantages over the SMRA. First, the exposure problem is eliminated: though a given pair of objects may be substitutes for one bidder but complements for another, bidders are allowed to place bid on packages. Second, the CCA eliminates incentives for demand reduction and for most gaming behavior. This is accomplished through an effective pricing rule (a version of the VCG auction that generates core payoffs), well-designed activity rules, and the fact that only aggregated information is revealed to bidders.

Even though the CCA potentially solves problems, it also has some drawbacks. Small bidders tend to obtain disproportionately smaller surplus and may be deterred from entering the auction. Bids that are unlikely to affect a bidder's own outcome can nonetheless influence the price another bidder pays. As a result, some participants may either refrain from making bids that would primarily benefit other bidders or the seller ("the missing-bid problem"), or else bid maliciously to make other bidders pay more (see Bichler et al., 2013; Levin and Skrzypacz, 2016). Unlike the single-object Second-price auction that inspires it, the CCA does not make truthful bidding weakly dominant. Moreover, it tends to have multiple equilibria, some of which are inefficient.

After the adoption of CCA for selling radio-spectrum licences in the U.K. in 2008, many countries followed suit, including Austria, Australia, Canada, Denmark, Ireland, the Netherlands, Romania, Slovakia, and Switzerland. For more details, see Bichler and Goeree (2017), Cave and Nichols (2017), Goeree and Holt (2010), and Monchon and Saez (2017).

Incentive Auctions

Milgrom led a team of economists advising the FCC to repurpose radio spectrum from broadcast television to wireless broadband services (see Milgrom et al., 2012). The resulting new *Incentive auction* was adopted by the FCC in 2017. This design combines two separate but interdependent auctions. The first is a *reverse auction* that determines a price at which the remaining over-the-air broadcasters voluntarily relinquish their existing spectrum-usage rights. The second is a *forward auction* of the freed-up spectrum. In 2017, the reverse auction removed 14 channels from broadcast use, at a cost of $10.1 billion. The forward auction sold 70 MHz of wireless internet licenses for $19.8 billion, and created 14 MHz of surplus spectrum. The two stages of the incentive auction thus generated just below $10 billion to U.S. taxpayers, freed up considerable spectrum for future use, and presumably raised the expected surpluses of sellers as well as buyers.

The complexity of the underlying economic problem was a particularly demanding aspect of the reverse auction design. The television stations remaining on air after the auction (about 95% of the pre-auction stations) needed to be "repackaged" in a way that satisfied more than a million constraints (Milgrom and Segal, 2017), a computationally difficult problem. The ultimate success raises the hope that an efficient reallocation of network capacity can be implemented through voluntary market solutions in other applications as well.

Practical Implementation

There is no silver bullet in auction design across contexts and market-specific details. However, the SMRA and the CCA formats both remain in widespread use. According to Koutroumpis and Cave (2018), dozens of countries have now used some version of them to allocate frequency bands.

Not all of these auctions have lived up to what they promised (see, e.g., the discussions in Bichler and Goeree, 2017; Binmore and Klemperer, 2002; Cramton and Schwartz, 2000; Doraszelski et al., 2019; Hazlett et al., 2012; Klemperer, 2002). A possible reason is that details of the auction design were flawed—e.g., excessively high reserve prices, poorly designed bidding credit schemes, and inflexible packages. But even with the right auction design, spectrum auctions may fail to fulfill their potential because of poor planning, inaccurate predictions, and various timing issues. For example, too long a time window between the decision to run an auction and the auction start may provide opportunities for collusion among participants, implementation of entry-deterrence strategies, and changes in market structure.

Problems associated with an auction may also reflect conflicting interests in society and politics. For example, private firms may manage to lobby regulators into tweaking auction-design details to serve firm interests. Alternatively, there may be agency problems between, on the one hand, the private sector and, on the other hand, powerful regulators or politicians who act in their own private interest. It is also conceivable that an academic auction designer may give advice where the public interest is compromised in favor of her private interest. Indeed, many of the major telephone companies relied on the advice of auction theorists already in the 1994 FCC auction—something which is described in detail by McMillan (1994). Feltri (2020a,b) and Weyl (2020) critically discuss the role of academic auction designers in the context of the 2017 Incentive auction (see Milgrom, 2020, for a detailed reply to Feltri and Weyl).

While these kinds of problems may limit the social gains from sophisticated new auction formats, the same underlying problems would presumably manifest themselves even more strongly under alternative modes of allocation. Carefully designed and planned auctions may thus still allocate part of the radio spectrum (and other publicly controlled resources) more efficiently than traditional administrative procedures and lotteries. At best, those primitive allocation mechanisms transfer public wealth to private hands without other distortions than losses of public revenue. At worst, inefficient secondhand markets or political distortions prevent the objects from being matched with the owner who can make the best use of them.

