Economics 703
Advanced Microeconomics **Prof. Peter Cramton**

Lecture Note 2: Dynamics[1]

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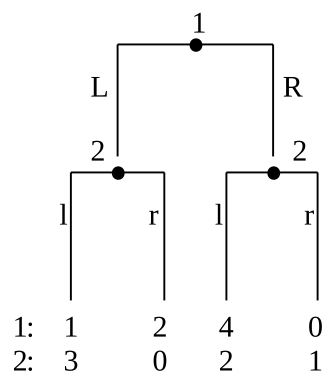
A. Theory

1. Subgame Perfection (Selten, 1965)

The central idea of a Nash equilibrium is that each player must act optimally given the other players' strategies, i.e., play a best response to the others' strategies. Nash, however, only applies this optimality condition at the beginning of the game. Hence, some Nash equilibria of dynamic games involve incredible threats. Consider the game below.



The pure strategy sets are S_1 = {L,R} and S_2 = {ll,lr,rl,rr} where xy means player 2 plays x following L by player 1 and y following R. The normal form is:

[1]These notes are based in large part on notes by Robert Gibbons, MIT.

		2			
		ll	lr	rl	rr
1	L	1, $\underline{3}$	$\underline{1}$, $\underline{3}$	2, 0	$\underline{2}$, 0
	R	$\underline{4}$, $\underline{2}$	0, 1	$\underline{4}$, $\underline{2}$	0, 1

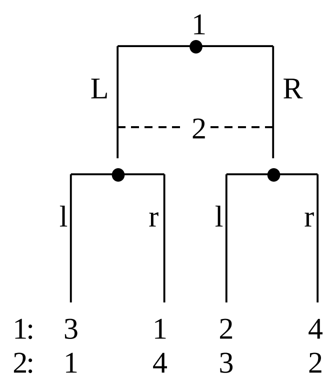
Three Nash equilibria in pure strategies: {R,ll}, {L,lr}, and {R,rl}, but the last two of these involve incredible threats. If it makes sense to require the players to act optimally at the beginning of the game, shouldn't we require that they act optimally at every point of the game (at each information set)? This is the idea of subgame perfection.

Backwards recursion (familiar from dynamic programming and decision analysis) determines credible behavior in finite-horizon extensive-form games. Player 2's only credible behavior is ll, since 3 is better than 0 and 2 is better than 1. Hence, {R,ll} is the only equilibrium that is free of incredible threats. Selten (1965) formalized this idea for games of perfect information (games in which each information set contains only one node).

Consider a game Γ of perfect information consisting of a tree T linking the information sets $i \in I$ (each of which consists of a single node) and payoffs at each terminal node of T. A *subtree* T_i is the tree beginning at information set i, and a *subgame* Γ_i is the subtree T_i and the payoffs at each terminal node of T_i.

Def: A Nash equilibrium of Γ is *subgame perfect* if it specifies Nash equilibrium strategies in every subgame of Γ. In other words, the players act optimally at every point during the game.

How does this extend to games of imperfect information, like the following game?



In a game of imperfect information, each information set consisting of a single node determines a subgame. Hence, there are no (proper) subgames in the previous example. Selten (1975) and Kreps and Wilson (1982) extend the spirit of subgame perfection to games of imperfect information by allowing subgames to start at entire information sets.

2. Repeated Games

Stage game: $G = \{A_1,..A_n; u_1,...,u_n\}$

outcome of G: $a = (a_1,...,a_n) \in A = A_1 \times ... \times A_n$

repeated game G(T): G is repeated T times

a^t outcome of t^{th} repetition of G

history prior to t^{th} repetition: $h^{t-1} = (a^1,...,a^{t-1}) \in A^{t-1}$

strategy for player i in G(T) is $s_i = (s_i^1,..., s_i^t,..., s_i^T)$,

where $s_i^t: A^{t-1} \rightarrow A_i$ maps the history into an action.

