Auctioning Many Similar Items

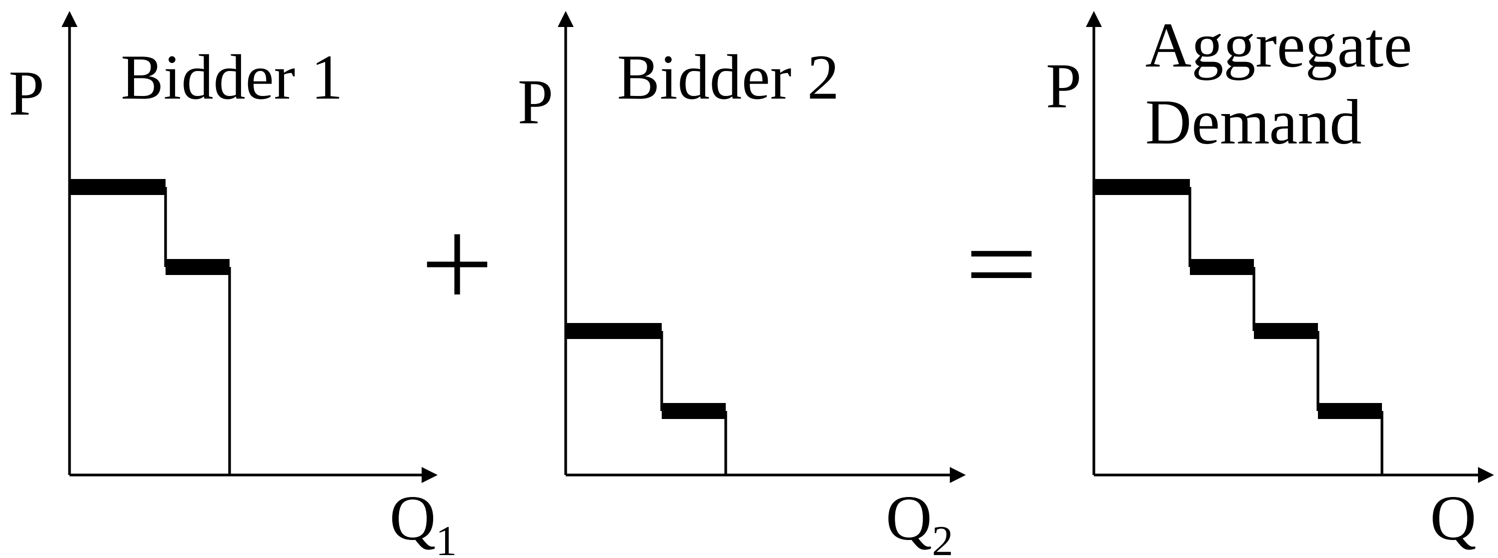
Lawrence Ausubel and Peter Cramton
Department of Economics
University of Maryland

Examples of auctioning similar items

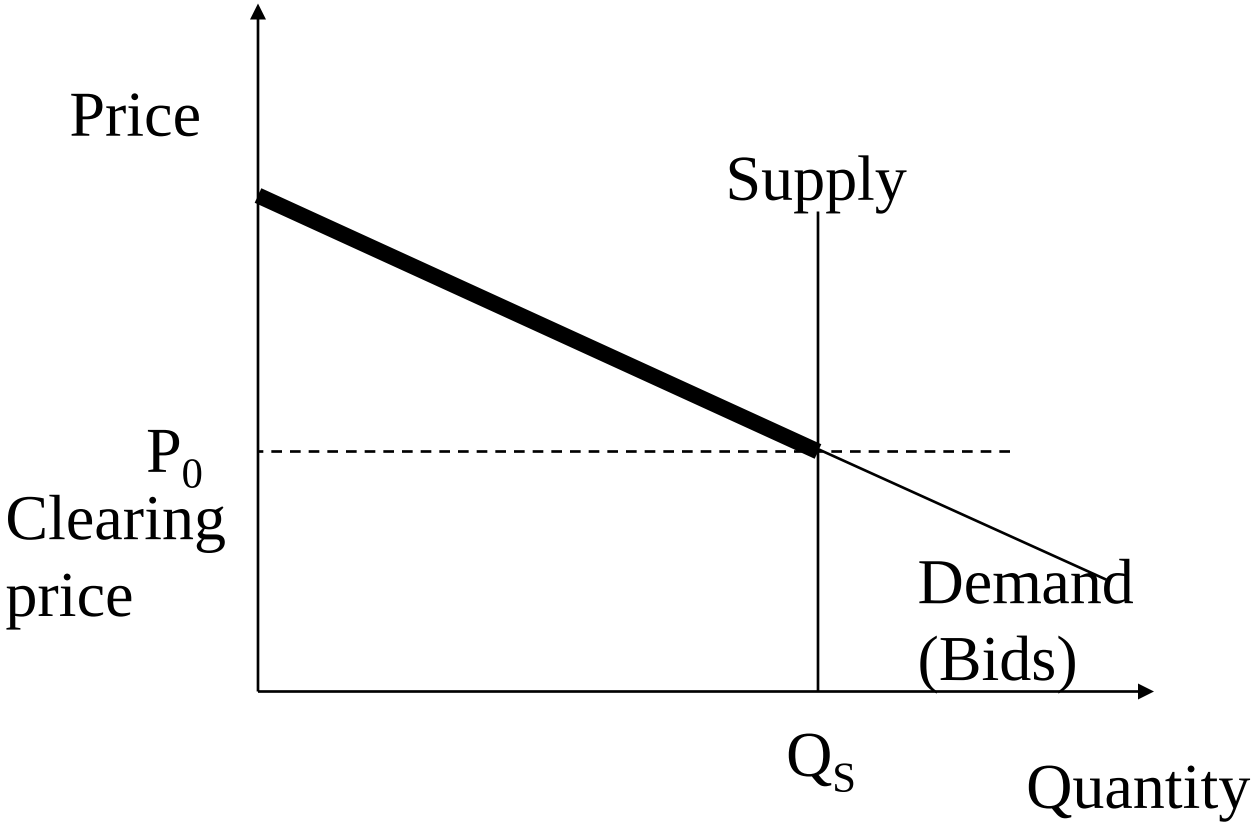
- Treasury bills
- Stock repurchases and IPOs
- Telecommunications spectrum
- Electric power
- Emission allowances

Ways to auction many similar items

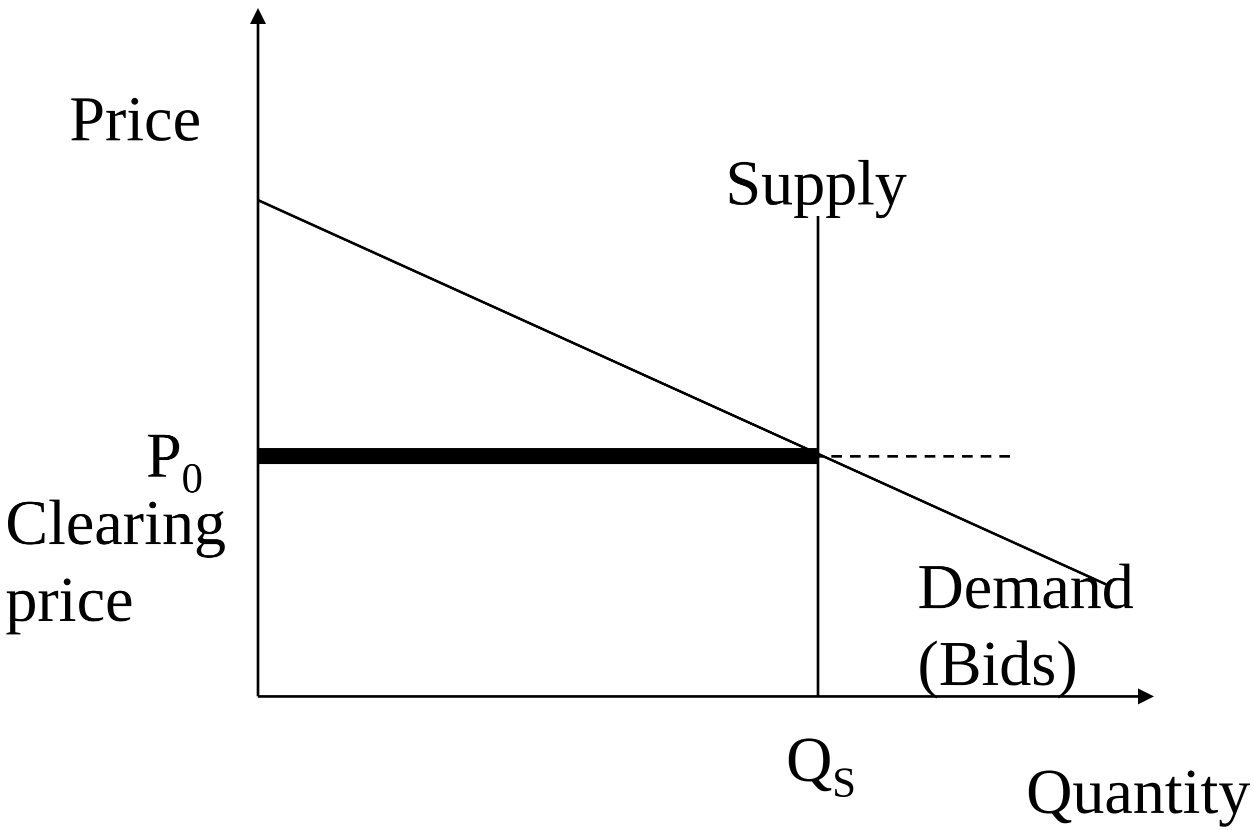
- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)



