

1	2	3	4	Total

Final Examination

Do all four problems. Show work on exam. Use back of page if more space is needed.

Question 1

- (a) State the Folk Theorem of infinitely repeated games, defining the terms arising in the definition. (There is no need to define basic concepts of equilibrium.)

- (b) Suppose the game G below is played twice, the strategies used the first time being observed before the second play. Total payoffs are the undiscounted sum of payoffs in the two plays. Call the repeated game G^2 .

Game G

	d	e	f
a	12, 12	0, 18	0, 0
b	18, 0	10, 10	0, 0
c	0, 0	0, 0	5, 5

- (i) Show that there is no pure-strategy subgame-perfect equilibrium of G^2 in which (a,d) occurs in the *first* play.

- (ii) Exhibit and explain a mixed-strategy subgame-perfect equilibrium in which play in the first period is (a,d), and on the equilibrium path, play in the second period is (b,e). (Thus, the mixing is *off* the equilibrium path only.)

Question 2

Consider a Cournot market game. Each firm has unit cost $C = \$10$. Market price is given by $P = 90 - 4Q$, where Q is the industry output. Each firm i chooses quantity q_i to maximize its profits. Quantity is a continuous variable, and is chosen simultaneously.

(a) Suppose that there are two firms, so $Q = q_1 + q_2$. Find the Nash equilibrium of the Cournot game. What are equilibrium profits for each firm?

(b) Suppose that there are n firms, so $Q = q_1 + \dots + q_n$. Find the Nash equilibrium of the Cournot game as a function of n . What are per firm equilibrium profits as a function of n ?

(c) Suppose that firms decide to enter this market sequentially, and that there is a fixed cost of entry of \$95 per firm. After entry decisions are made, the firms that enter *simultaneously* select quantity, as in the Cournot game in part (b). How many firms will enter?

Question 3

Consider a Rubinstein bargaining game over a pie worth 10, but with an outside option for player 1. Players 1 and 2 alternate making offers until an offer is accepted or player 1 terminates negotiations. In the event of termination, player 1 receives an outside option of 4 and player 2 receives 0. Player 1 can terminate only when responding to an offer by player 2. Payoffs are discounted according to the discount factor $\delta < 1$. You may assume that δ is close to 1. Your task is to determine the bargaining outcome under two different interpretations of the discount factor.

(a) *Shrinking Pie.* Suppose that δ represents the fraction of the pie that remains after one period of disagreement (x tomorrow is worth δx today). Both the pie and the outside option are discounted. For example, if 1's initial offer of x is rejected and 2 offers y , which 1 accepts, then the discounted payoffs are $(\delta y, \delta(10 - y))$; whereas, if 1's initial offer of x is rejected and 2 offers y and then 1 terminates negotiations, then the discounted payoffs are $(4\delta, 0)$. What is a subgame perfect equilibrium in this bargaining game (be sure to fully specify strategies for each player)?

(b) *Random Breakdown.* Suppose that δ is the probability that negotiations are allowed to continue one more period. Hence, after each rejection, with probability $1 - \delta$ negotiations break down and player 1 gets her *undiscounted* outside option of 4 (player 2 gets 0). What is a subgame perfect equilibrium in this bargaining game?

(c) Interpret any difference in (a) and (b) or explain why there is no difference.

Question 4

Consider an independent private value auction with two bidders. The seller has two identical units to sell, and values each at zero. Bidder 1 wants just a single unit, and has a value u , uniformly distributed on $[0,1]$. Bidder 2 wants up to two units and has a constant marginal value v for each unit, uniformly distributed on $[0,1]$. Values u and v are independent, and both bidders are risk neutral. Each bidder submits two bids, the first bid for the first unit it wins and the second for the second unit it wins.

(a) Suppose a third-price auction is used: the highest two bids win and both winning units are priced at the highest-rejected bid (i.e., the third-highest bid). What is the best bid for each bidder for the first unit and why?

(b) What is Bidder 2's optimal bid b for the second unit in the third-price auction?

(c) Suppose the seller sets a reserve price $r = 1/2$. You can think of this as the seller submitting two bids of her own $(1/2, 1/2)$. Again, the two biggest of the six bids wins and the unit price is the third-highest bid; if there is a tie between the seller and a bidder, the good is awarded to the bidder. How does this change the answer to (a) and (b) above?