Stage-game payoffs: $\{u_i(a^1),...,u_i(a^T)\}$

Payoffs for G(T):

1. average $$U_i(h^T) = \frac{1}{T}\sum_{t=1}^{T} u_i(a^t)$$

2. discounted sum $$U_i(h^T) = \sum_{t=1}^{T} d^{t-1} u_i(a^t), \quad \text{for } d \in [0,1]$$

Def: A strategy profile is a *subgame-perfect equilibrium* of G(T) if:

(i) s is a Nash equilibrium of G(T), and

(ii) for every $t < T$ and all $h^t \in A^t$, $s[h^t]$ is a Nash equilibrium of G(T-t), where $s[h^t]$ is the strategy profile for the game G(T-t) specified by s following the history h^t.

Players condition their behavior in period t on the past history h^{t-1}. This may allow the creation of reputations and cooperation that is not possible in the stage game.

3. Finitely Repeated Games (Benoit and Krishna, *Econometrica* 1985)

Consider the prisoners' dilemma:

		2	
		Mum	Fink
1	Mum	–1, –1	–5, 0
	Fink	0, –5	–4, –4

(Fink,Fink) is the unique Nash equilibrium of this stage game G. What if we repeat G many times? By backward recursion, at the last stage both will fink regardless of the past history (fink is a dominant strategy), hence in the next to last stage both will fink regardless of the history, and so on. Therefore, G(T) has a unique subgame perfect equilibrium in which (Fink,Fink) is played in each stage. The problem here is that second period play cannot reward cooperative first period behavior, since the players cannot commit to playing Mum. This is true for any stage game with a unique equilibrium.

What if G has several Nash equilibria? Let the payoff for G(T) be given by the average of the stage payoffs. Clearly any sequence of length T of Nash equilibria of G is a subgame-perfect equilibrium of G(T). Can we support actions that are not Nash equilibria of G to be played in a subgame-perfect equilibrium of G(T)? Consider the following stage game G.

		2		
		x_2	y_2	z_2
	x_1	<u>1</u>, <u>1</u>	<u>5</u>, 0	0, 0
1	y_1	0, <u>5</u>	4, 4	0, 0
	z_1	0, 0	0, 0	<u>3</u>, <u>3</u>

G has two pure-strategy equilibria: (x_1,x_2) and (z_1,z_2), but (y_1,y_2) which Pareto dominates either equilibrium is not an equilibrium. If we repeat G twice, we can achieve (4,4) in the first period by punishing any deviation from (y_1,y_2) by reverting to (x_1,x_2):

$$s_i^1 = y_i, \text{ and } s_i^2(h^1) = \begin{cases} z_i & \text{if } h^i = (y_1, y_2), \\ x_i & \text{otherwise.} \end{cases}$$

This is a subgame-perfect equilibrium of G(2), since the best deviation yields (5 + 1)/2, which is worse than the equilibrium payoff of (4 + 3)/2.

Abreu (1983) has made clear that the most cooperative equilibrium behavior is supported by the strongest possible (credible) punishments. Benoit and Krishna show what can happen as T gets large? They show that anything that is feasible and individually rational can be achieved in a subgame perfect equilibrium.

4. Infinitely Repeated Games (Fudenberg and Maskin, *Econometrica* 1986)

stage game $G = \{A_1,...,A_n; u_1,...,u_n\}$

infinitely repeated game $G(\infty)$

strategy $s_i = (s_i^1,..., s_i^t,...)$ where $s_i^t: A^{t-1} \to A_i$.

action set A_i = set of all mixed strategies and strategies based on public randomization devices

minimax payoff $v_j^* = \min_{a_{-j}} \max_{a_j} u_j(a_j, a_{-j})$

Player j can guarantee a payoff of at least v_j^* given any strategy of others.

minimax strategy $M_{-j}^j \in \arg\min_{a_{-j}} \max_{a_j} u_j(a_j, a_{-j})$

If the others play M_{-j}^j they can hold player j to no more than v_j^*.

Payoffs $v_j \geq v_j^*$ are said to be *individually rational* for player j. In any equilibrium of G(T), j's expected payoff must be at least v_j^*.

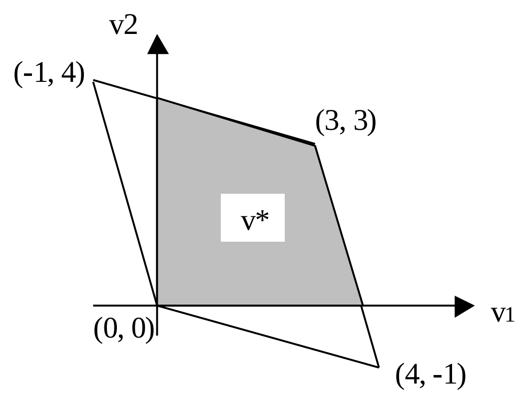
Normalize payoff functions $\{u_1,...,u_n\}$ so that $v_i^* = 0$ for all i.