Pay-as-bid Auction:
All bids above P_0 win and pay bid

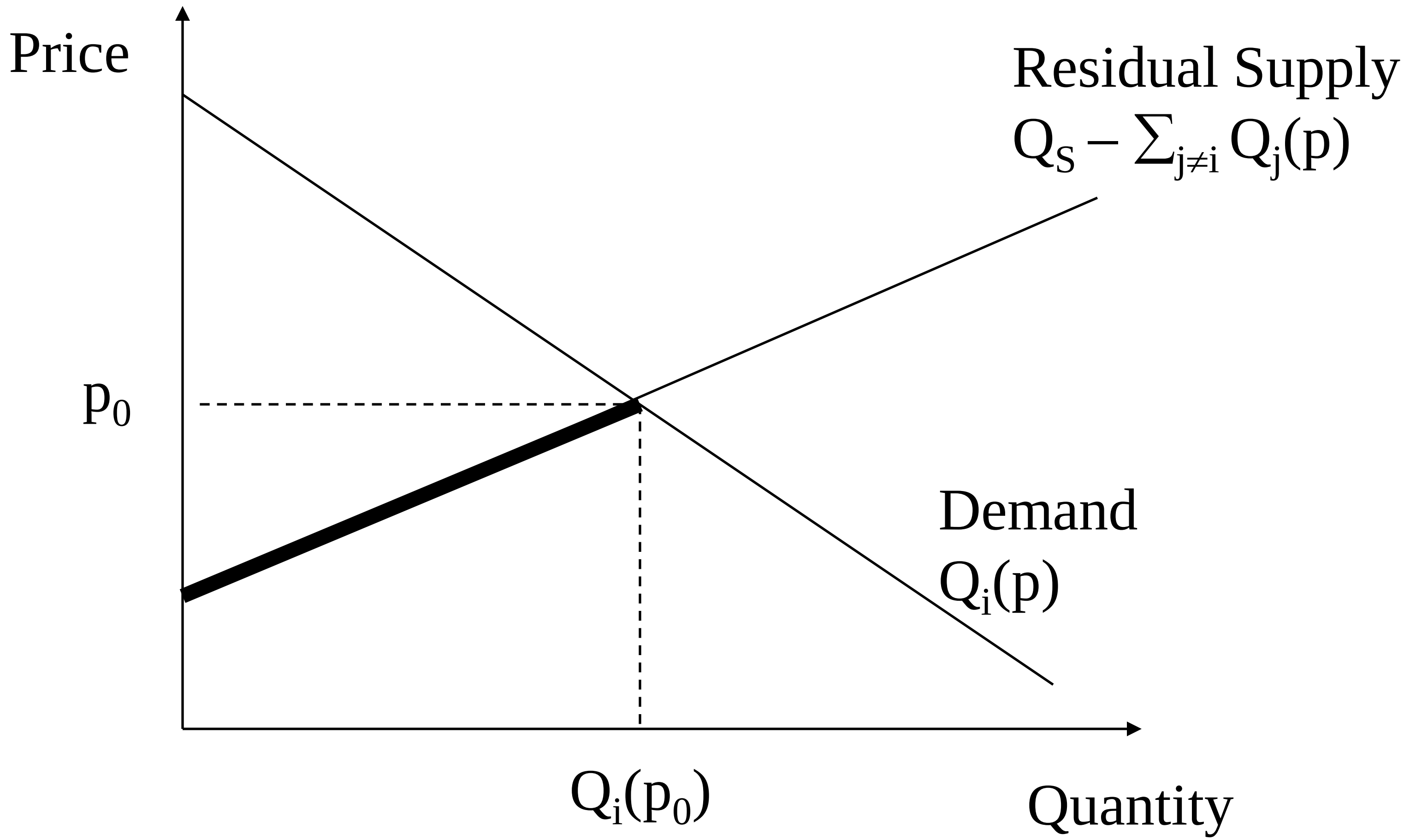


Uniform-Price Auction:
All bids above P_0 win and pay P_0



Vickrey Auction:

All bids above P_0 win and pay opportunity cost

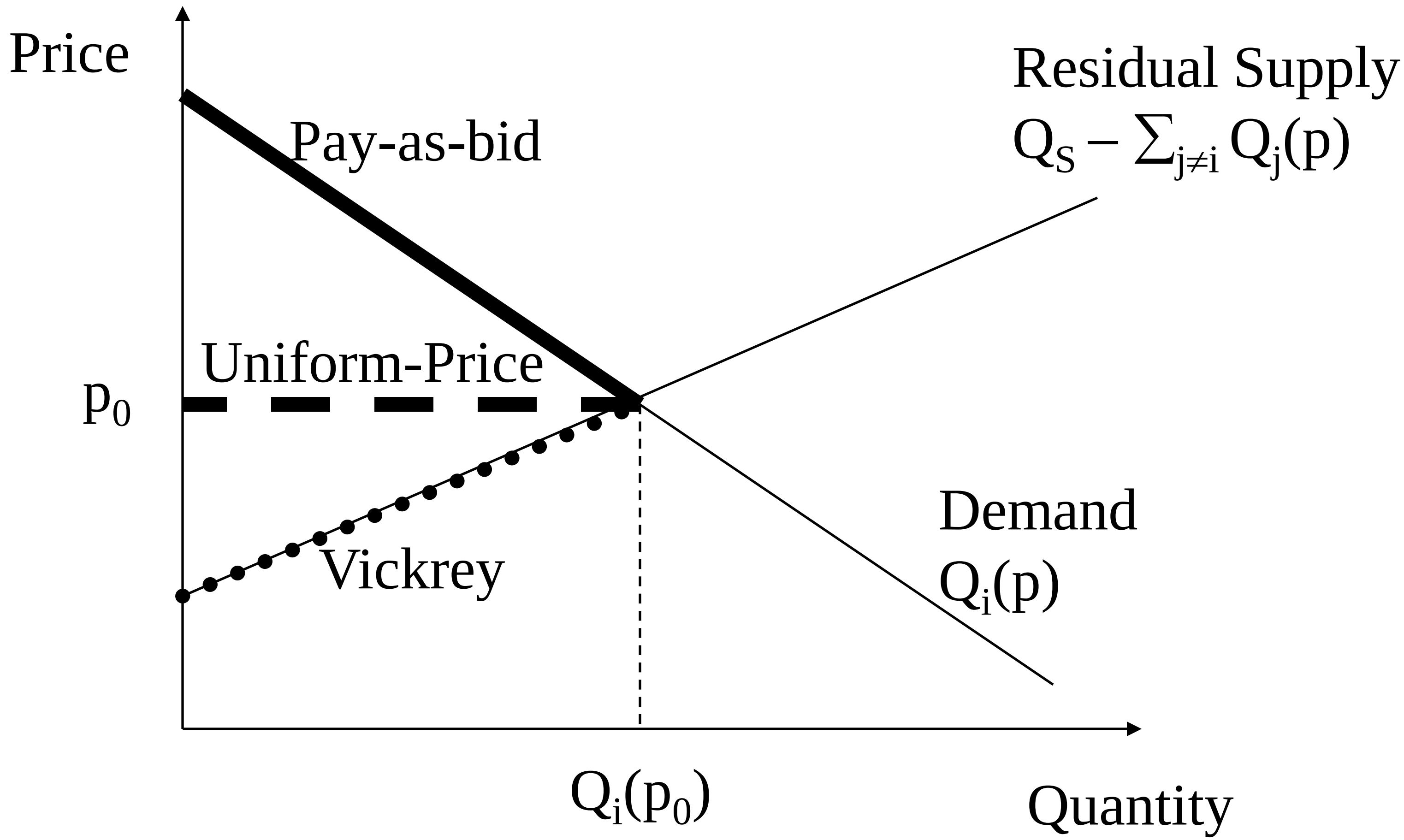


Vickrey Auction: m Discrete Items

- Allocate m items efficiently: m highest marginal values
- Winning bidder pays k^{th} highest *losing* bid of *others* on k^{th} item won
- Payment = social opportunity cost of items won

3 bidders, 3 items marginal values			
	A	B	C
1^{st}	10 5	8 6	4
2^{nd}	6	7 4	2
3^{rd}	3	5	1