3.4 Other Prominent Auctions

While this year's Laureates have played a major role in inventing the new auctions mentioned in Subsection 3.3, several influential designs for the sale of multiple items—other than radio spectrum—have been developed by others. This brief subsection mentions a couple of these.

Product-Mix Auctions

A prominent example of a new format that allows package bidding is the *product-mix auction*, a single-stage mechanism designed by Paul Klemperer in response to the 2007 Northern Rock bank run (see Klemperer, 2010). Like SMRA, this auction identifies equilibrium allocations consistent with the preferences revealed by the participants' bids. While sacrificing the price discovery associated with multi-stage mechanisms, it is less vulnerable to collusion and much faster than SMRA.

The product-mix auction is thus better suited for financial markets where instant market clearing is an important concern. The Bank of England has successfully continued to use this auction format to sell batches of troubled debt.

Position Auctions

Another example of new designs for the auction of interrelated objects is the so-called *position auction*. This class of auctions includes, for example, the generalized Second-price auction by Edelman et al. (2007) and Varian (2007). In terms of important real-world applications, Google has relied on this kind of format for selling keywords in Internet searches.

4 Impact on Other Fields

In this penultimate section, we shift the focus from research on auctions to research in other fields. We first touch upon how the findings in auction theory—especially those made by Milgrom and Wilson—have enriched other fields. Then, we briefly account for a few of the most important contributions the Laureates have made in fields adjacent to auctions.

Spillovers from Auction Theory

In addition to the direct findings discussed in Sections 2 and 3, auction theory has had substantial indirect impacts on other research areas. Three instances, out of many such spillovers, are mentioned below.

First, the analysis of bidding behavior in particular auction settings has been extended to the analysis of more general mechanisms for trade. In this way, insights from auction theory have helped unify the analysis of different trading institutions. For example, the problem of determining an optimal reserve price in a single-object auction turns out to be analogous to finding a seller-optimal trading mechanism in the case of a single buyer. Less obviously, the best outcomes that can be attained in bilateral bargaining between privately informed parties are tightly connected to equilibrium outcomes of double auctions (see below), where both the buyer and the seller submit bids.

Second, the notion of affiliation and related concepts defined, refined, and popularized by Milgrom and his coauthors—such as the monotone likelihood ratio property and supermodularity[24]—have become general tools for finding equilibria and for carrying out comparative statics. Therefore, these tools have become important also in general information economics and organizational economics, as well as other fields, especially when incomplete information and/or complementarities play important roles in the application at hand.

Third, models of auctions have demonstrated the empirical relevance of game theory in well-defined situations where players, strategies, and outcome functions are easy to identify. This has strengthened the hope that game theory may help us understand other social phenomena, if only we can clarify the essentials of the situation that people face: their preferences, their information, their understandings, and their possible actions. While it is hard to speculate about counterfactuals, game theory's reshaping of social science would in all likelihood have been slower without its successful application in the auction domain.

Information in Securities Markets

A long-standing question in economics is to what extent market prices serve to aggregate information, which is incomplete and dispersed among economic actors. Wilson (1977) found conditions under which the price in a sealed-bid auction with common values approaches the true value as the number of bidders grows large. These results were extended by Milgrom (1979).

Double auctions are two-sided markets, where many buyers and many sellers are treated symmetrically, as in stock markets. These types of auctions were first studied by Chatterjee

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[24] For detailed analyses of supermodularity, see Milgrom and Roberts (1990), who build on Topkis (1978), Bulow et al. (1985) and Fudenberg and Tirole (1984).

and Samuelson (1983), but they restricted their attention to the case of a single buyer and a single seller. In a pioneering study, Wilson (1985) considered multiple buyers and sellers and showed how—with a proper design of the market—the true value of a security can be efficiently discovered through the interaction of market participants with idiosyncratic information.

The celebrated *no-trade theorem* by Milgrom and Stokey (1982) says that without efficiency motives, there can be no speculative trade based on asymmetric information. The result may appear counterintuitive. Surely, someone who has favorable information about the value of some security would want to buy it from traders with less (or no) information? However, that intuition ignores the (Bayesian) inference about the value of the security that a less-informed trader would draw from the informed trader's willingness to buy. When one properly accounts for this inference, the less-informed trader's value of the security goes up to such an extent that it eliminates any perceived gains from trade.