Attainable payoffs $U = \{(v_1,...,v_n) \mid \exists\, a \in A \text{ s.t. } u_i(a) = v_i \text{ for each } i\}$.

V = convex hull of U. V is set of payoffs achievable via correlated strategies.

Feasible individually rational payoffs: $V^* = \{(v_1,...,v_n) \in V \mid v_i > 0\ \forall\ i\}$

		2	
		Mum	Fink
1	Mum	3, 3	-1, 1
	Fink	4,-1	0, 0



payoff in G(∞): $U_i(h^\infty) = \sum_{t=1}^{\infty} \delta^{t-1} u_i(a^t)$.

Then $(1 - \delta)U_i(h^\infty)$ is the average payoff in the sense that if it were received every period then the same total payoff would result.

Folk Theorem. For any $\{v_1,...,v_n\} \in V^*$ if players discount the future sufficiently little ($\exists\, \underline{\delta} \in (0,1)$ s.t. $\forall\ \delta \in (\underline{\delta}, 1)$), there exists a Nash equilibrium of G(∞) where for all i, i's average payoff is v_i.

Proof: Find a vector of actions $a = (a_1,...,a_n) \in A$ such that $v_i(a) = v_i$ for each i. Let i's strategy be "play a_i until some j deviates from a_j; thereafter play M_i^j." (Note that only unilateral deviations need be considered.) These unrelenting punishments are called *grim strategies*. They form an equilibrium because the players must trade off a one-shot, finite gain against an infinitely long stream of losses, so for sufficiently large δ they will conform.

Problem: This is not subgame perfect.

Perfect Folk Theorem (Aumann-Shapley; Rubinstein). For any $(v_1,...,v_n) \in V^*$, there exists a perfect equilibrium of G(∞) with no discounting where, for all i, player i's expected payoff is v_i.

Payoff without discounting: $U_i(h^\infty) = \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} U_i(a^t)$.

Proof Sketch: Begin by playing $a = (a_1,...,a_n)$ where $u_i(a) = v_i$. If j deviates then the others play M_{-j}^j, but only long enough to wipe out j's gain from deviating. Afterwards go back to a. To induce the punishers to administer the punishment, the punishers are themselves threatened with punishment if they fail to punish the original deviator. As before, these punishments for failed punishers are only long enough to wipe out the gain from deviating. This leads to an infinite sequence of punishments, each just long enough to deter deviations from the one before. These punishments may need to become increasingly harsh. Without discounting this can be done by making them increasingly long.

Folk Theorem with Nash Threats (Friedman). If $(v_1,...,v_n) \in V^*$ Pareto dominates the payoffs of a Nash equilibrium of the stage game G, then if the players discount the future sufficiently little, there exists a perfect equilibrium of $G(\infty)$ where for all i, i's average payoff is v_i.

Proof: Same as the Folk Theorem, except deviations lead to Pareto-dominated Nash equilibrium forever, instead of the minimax strategies.

Example (Prisoners' Dilemma)

		2	
		Mum	Fink
1	Mum	3, 3	–1, $\underline{x}$
	Fink	$\underline{x}$, –1	$\underline{0}$, $\underline{0}$

Let x > 3, so that (F,F) is an equilibrium in dominant strategies. The strategies above support (3,3) as the average payoff in a subgame perfect equilibrium provided

$$\frac{3}{1-\delta} \geq x,$$

so if $\delta = .9$, then $x \leq 30$ works, and if x = 4, $\delta \geq 1/4$ works.