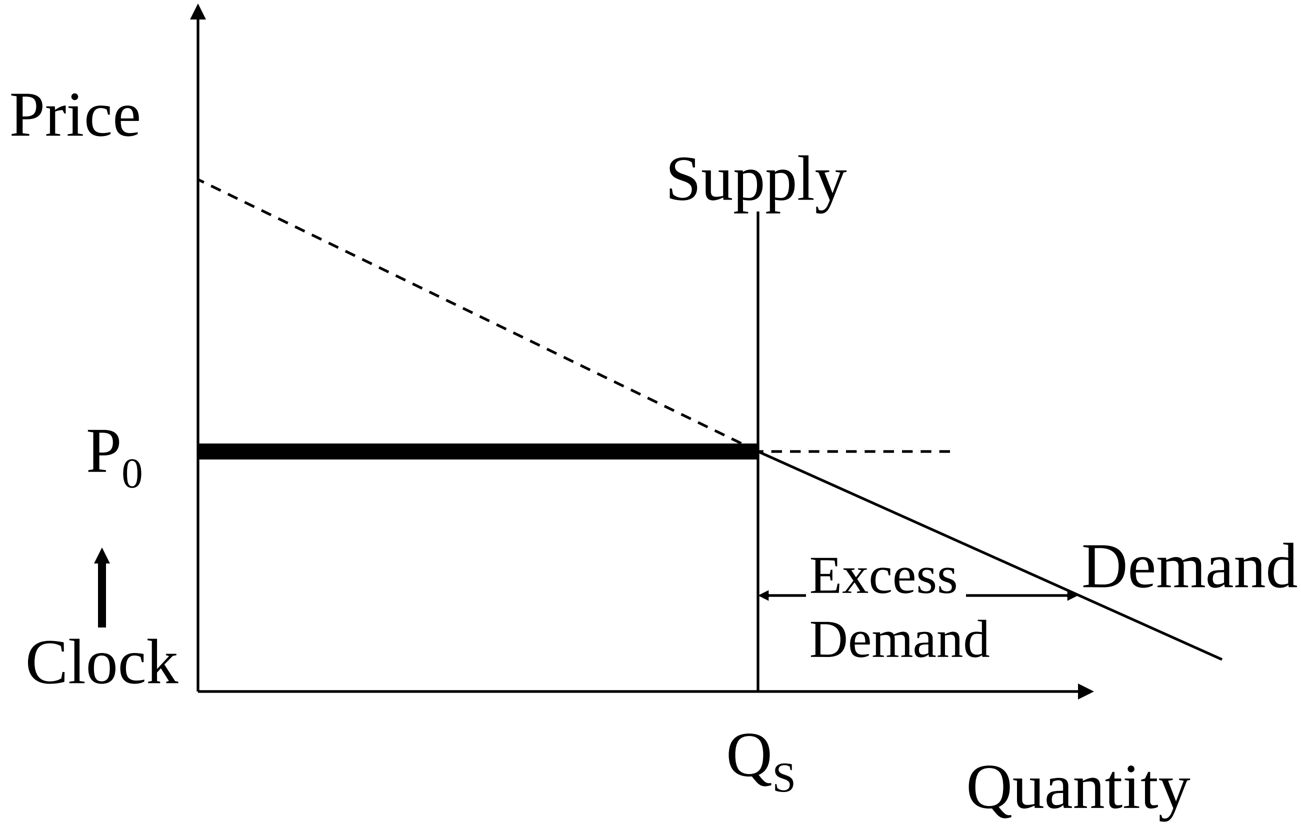
Payment rule affects behavior



More ways to auction many similar items

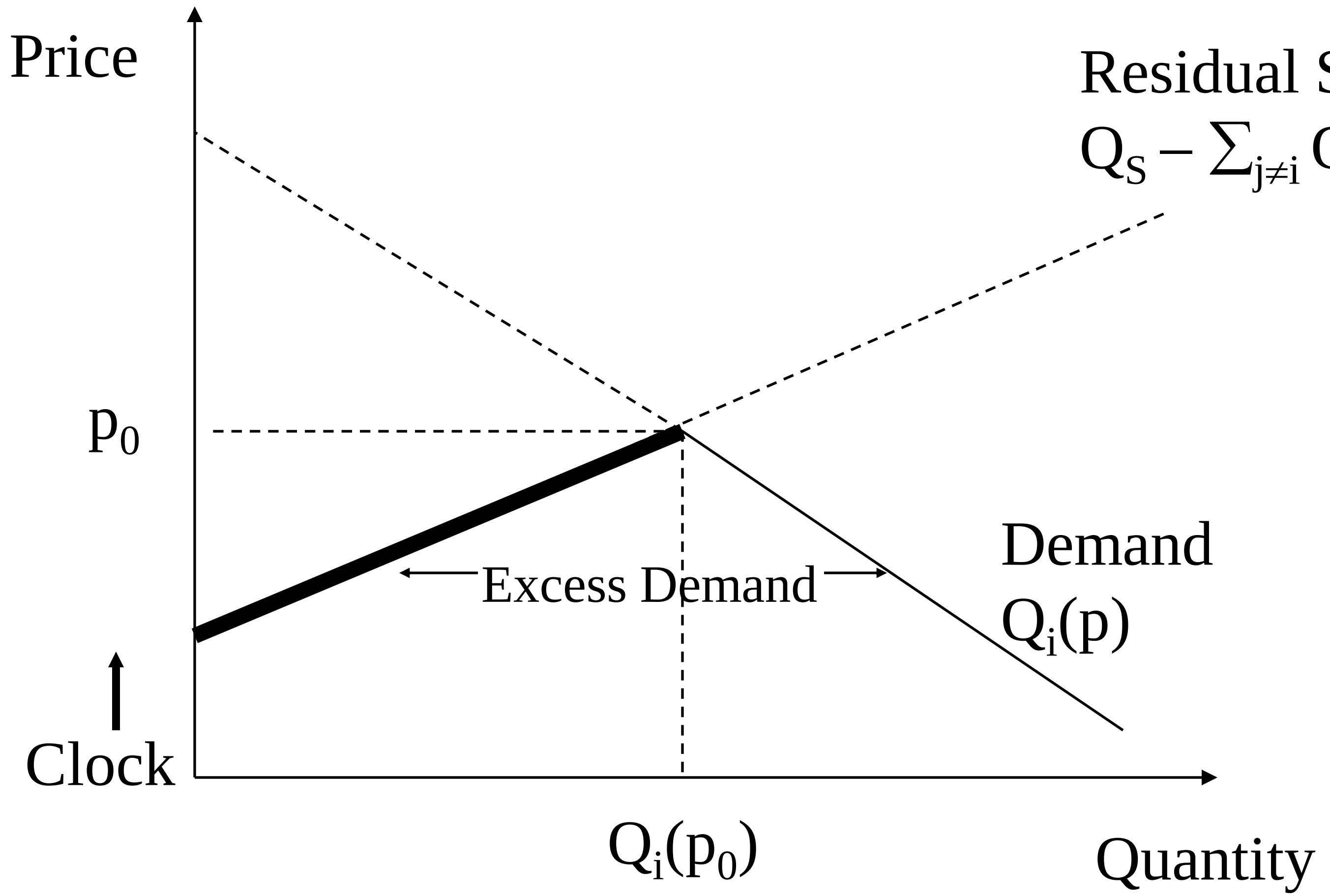
- Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  - Standard ascending-bid
  - Ausubel ascending-bid (Ausubel 1997)

Standard Ascending-Bid Auction:
All bids at P_0 win and pay P_0



Ausubel Ascending-Bid:

All bids at P_0 win and pay price at which clinched



More ways to auction many similar items

- Ascending-bid
  - Simultaneous ascending auction (FCC spectrum)
- Sequential
  - Sequence of English auctions (auction house)
  - Sequence of Dutch auctions (fish, flowers)
- Optimal auction
  - Maskin & Riley 1989

Research Program
How do standard auctions compare?

- Efficiency
  - FCC: those with highest values win
- Revenue maximization
  - Treasury: sell debt at least cost

Efficiency
(not pure common value; capacities differ)

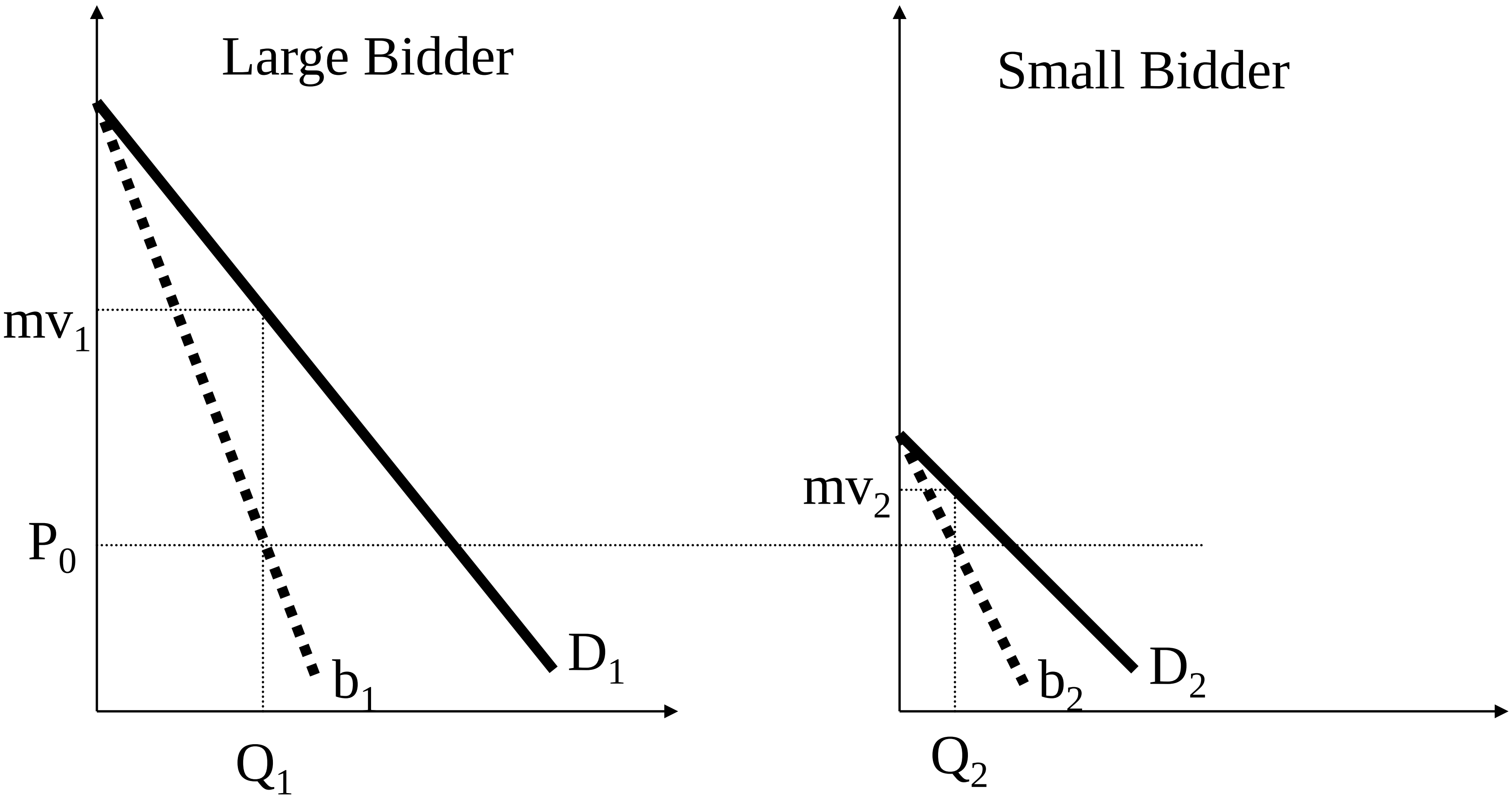
- Uniform-price and standard ascending-bid
  - Inefficient due to demand reduction
- Pay-as-bid
  - Inefficient due to different shading
- Vickrey
  - Efficient in private value setting
  - Strategically simple: dominant strategy to bid true demand
  - Inefficient with affiliated information
- Ausubel ascending-bid
  - Same as Vickrey with private values
  - Efficient with affiliated information

Inefficiency Theorem

In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values.

- Winning bidder influences price with positive probability
- Creates incentive to shade bid
- Incentive to shade increases with additional units
- Differential shading implies inefficiency

Inefficiency from differential shading

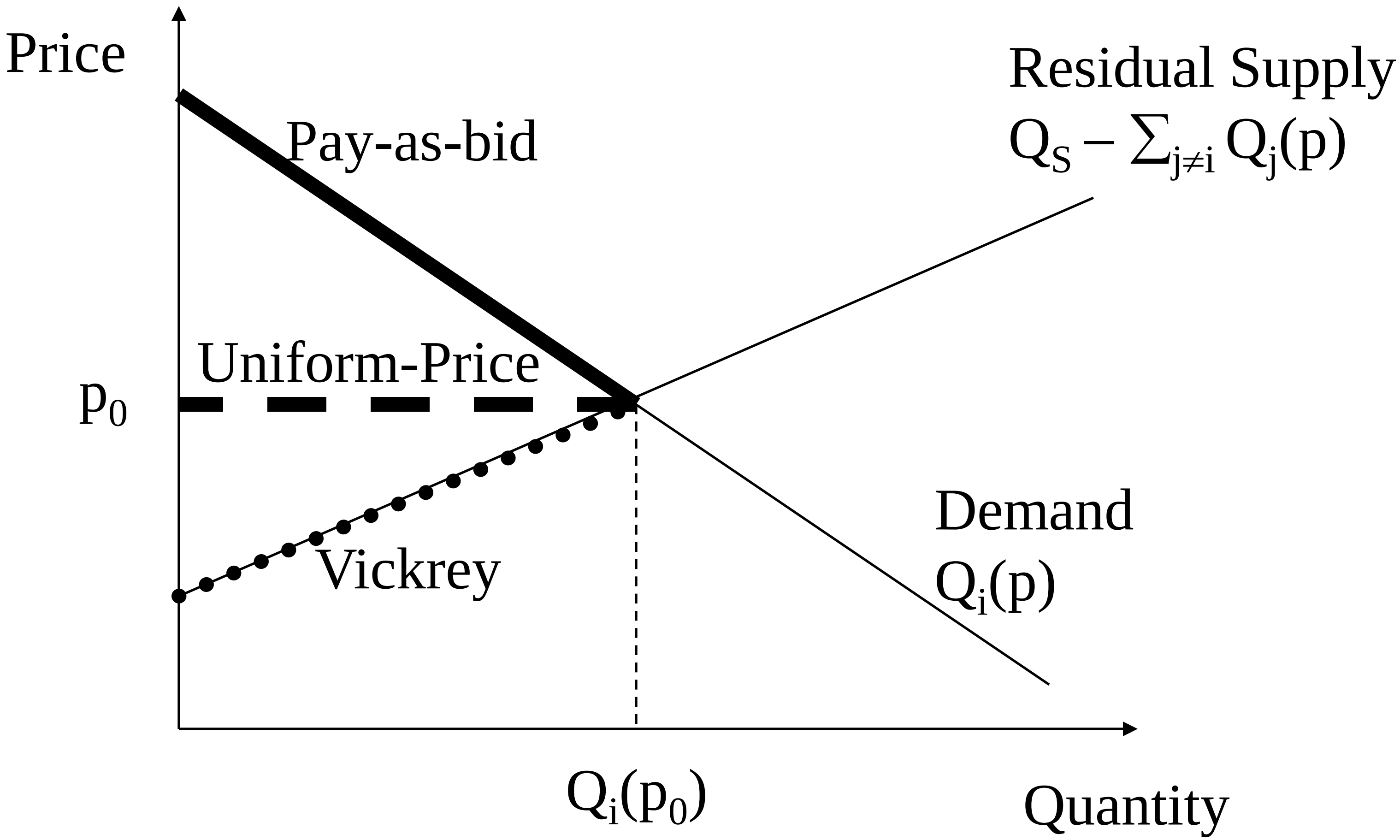


Large bidder makes room for smaller rival

Vickrey inefficient with affiliation

- Winner's Curse in single-item auctions
  - Winning is bad news about value
- Winner's Curse in multi-unit auctions
  - Winning more is worse news about value
  - Must bid less for larger quantity
  - Differential shading creates inefficiency in Vickrey

What about seller revenues?