Glosten and Milgrom (1985) formulated a pioneering model of financial markets under adverse selection. Specifically, they introduced a framework of sequential trading to study how a market maker who faces a pool of differently informed traders will dynamically set prices of a security. Such a market maker optimally adjusts bid and ask prices with the goal of breaking even. Glosten and Milgrom found that a market maker who wants to protect herself against adverse selection among traders has to create a bid-ask spread, which serves as a tax on buying and selling by uninformed traders. Their analysis explains how adverse selection in financial markets reduces liquidity.

Matching Theory

Auctions make use of price mechanisms, but societies sometimes refrain from using such mechanisms on distributional or moral grounds. The field of matching theory, which was the subject of the 2012 Prize in Economic Sciences awarded to Alvin E. Roth and Lloyd S. Shapley, deals with this topic. Milgrom is a leading explorer of the linkages between auction theory, matching theory, and general-equilibrium theory. For example, building on previous work by Kelso and Crawford (1982), Hatfield and Milgrom (2005) introduced a general model of *matching with contracts* that encompasses several known two-sided matching models and auction models (see also Andersson and Svensson, 2014; Echenique, 2012; Fleiner, 2003). This work sparked new applications of market design, including regionally capped residency matching of doctors and hospitals (Kamada and Kojima, 2012), and cadet matching to military specialties (Sönmez, 2013; Sönmez and Switzer, 2013).

Utility Pricing

Wilson derived a set of important results on public-utility pricing, which have been practically important to regulators. Many of the findings are summarized in his book *Nonlinear Pricing* (Wilson, 1993), which offers a comprehensive analysis of tariff design in industries such as electricity, telecommunications, and transportation. As the title suggests, a common theme is the desirability of nonlinear pricing. Such pricing means that the customers—say, of an electricity or a phone service—do not face a tariff strictly proportional to the quantity purchased. Instead, they are offered a menu of quantities with corresponding two-part charges. Such tariffs contribute to efficient resource use by balancing cost recovery of the regulated utility and exercise of monopoly power against rewards for customer loyalty and other social goals, such as cheaper services for specific consumer groups.

Game Theory

Auctions usually involve asymmetric information, and many auctions are sequential. To formally model such situations, general solution concepts had to be developed for extensive-form games with incomplete information, concepts which could also be applied outside the field of auctions.

Kreps and Wilson (1982b) proposed the first successful solution concept: *sequential equilibrium.* The idea here is to require every player's behavior to be sequentially rational: i.e., each player's action is optimal at any decision node, given her beliefs and the strategies of other players. Sequential equilibrium has found applications far beyond auctions, especially to model the role of reputations for toughness or benevolence in predatory pricing, price wars, and other market battles.

The most well-known application is the *chainstore game*, where an incumbent monopolist has branches in a number of different towns and potential local competitors can enter markets in sequential order. Upon entry, the incumbent can decide whether to fight and set a low price (creating a loss for both parties), or accommodate and set a high price (yielding a positive profit for both). The 1994 Laureate Reinhard Selten observed that the outcome under perfect information is entry in each market (Selten, 1978). The logic is simple: because the incumbent has no reason to fight in the last market, entry will occur in that market. But knowing this, the incumbent has no reason to fight in the second-last market, and so on. This is known as the *chain-store paradox.*

Kreps and Wilson (1982a) showed that the paradox hinges crucially on the perfect-information assumption. Even a slight uncertainty about the incumbent's willingness to fight (the incumbent is "strong") can suffice to deter entry. In a sequential equilibrium, potential entrants in early markets refrain from entering because they anticipate that even a weak incumbent prefers to fight rather than accommodate and reveal weakness. Indeed, there cannot be an equilibrium where only a strong incumbent is prepared to fight early on: if one round of fighting would suffice to deter all future entry, a weak incumbent would definitely fight if challenged! Using a closely related argument, Kreps et al. (1982) demonstrated how small informational asymmetries can generate cooperative behavior in a finitely repeated prisoners' dilemma.

Both Milgrom and Wilson have continued to work on the foundations of game theory throughout their careers. Recent examples of their persistent research to find justified refinements of the Nash equilibrium concept include Govindan and Wilson (2009) and Milgrom and Mollner (2018).

5 Conclusion

For five decades—and still counting—this year's Laureates in Economic Sciences have produced research results that have deepened our understanding of how auction markets function in the presence of private information. Their findings have allowed trained analysts to design new auctions and practitioners to choose more wisely among existing auction formats. Milgrom and Wilson have brought theory and practice tightly together, as few other economists have done. In sum, their research on auction theory and auction design has been instrumental in gradually replacing a process of intuitive trial and error with a more rigorous scientific approach.

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