Fudenberg and Maskin generalize Friedman's Theorem to the entire set of feasible, individually rational payoffs, V^*. Two player case:

Theorem 1 (Fudenberg-Maskin). For any $(v_1,v_2) \in V^*$, there exists $\underline{\delta} \in (0,1)$ s.t. for all $\delta \in (\underline{\delta}, 1)$ there exists a subgame perfect equilibrium of $G(\infty)$ in which i's average payoff is v_i when the players discount the future according to δ

Proof: Let M_i be i's minimax strategy against j

$$M_i \in \arg\min_{a_i} \max_{a_j} u_j(a_i, a_j),$$

and let

$$\overline{v}_i = \max_{a_1,a_2} u_i(a_1, a_2).$$

It suffices to show that there exists a positive integer N and $\underline{\delta} \in (0,1)$ such that for each i,

(1) $v_i > \overline{v}_i(1 - \underline{\delta}) + \underline{\delta} p_i$, and

(2) $p_i \equiv (1 - \underline{\delta}^N) u_i(M_1,M_2) + \underline{\delta}^N v_i > 0.$

(1) guarantees that player i prefers receiving v_i forever than receiving $\overline{v}_i$ once and then $u_i(M_1,M_2)$ for N

periods, and then v_i thereafter. [Note that we are giving the deviator the benefit of the doubt here by given him $\bar{v}_i$.]

(2) guarantees that being punished for deviating (and cooperating in the punishment by playing M_i) is individually rational, since it exceeds the reservation value of 0.

If (1) and (2) hold then the following strategies are subgame perfect:

(i) play a_i provided (a_1,a_2) was played the previous period. After a deviation (ii) play M_i for N periods and then start (i) again. After a deviation from (ii) start (ii) again.

Condition (1) guarantee that deviation is not profitable in phase (i). In phase (ii), i receives at least $p_i > 0$ from not deviating and at most δp_i from deviating (zero immediately since j plays M_j and p_i thereafter), so deviation is not profitable in phase (ii).

Does there exist N and $\underline{\delta}$ satisfying (1) and (2)?

Recall that $v_i \in (0, \bar{v}_i]$ and $u_i(M_1,M_2) \le 0$. Hence, there exists $\underline{\delta} \in (0,1)$ such that

$$\underline{\delta} > 1 - \frac{v_i}{\bar{v}_i} \qquad \text{and} \qquad \underline{\delta}^2 > \frac{-u_i(M_1,M_2)}{v_i - u_i(M_1,M_2)}.$$

The latter guarantees that (2) holds for N=1, and the former ensures that (1) holds if p_i is sufficiently small because $v_i > \bar{v}_i(1 - \underline{\delta})$. If (1) fails because p_i is too large, consider raising N. This will reduce p_i because $u_i(M_1,M_2) \le 0 < v_i$. For δ sufficiently close to 1, (2) will still be satisfied. Q.E.D.

Theorem 2 (Fudenberg and Maskin). Assume the dimensionality of $V^* = n$ (i.e., the interior of V^* relative to $\Re^n$ is non-empty). Then for any $(v_1,...,v_n) \in V^*$, there exists $\underline{\delta} \in (0,1)$ such that for all $\delta \in (\underline{\delta},1)$ there exists a subgame-perfect equilibrium of $G(\infty)$ in which i's average payoff is v_i when players have discount factor δ.

The extra hypothesis is needed since discounting may prevent the creation of sufficiently strong punishments to punish those who fail to punish a deviator, so instead those who successfully administer a punishment are rewarded. If the dimensionality condition is not met then there will exist a player k such that if k deviates it will not be possible to reward k's punishers without rewarding k.

B. Applications

1. Cartel Maintenance (Porter, *Jet* 1983)

Static oligopoly model:

firms $i \in (1,...,N)$

simultaneous selection of quantity q_i of homogenous good

Quantity vector: $q=(q_i,q_{-i})$; total production $Q = q_1 + ... + q_n$

price $\tilde{p}(Q) = \tilde{\theta} p(Q)$ where $p(Q) = a - bQ$ and

$\tilde{\theta}$ has distribution $F(\theta)$ on $[0,\infty)$ and mean μ.

costs $C(q_i) = c_0 + c_1 q_i$ where $0 < c_1 < ma$.

profit $p_i(q) = m[a - b(q_i + Q_{-i})]q_i - c_0 - c_1 q_i$.