Exercise

- 2 bidders (L and R), 2 identical items
- L has a value of $100 for 1 and $200 for both
- R has a value of $90 for 1 and $180 for both
- Uniform-price auction
  - Submit bid for each item
  - Highest 2 bids get items
  - 3rd highest bid determines price paid
- Ascending clock auction
  - Price starts at 0 and increases in small increments
  - Bidders express how many they want at current price
  - Bidders can only lower quantity as price rises
  - Auction ends when no excess demand (i.e. just two demanded); winners pay clock price

Uniform price may perform poorly

- Independent private values uniform on [0,1]
- 2 bidders, 2 units; L wants 2; S wants 1
- Uniform-price: unique equilibrium
  - S bids value
  - L bids value for first and 0 for second
  - Zero revenue; poor efficiency
- Vickrey
  - price = $v_{(2)}$ on one unit, zero on other

Standard ascending-bid may be worse

- 2 bidders, 2 units; L wants 2; S wants 2
- Uniform-price: two equilibria
  - Poor equilibrium: both L and S bid value for 1
    - Zero revenue; poor efficiency
  - Good equilibrium: both L and S bid value for 2
    - Get $v_{(2)}$ for each (max revenue) and efficient
- Standard ascending-bid: unique equilibrium
  - Both L and S bid value for 1
    - S's demand reduction forces L to reduce demand
    - Zero revenue; poor efficiency

Efficient auctions tend to yield high revenues

Theorem. *With flat demands drawn independently from the same regular distribution, seller's revenue is maximized by awarding good to those with highest values.*

Generalizes to non-private-value model with independent signals:

$v_i = u(s_i, s_{-i})$

Award good to those with highest signals if downward sloping MR and symmetry.

Downward-sloping demand: $p_i(q_i) = v_i - g_i(q_i)$

Theorem. *If intercept drawn independently from the same distribution, seller's revenue is maximized by*

- *awarding good to those with highest values if constant hazard rate*
- *shifting quantity toward high demanders if increasing hazard rate*

- Note: uniform-price shifts quantity toward *low* demanders

But uniform price has advantages

- Participation
  - Encourages participation by small bidders (since quantity is shifted toward them)
  - May stimulate competition
- Post-bid competition
  - More diverse set of winners may stimulate competition in post-auction market

Auctioning Securities

A pure common-value model with affiliation

- n risk-neutral symmetric bidders
- Each bidder has pure common value V for security and can purchase any quantity (flat demand curve w/o capacity)

Models

- Common uncertainty
  - Bidders have no private information
- Affiliated private signals
  - Bidder i gets signal S_i
  - Random variables $V, S_1, \ldots, S_n$ are affiliated

Results: Common Uncertainty

Proposition. (Wilson '79; Maxwell '83; Back & Zender '93)

- *Wide range of prices can be supported as equilibrium in uniform-price auction, even if supply is stochastic; highest yields EV*

Proposition. (Wang & Zender '02)

- *Many equilibria in pay-as-bid auction, even if supply is stochastic; highest yields EV*
- *Indeterminacy avoided if set reserve price (even 0)*

Results: Common Uncertainty

Theorem.

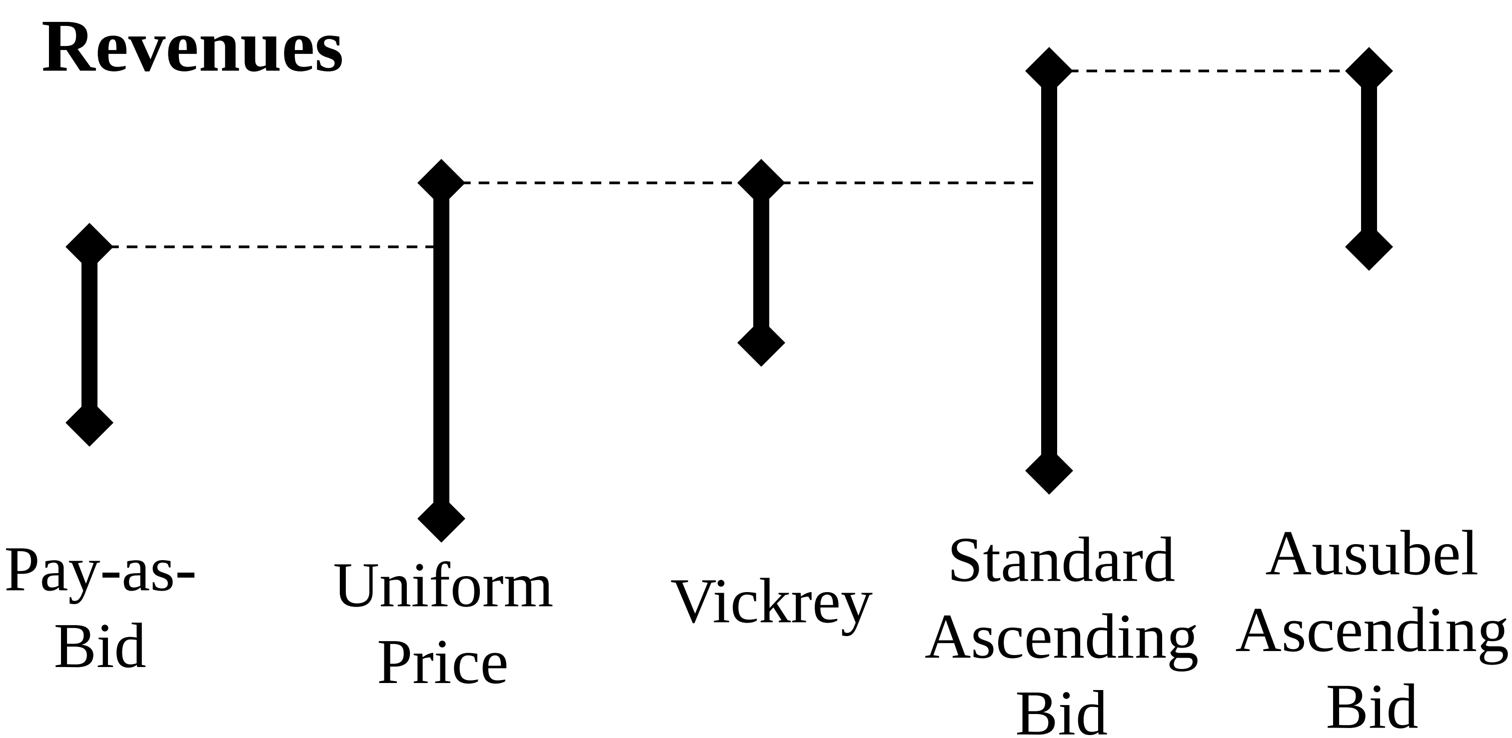
- *Vickrey auction has a unique equilibrium that survives elimination of weakly-dominated strategies*
- *Vickrey auction has a unique symmetric equilibrium consistent with stochastic supply*
- *This equilibrium revenue-dominates all equilibria of **all** auction formats consistent with voluntary bidder participation*

Results: Affiliated Private Signals

- With affiliated signals, each auction format has a “simple equilibrium” where bidders submit flat demand curves
- Conjecture: These simple equilibria provide upper bounds on revenues from each format
- Alt. ascending-bid > Vickrey > Pay-as-bid
- Std. ascending-bid > Uniform > Pay-as-bid

Results: Affiliated Private Signals

Vickrey and Ausubel ascending-bid eliminate bottom end of revenue indeterminacy:



Conclusion

- Efficient auctions should be favored
- Treasury should try Ausubel ascending-bid
- IPOs should be auctioned

Competitive Bidding Behavior in Uniform-Price Auction Markets

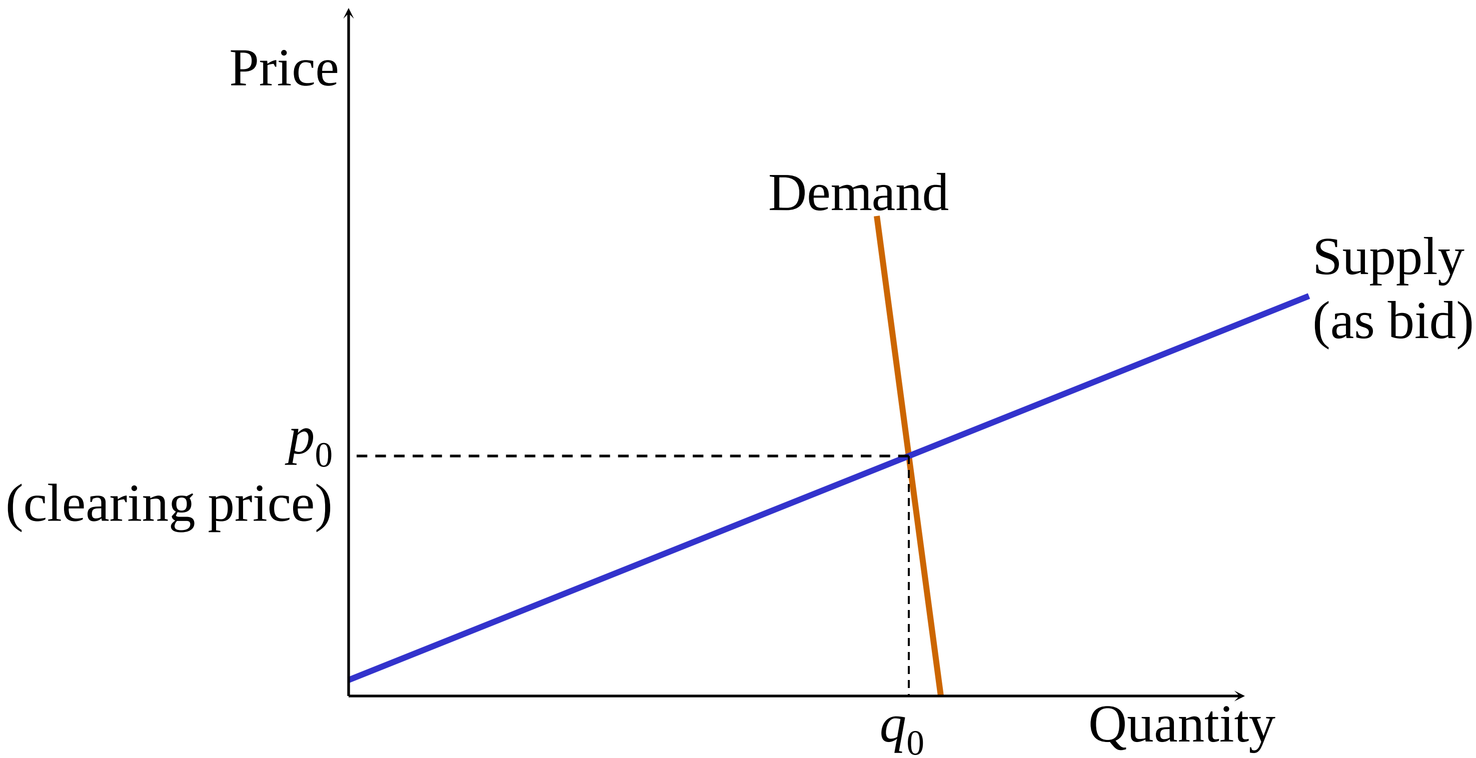
Peter Cramton
University of Maryland
6 January 2004

Summary

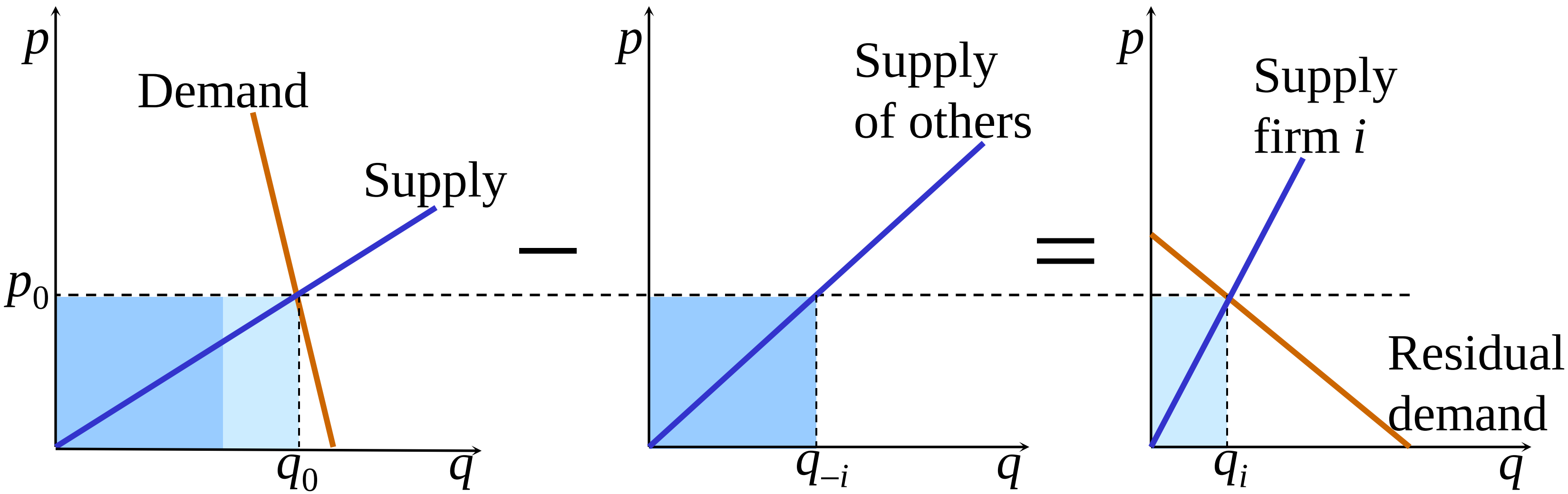
- Marginal cost bidding is a useful benchmark, but not a norm of behavior
- Profit maximization is an appropriate norm of behavior in markets
- Profit maximization should be expected and encouraged
- Market rules should be based on this norm

Uniform-price auction:

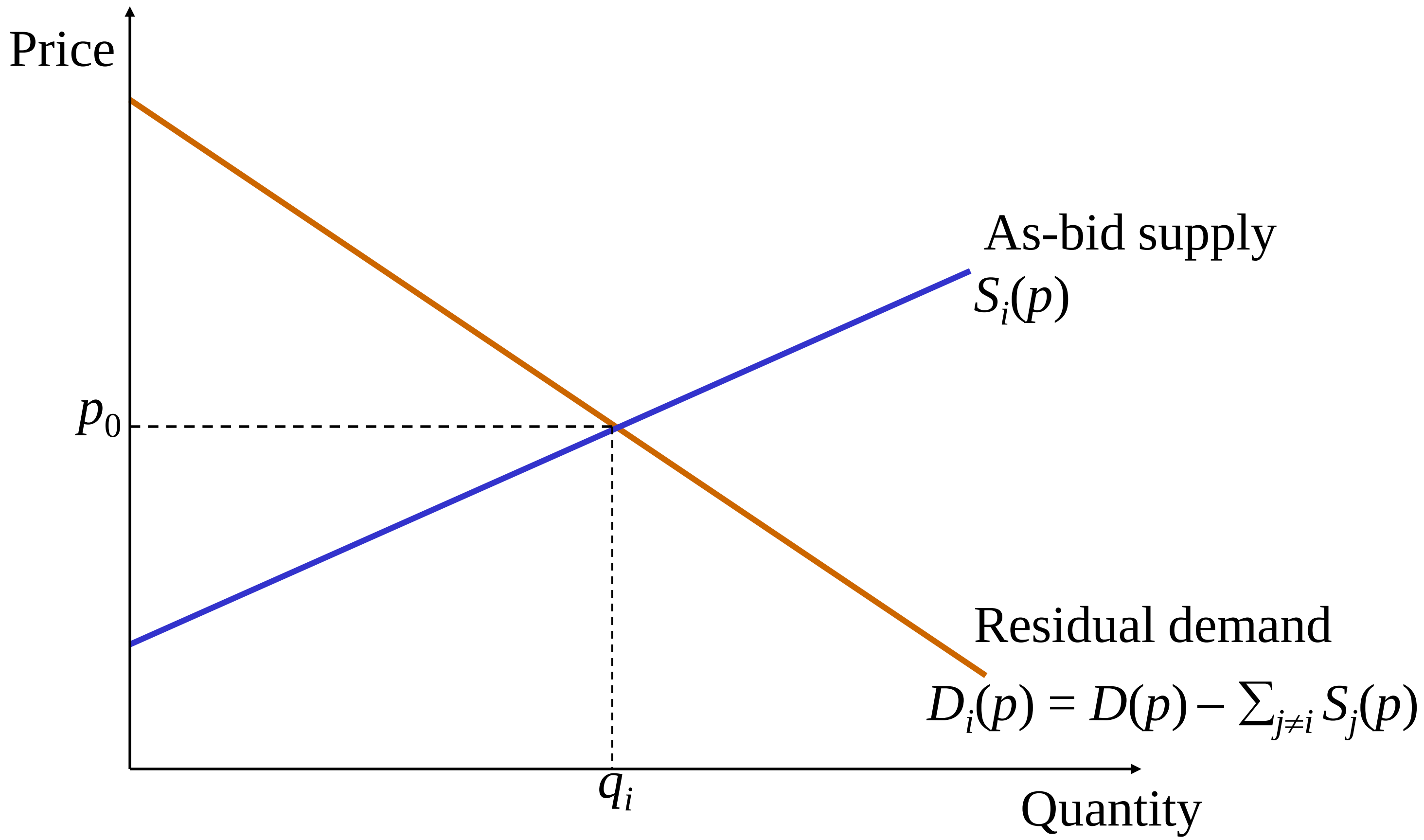
All bids below p_0 win and get paid p_0



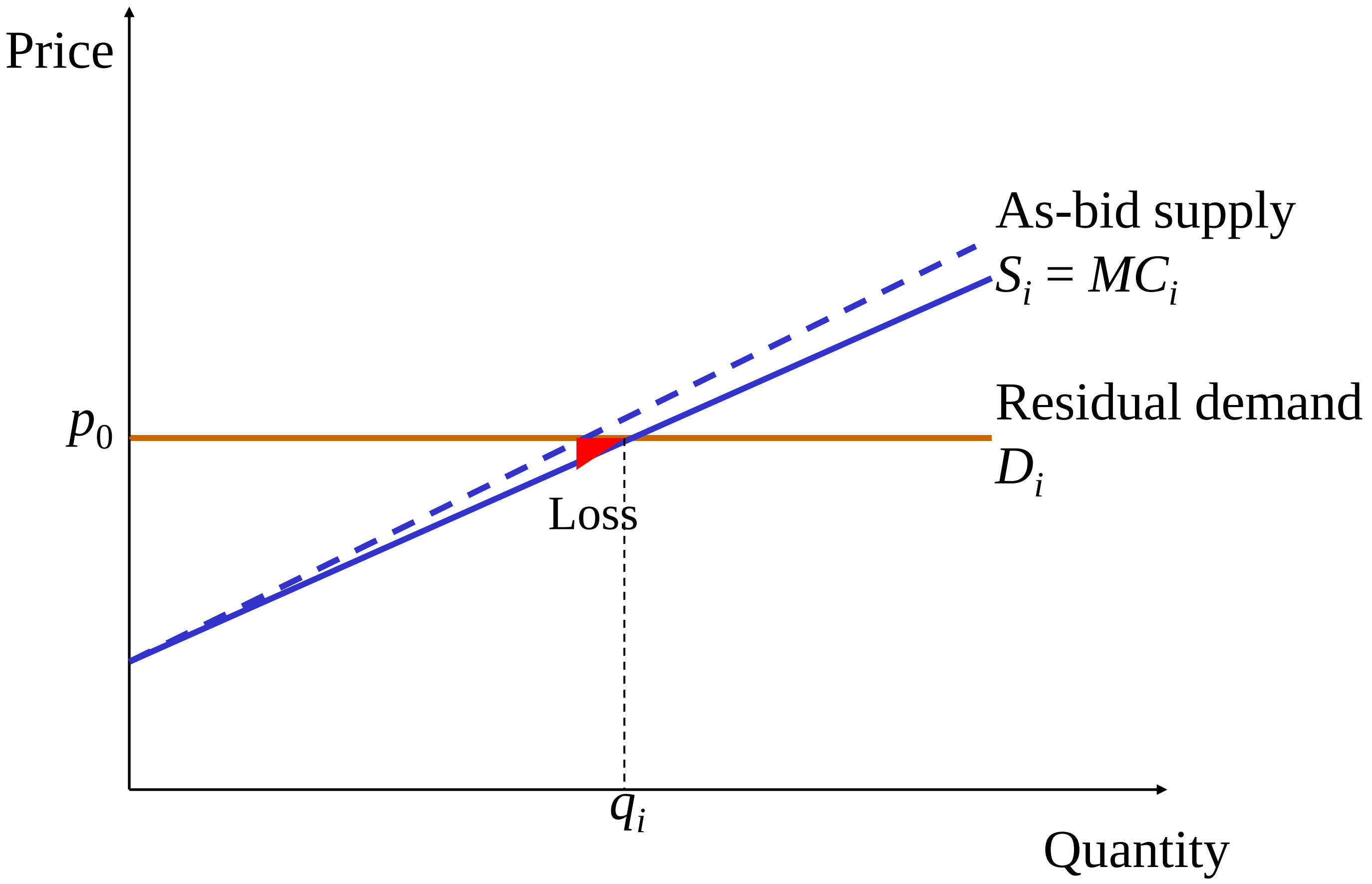
Residual demand removes supply of other bidders