Have game in normal form $\Gamma = \{S_1,...,S_n; \pi_1,...,\pi_n\}$ where $S_i = [0,\infty)$ for all i. $\bar{q} = (\bar{q}_1,...,\bar{q}_n)$ is a (Cournot) Nash equilibrium if

$$p_i(\bar{q}_i, \bar{q}_{-i}) \geq p_i(q_i, \bar{q}_{-i}) \text{ for all i and } q_i \geq 0.$$

Differentiating yields the equivalent requirement that

$$\frac{\partial p_i(q_i, \bar{q}_{-i})}{\partial q_i} = 0.$$

A little algebra shows that the unique Cournot-Nash equilibrium satisfies

$$\bar{q}_i = \frac{ma - c_1}{mb(N+1)} = \frac{A}{B(N+1)} \equiv s \quad \text{where } A = ma - c_1 \quad \text{and } B = mb.$$

Denote equilibrium profits by $\pi_i(s) = [A^2/B(N+1)^2] - c_0$ and assume $\pi_i(s) > 0$.

If firms could collude, they could maximize aggregate profits subject to the constraint that every firm stay in operation:

$$\max_q \sum_{i=1}^{N} p_i(q).$$

The solution is for each firm to produce $r \equiv A/2BN$, resulting in profits $\pi_i(r)$. $(r,...,r)$ is not an equilibrium in Γ. How close to the cooperative outcome can we get in the infinite-horizon repeated game?

periods $\{1,...,t,...\}$

firm i selects q_i^t and then observes p_t but not q^t_{-i}.

random disturbance $\tilde{q}_t$ is iid according to $F(q)$.

expected payoff $E\left\{\sum_{t=0}^{\infty} b^t p_i(q^t)\right\}$, where the discount factor is $b \in (0,1)$.

No proper subgames here: no firm reaches a singleton information set because no firm ever learns the complete history of the game.

Trigger-price strategy (q,p,T), where for all i:

(a) i plays q_i in period 0;

(b) if q_i was played in period t and $p_t \geq p$ then i plays q_i in period t+1; and

(c) if q_i was played in period t and $p_t < p$ then i plays s for the T-1 periods t+1, t+2, ... , t+T-1, and then plays q_i in period t+T.

Value function:

$$V_i(q) = \pi_i(q) + \Pr(p_t \geq p)\beta V_i(q) + \Pr(p_t < p)\left[\sum_{t=1}^{T-1}\beta^t \pi_i(s) + \beta^T V_i(q)\right].$$

Solving the value function yields

$$V_i(q) = \frac{\pi_i(s)}{1-\beta} + \frac{\pi_i(q) - \pi_i(s)}{1-\beta+(\beta-\beta^T)\Pr(p_t < p)},$$

where $\Pr(p_t<p) = \Pr(\theta_t p(Q)<p) = \Pr(\theta_t < p/p(Q)) = F(p/p(Q))$.

The trigger-price strategy (q,p,T) is a Nash equilibrium if for all i

$V_i(q_i, q_{-i}) \geq V_i(\tilde{q}_i, q_{-i})$ for all $\tilde{q}_i \geq 0$.

Denote the solution by

$q^*(p,T) = \{q_1^*(p,T),...,q_N^*(p,T)\}$ and value function by $V_i^*(p,T) \equiv V_i(q^*(p,T))$.

Proposition 1. Cournot-Nash quantity $q_i^* = s$ for all i is a Nash equilibrium for all p,T.

Proposition 2. Given p and T, $q_i^* = q_j^*$ for all i,j.

Proposition 3. For all p and T, $q_i^* \in (s/N, s]$ where $s/N < r < s$.

Proposition 4. If $F(\theta)$ is convex then $V_i(q)$ is concave in q_i, so the first-order condition is sufficient.

Trigger-price equilibrium (q^*,p,T) will exhibit price wars of fixed duration at random intervals. These will be caused (along the equilibrium path) by low realizations of θ_t, not by deviations from q^*. Compare this with Fudenberg-Maskin result in which there is no noise and q^t is observable after each period: no punishments are ever administered.