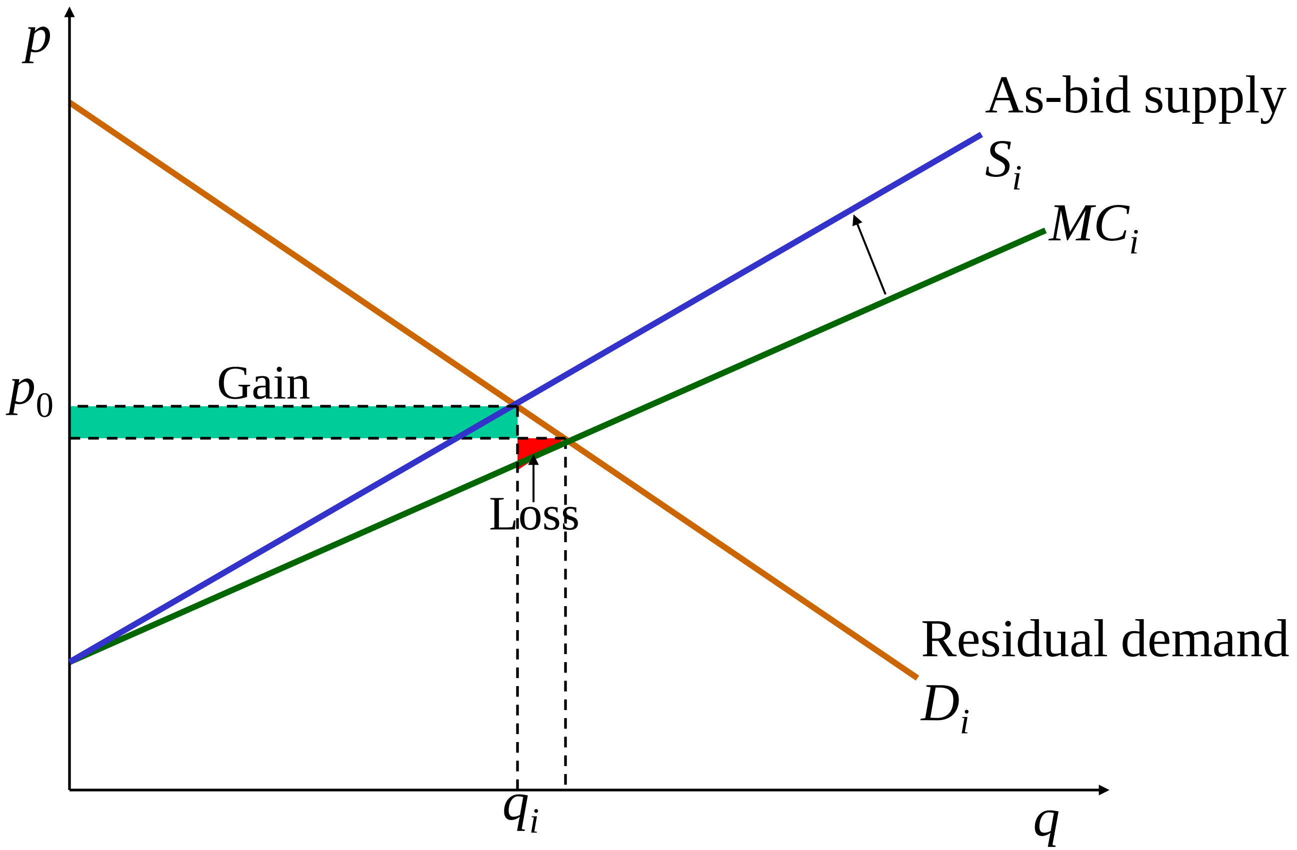
Residual demand curve



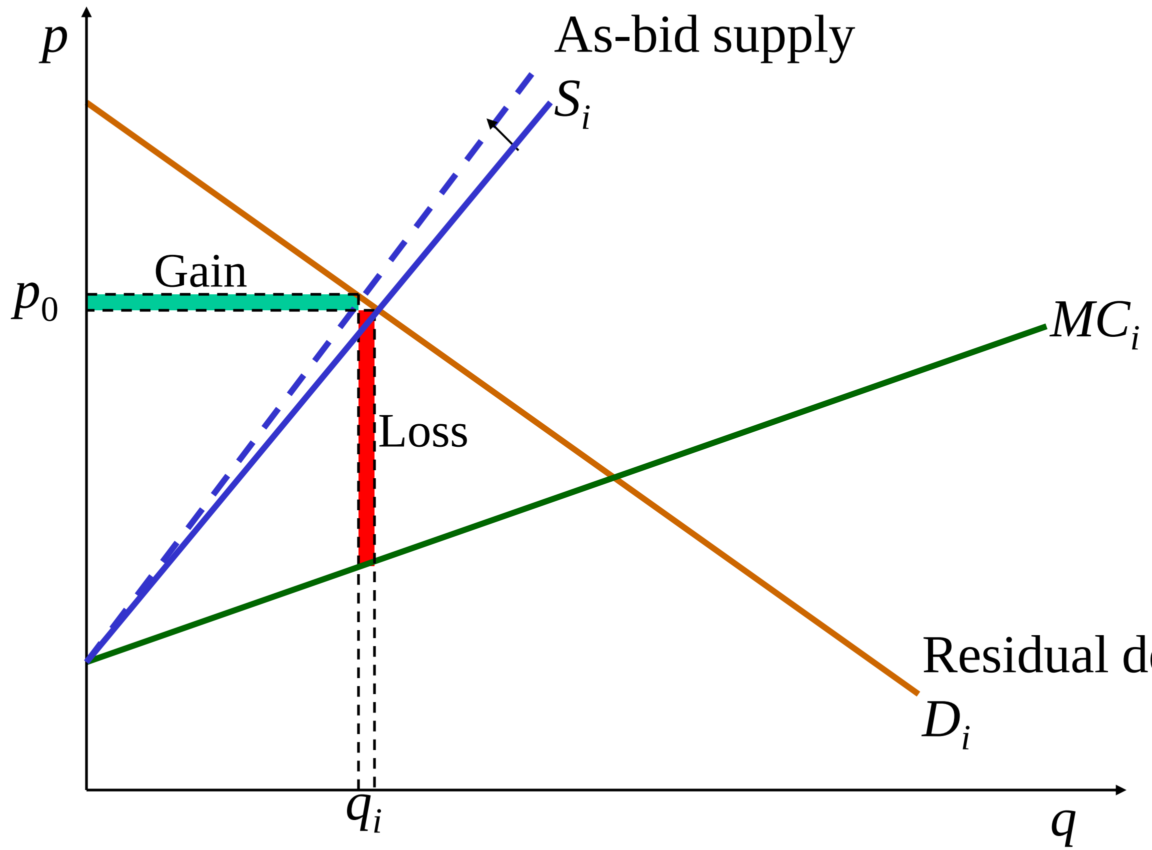
Bidding strategy with perfect competition



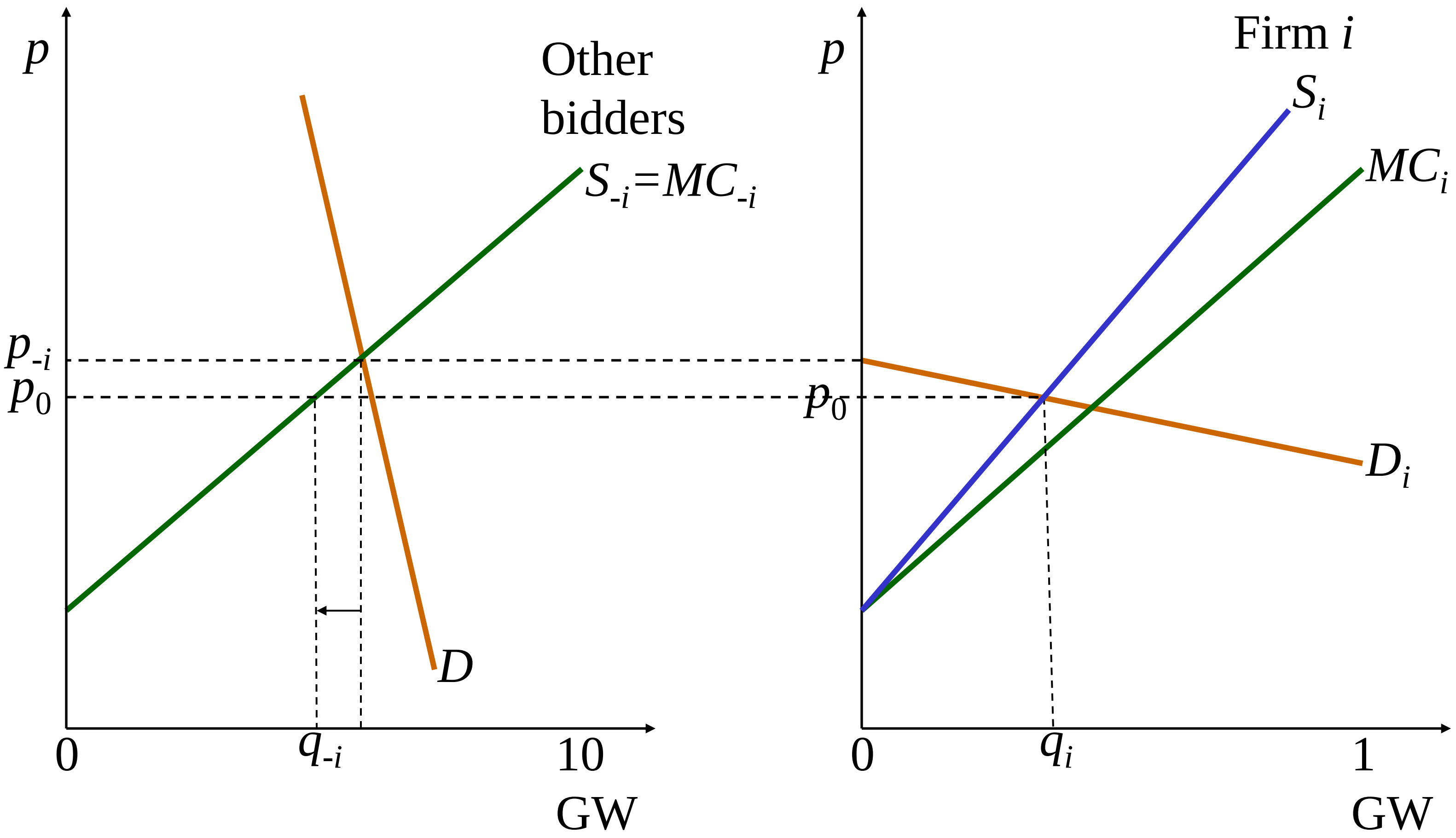
Incentive to bid above marginal cost: tradeoff higher price with reduced quantity



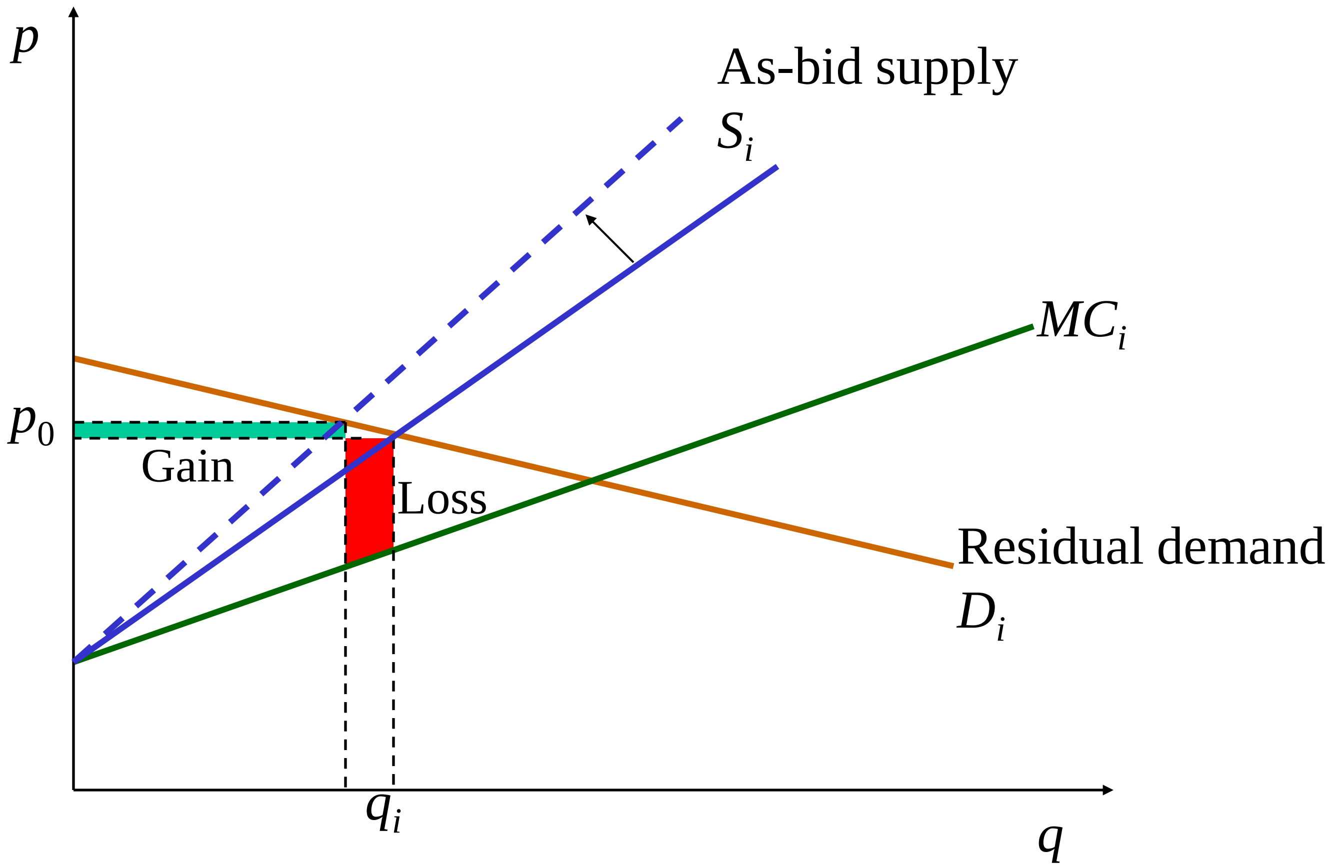
Optimal bid balances marginal gain and loss



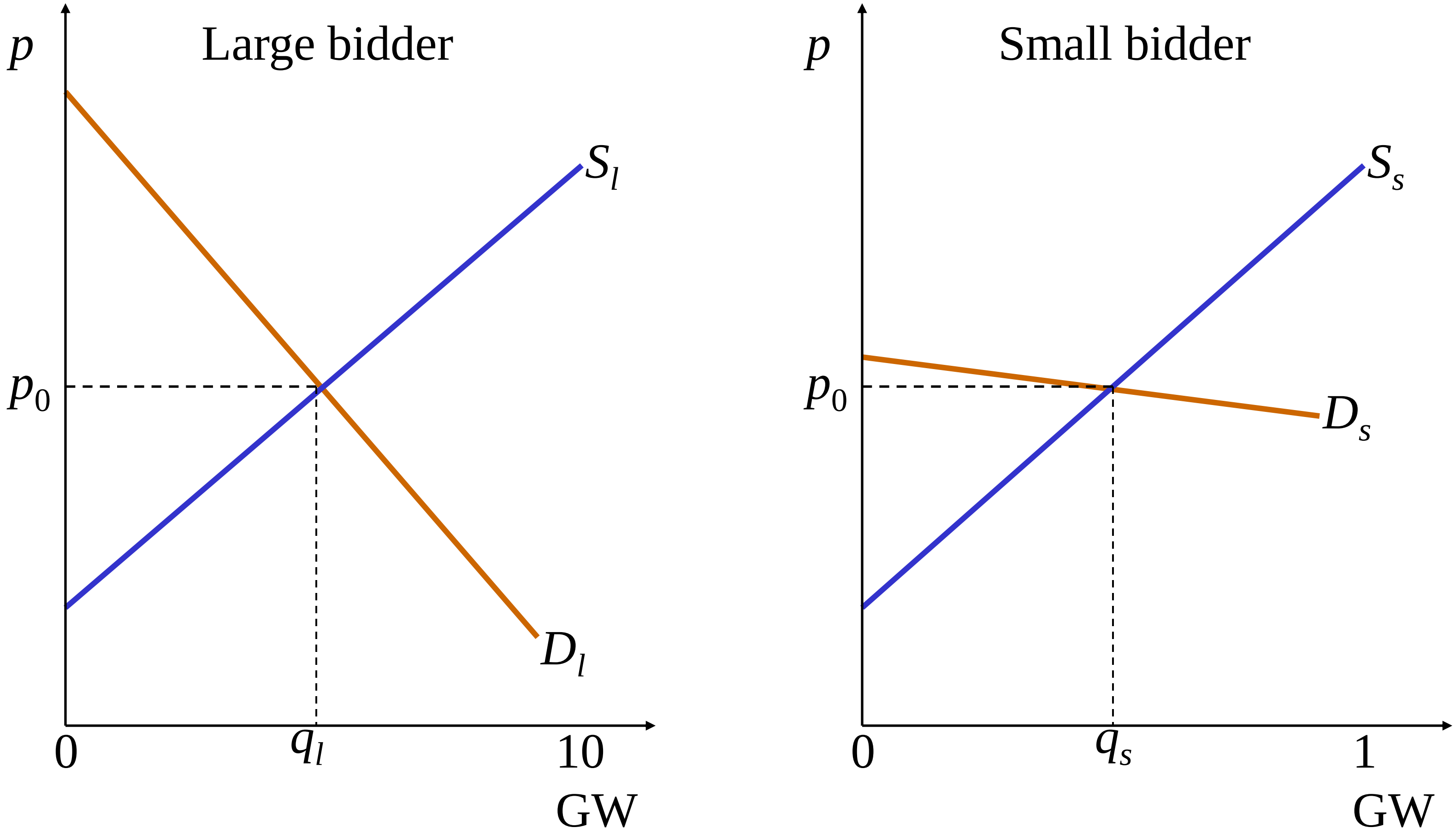
Still bid above marginal cost when others bid marginal cost



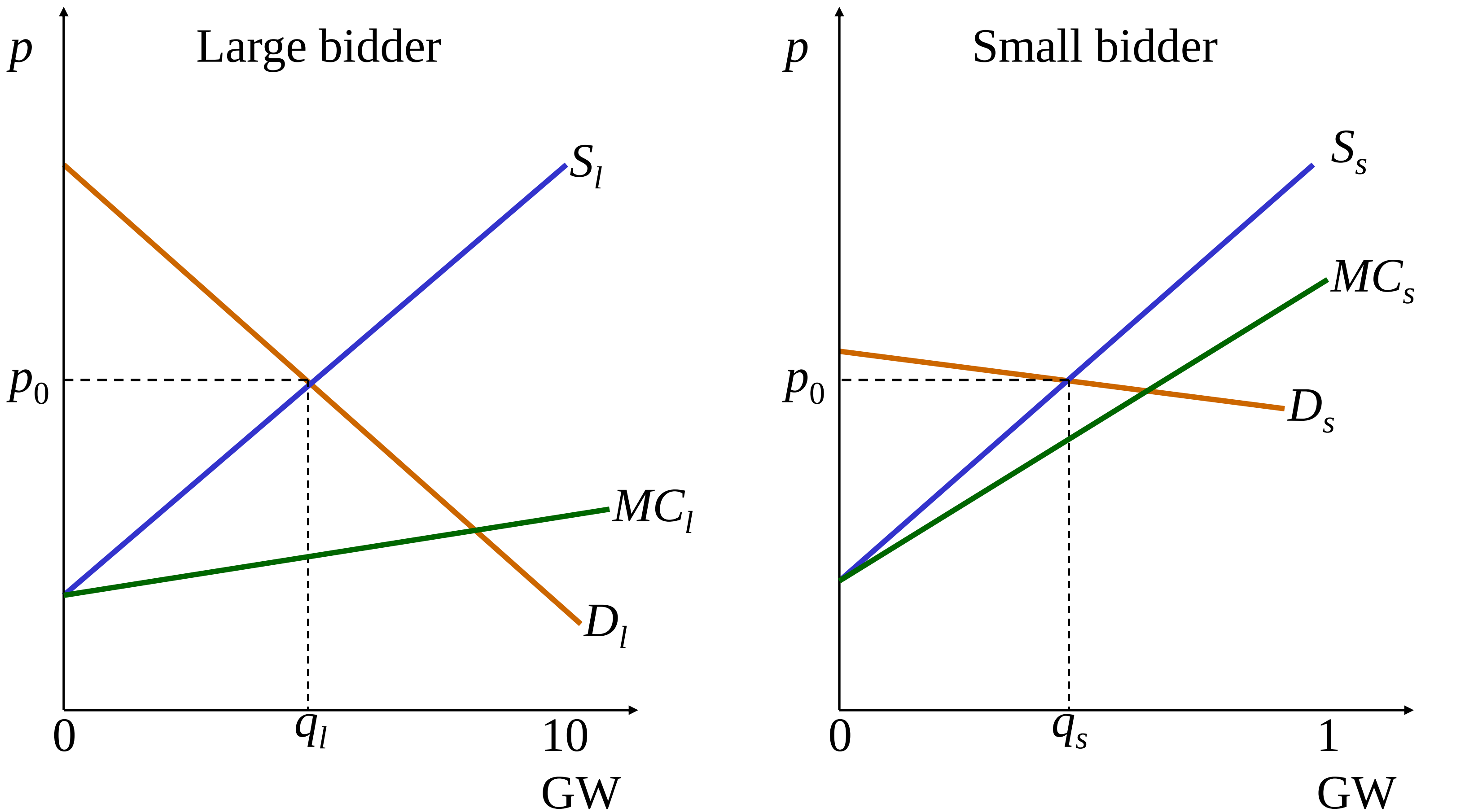
Residual demand response reduces incentive to inflate bids