Now consider optimizing p and T. Interior solutions are characterized by

$$\frac{\partial V_i^*}{\partial p} = \frac{\partial V_i^*}{\partial T} = 0.$$

Proposition 5. For interior p^* and T^*,

$$q_i^* = \begin{cases} r\left(\dfrac{N+\eta^*+(N+1)(a/A)}{N+\eta^*+1}\right) & \text{for } \eta^* > \eta^0 \\ s & \text{otherwise,} \end{cases}$$

$$\text{where} \quad \eta^* = \frac{f(p^*/p(Nq_i^*))}{F(p^*/p(Nq_i^*))} \cdot \frac{p^*}{p(Nq_i^*)}$$

$$\text{and} \quad \eta^0 = \frac{N+1}{N-1} \cdot [(N+1)(a/A) - N].$$

This proposition has three immediate corollaries: (i) $q_i^* \in (r,s]$; (ii) $dq_i^*/d\eta^*$ is less than (equals) zero when η^* is greater than (is less than or equal to) η; (iii) as $\eta^* \to \infty$, $q_i^* \to r$. Compare this to Friedman's Theorem using Nash threats. Here η^* is an *endogenous* measure of noise; as the noise disappears, a deviation from q_i^*

will almost surely trigger a punishment phase. Note that the limiting $T^*(\eta^*)$ may be finite: Friedman's grim strategies are not needed to support r using s as a threat.

2. Sequential Bargaining (Rubinstein, *Econometrica* 1982)

A classic problem in economics is bargaining between bilateral monopolists. Here two players bargain over the division of a pie. They alternate in making offers: player 1 makes an offer which player 2 can accept or reject; if 2 rejects then 2 makes a proposal which 1 can accept or reject; and so on. Each offer takes one period and the players are impatient: they discount payoffs received in period t by the discount factors δ_1^t and δ_2^t, respectively. Let all proposals represent player 1's share of the pie. So if 1 offers s_1 in the first period, 2 rejects it and offers s_2 in the second period, and 1 accepts, then the payoffs are $\delta_1 s_2$ for 1 and $\delta_2(1 - s_2)$ for 2.

Although any division of the pie is a Nash equilibrium, the insight of Rubinstein [1982], is that if the players discount future payoffs, then the game has a unique subgame-perfect equilibrium in which trade occurs immediately. This efficient bargaining outcome is due to the assumption of perfect information: the players, being fully informed, are able to anticipate what would happen in the course of the game, and thus are prepared to make and accept a reasonable initial offer and thereby avoid any costs of delay.

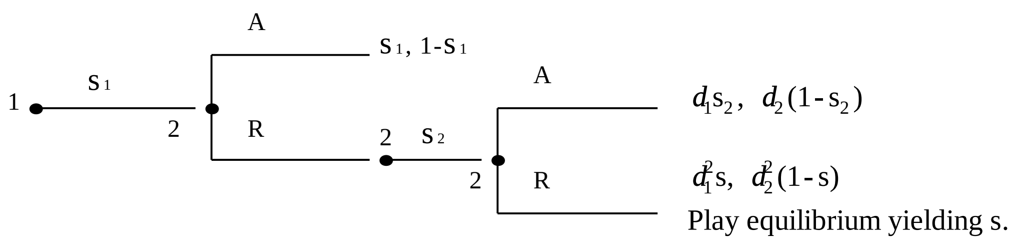
The uniqueness of the subgame-perfect equilibrium should not be surprising due to the complete information and sequential offers. At least in the finite-horizon game, the intuition is clear: the game is one of perfect information and a (generically) unique equilibrium is found by backward induction in the game tree, where at each information set a player has a dominant strategy.

Rubinstein shows that the infinite-horizon game has a unique subgame-perfect equilibrium outcome: 1 offers $s_1 = (1 - \delta_2)/(1 - \delta_1\delta_2)$, and 2 accepts, ending the game. This outcome has nice comparative static properties. For example, i's payoff is increasing in δ_i and decreasing in δ_j. As $\delta_1 \to 1$ or $\delta_2 \to 0$, player 1 gets all the pie. There is a first-mover advantage, but as the time between offers goes to zero, this advantage disappears: all that matters is the relative impatience of the players, and if they are equally impatient then the pie is divided equally.

While the model does explain how gains should be divided as a function of the players' impatience, it does not explain costly bargaining delays, such as strikes, which are frequently observed. For this we need a bargaining model with incomplete information, which will be taken up later in the term.

The proof of Rubinstein's bargaining solution proceeds in two steps. The first shows that if there exists a subgame-perfect equilibrium payoff, then it is unique, and the second exhibits subgame-perfect strategies that achieve this unique payoff.