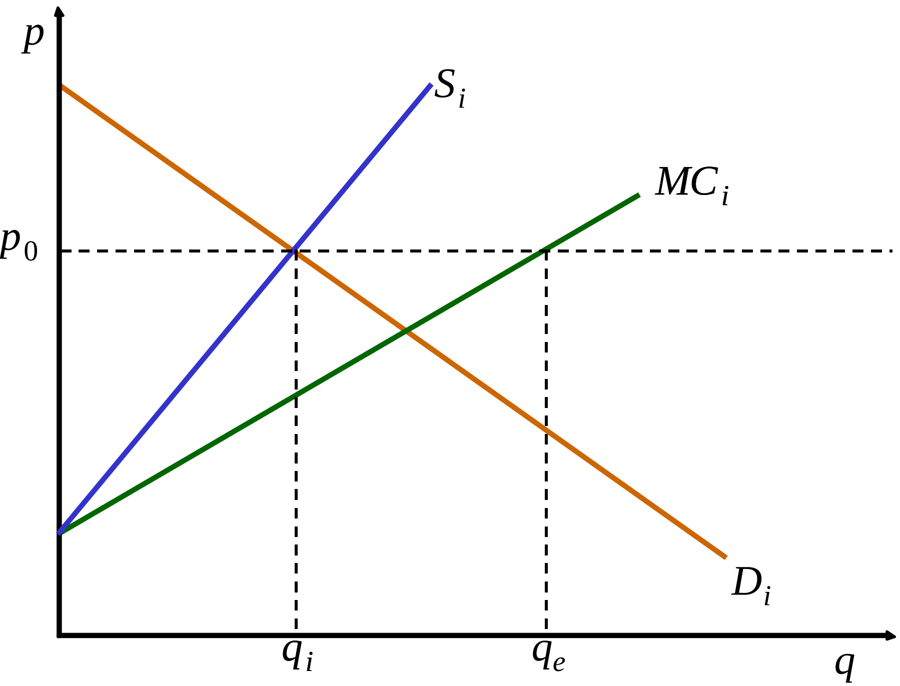
Residual demand is steeper for large bidders

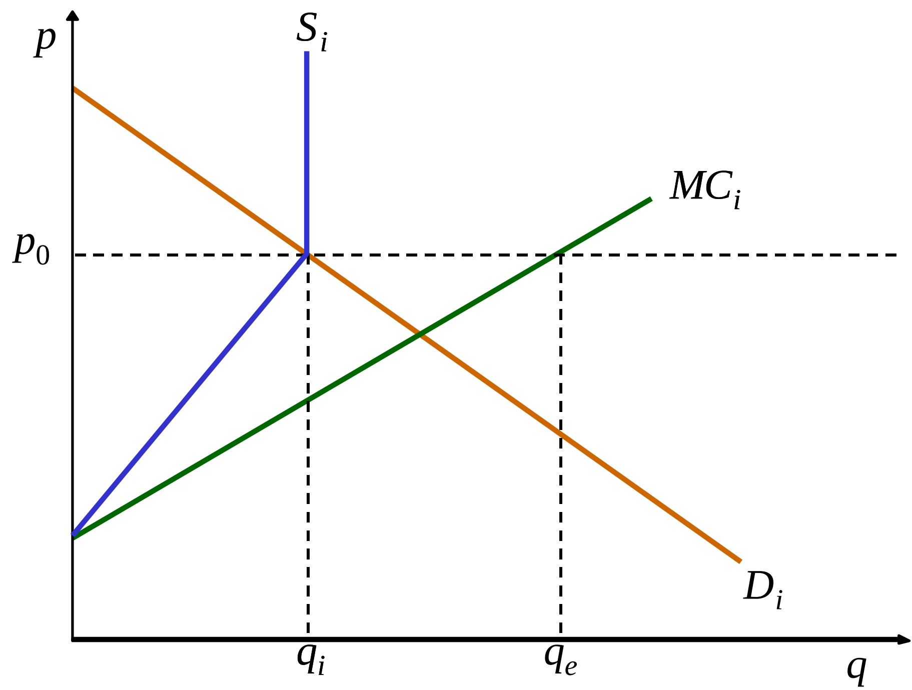


Large bidder makes room for its smaller rivals

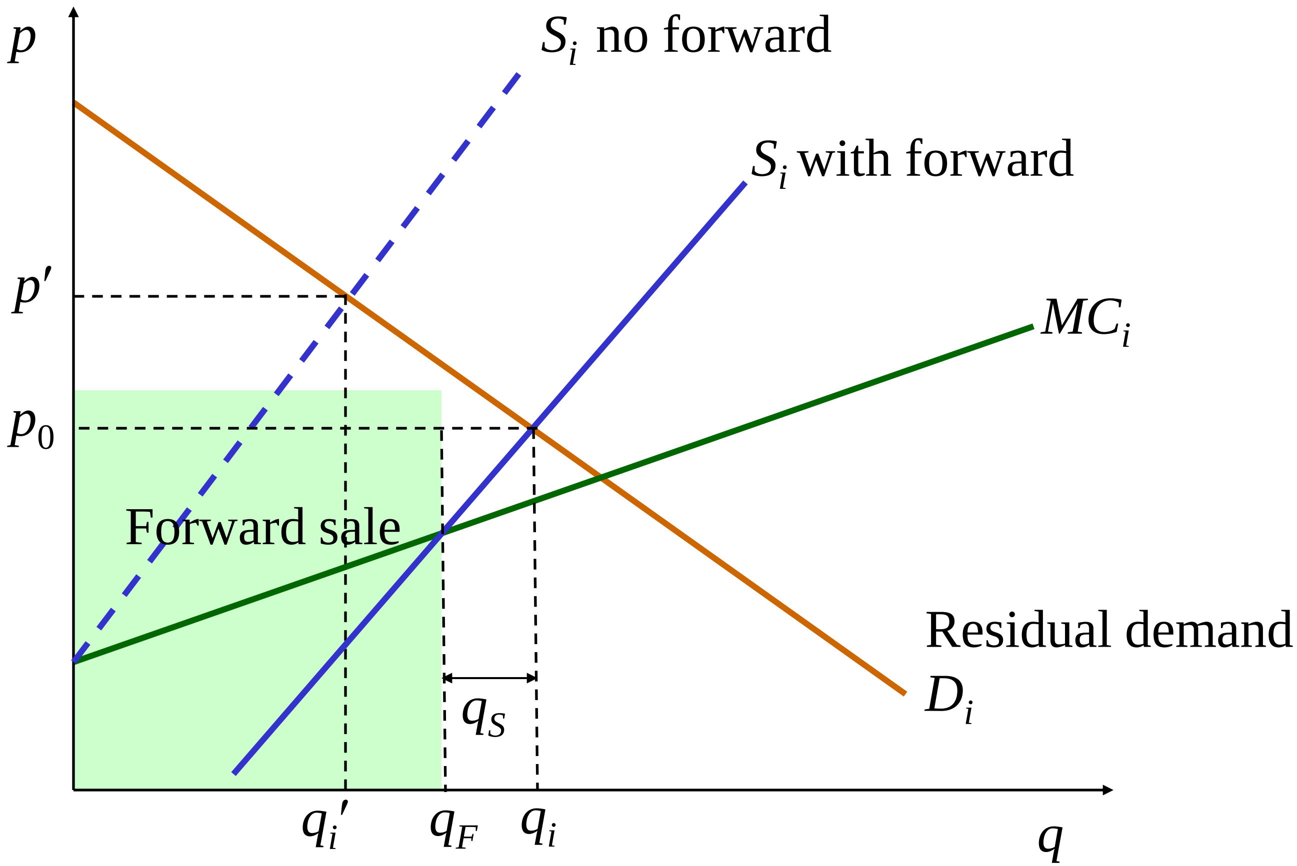


Economic vs. Physical Withholding





Forward contracts mitigate incentive to bid above marginal cost



California not more concentrated

California		New York		PJM		New England	
Owner	Share	Owner	Share	Owner	Share	Owner	Share
PG&E	17%	NYPA	17%	PSE&G	20%	PG&E NEG	17%
AES	9%	NRG Power	12%	PECO	17%	NRG	8%
Reliant	8%	LIPA	12%	PP&L	16%	Sithe	7%
Mirant	8%	Reliant	7%	GPU	13%	Notheast Util	6%
Duke	7%	Keyspan	6%	PEPCO	11%	Northeast Gen Serv	6%
SCE	6%	Constellation	5%	BG&E	11%	FP&L Energy	5%
Dynergy	6%	Entergy	5%	Connectiv	9%	Mirant	5%
Other	39%	Mirant	5%	Other	3%	Calpine	4%
		Dynergy	5%			Wisvest	4%
		AES	4%			Duke Energy	4%
		Sithe	3%			Other	33%
		Other	20%				
Total MW	44,682	Total MW	36,342	Total MW	65,067	Total MW	26,441
as of July 1999		as of January 2002		as of January 2000		as of January 2001	

Sources

Borenstein et al. (2002) NYISO Load and Capacity Data Singh and Jacobs (2000) Bushnell and Saravia (2002)