The key to the proof is to recognize that all the subgames beginning with an offer by 1 in the third period are identical to the original game. So if there exists a subgame-perfect equilibrium payoff, say s, then we can use it to summarize behavior in these subgames, as in the figure below.



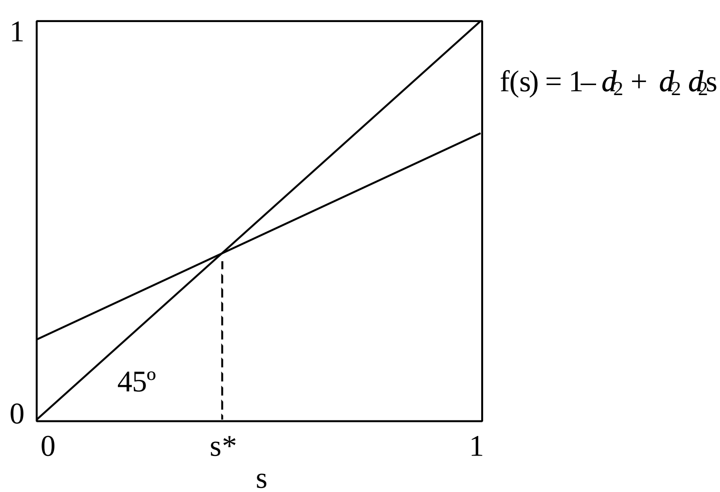
Suppose there exists an equilibrium in which 1 receives s if the game reaches period three. Then in period two, 1 will only accept offers that are at least as great as ds, so 2's best response is to offer $s_2^* = d$s, since 1 - ds > d(1 - s). Hence, in period one, 2 will accept 1's offer of s_1 if and only if 1 - $s_1 \geq d$(1 - ds), so that 1's best response is to offer s_1^* = 1 - d + $d\,d$s. So given any subgame-perfect equilibrium payoff s, there exists another subgame-perfect equilibrium payoff

$$s' = f(s) = 1 - d + d\,d s.$$

More importantly, given s there also exists

$$s'' = f^{-1}(s) = \frac{s + d_2 - 1}{d_1 d_2}$$

where s'' supports s just as s supports s'. The picture below shows that only one value of s is possible; namely, the fixed point s^* that satisfies $s^* = f(s^*)$. Simple algebra shows that s^* = (1 - d)/(1 - $d\,d$), which is Rubinstein's solution. No other s can work because it requires s'' outside of [0,1] to support it (by repeated applications of the s'' argument, since the slope of $f^{-1}(s)$ is greater than 1).f(s) = 1– d_2 + $d_2\,d_2$s



Part two of the argument goes as follows. Consider the following strategies:

1 always offers $x = \frac{1 - d_2}{1 - d_1 d_2}$, 2 always offers $y = 1 - \frac{1 - d_1}{1 - d_1 d_2}$

1 only accepts offers of y or more; 2 only accepts offers of x or less.

The third-period subgames are identical to the original game and hence it should be no surprise that the strategies are identical. But observe also that the second-period subgames are identical to the original game with interchanged discount factors. The strategies above reflect this, since in every period player i makes an offer that yields the payoff of $(1 - d)/(1 - d d)$ if it is accepted.

Recall that subgame perfection means that strategies prescribe Nash behavior in every subgame. Since the strategies are the same in every subgame, it suffices to prove that the strategies above are simply Nash. Given player 2's strategy, player 1 can get x in any odd period or y in any even period. Since $x > dy$, player 1's best response is to get x immediately. And given player 1's strategy, player 2 can get 1 - x in any odd period or 1 - y in any even period. Since $(1 - x) = d(1 - y)$, player 2 is just as well off taking 1 - x immediately as waiting a period for 1 - y.

Notice that the equilibrium is determined from two indifference equations:

(i) 2 is indifferent between 1 - x today and 1 - y tomorrow, and

(ii) 1 is indifferent between y today and x tomorrow.

These are necessary conditions for an equilibrium for if one player was not indifferent, then either (1) the offer would not be acceptable (so acceptance is not a best response) or (2) an offer that is more attractive to the offeror would also be acceptable (so the offer is not a best response).