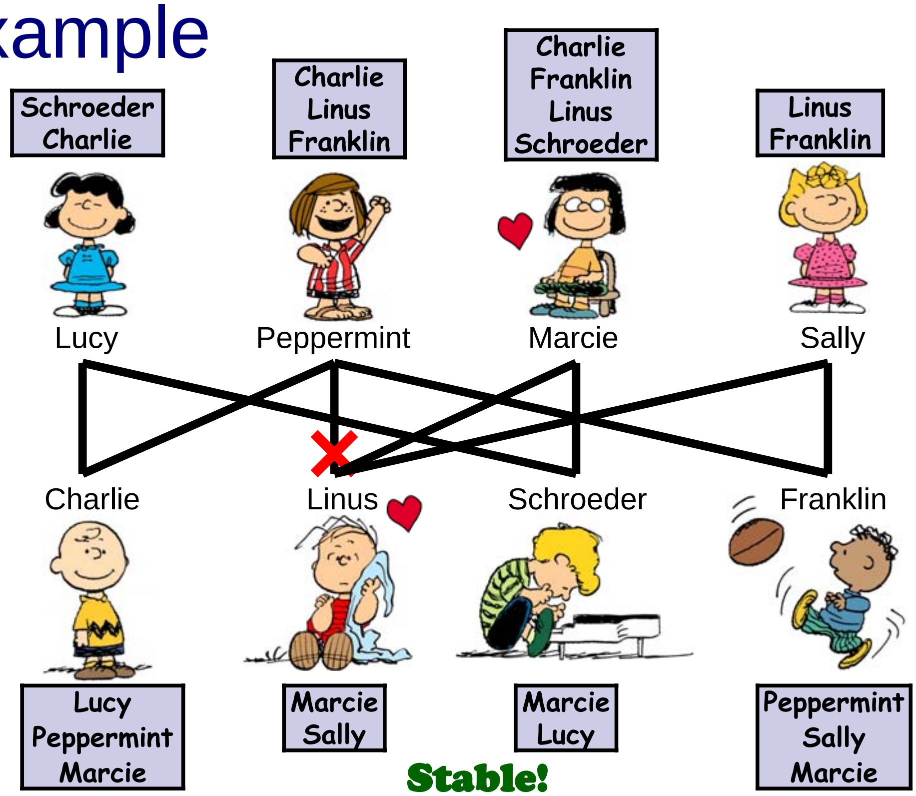
Stable Marriage

- Consider a set of *n* women and *n* men.
- Each person has an ordered list of some members of the opposite sex as his or her *preference list*.
- Let *μ* be a matching between women and men.
- A pair *(m, w)* is a *blocking pair* if both *m* and *w* prefer being together to their assignments under *μ*. Also, *(x, x)* is a blocking pair, if *x* prefers being single to his/her assignment under *μ*.
- A matching is *stable* if it does not have any blocking pair.

Example



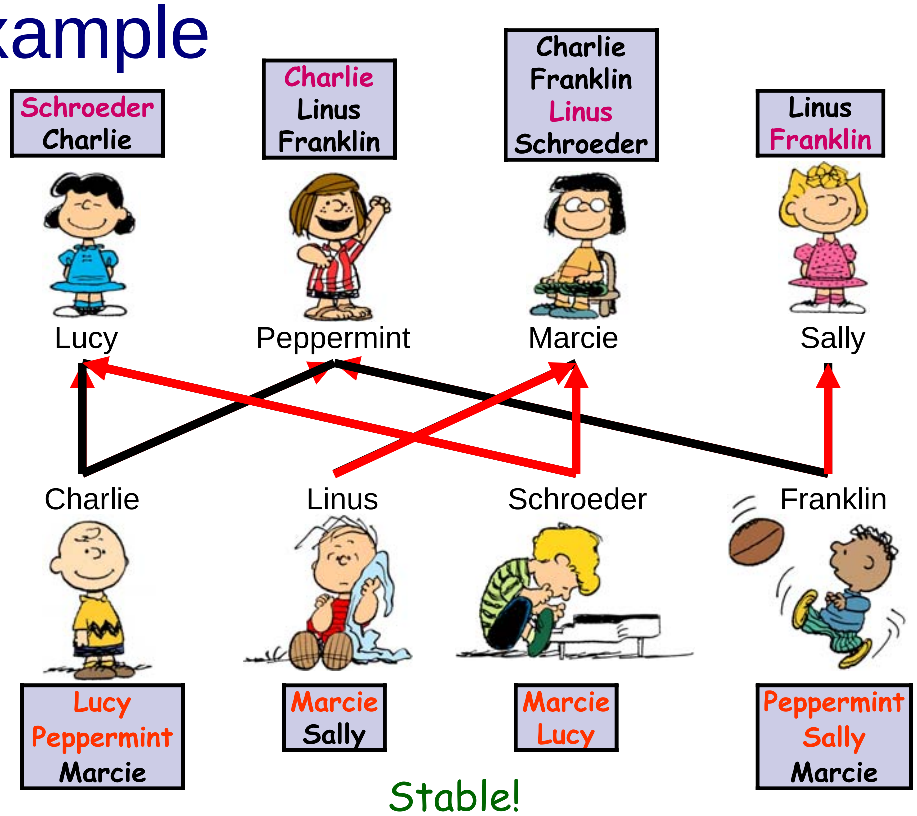
Deferred Acceptance Algorithms

(Gale and Shapley, 1962)

- In each iteration, an unmarried man proposes to the first woman on his list that he hasn't proposed to yet.
- A woman who receives a proposal that she prefers to her current assignment accepts it and rejects her current assignment.

This is called the men-proposing algorithm.

Example



Classical Results

- **Theorem 1.** The order of proposals does not affect the stable matching produced by the men-proposing algorithm.
- **Theorem 2.** The matching produced by the men-proposing algorithm is the *best* stable matching for men and the *worst* stable matching for women.

  This matching is called the *men-optimal* matching.
- **Theorem 3.** In all stable matchings, the set of people who remain single is the same.

Applications of stable matching

- Stable marriage algorithm has applications in the design of centralized two-sided markets. For example:
  - National Residency Matching Program (NRMP) since 1950's
  - Dental residencies and medical specialties in the US, Canada, and parts of the UK.
  - National university entrance exam in Iran
  - Placement of Canadian lawyers in Ontario and Alberta
  - Sorority rush
  - Matching of new reform rabbis to their first congregation
  - Assignment of students to high-schools in NYC
  - …

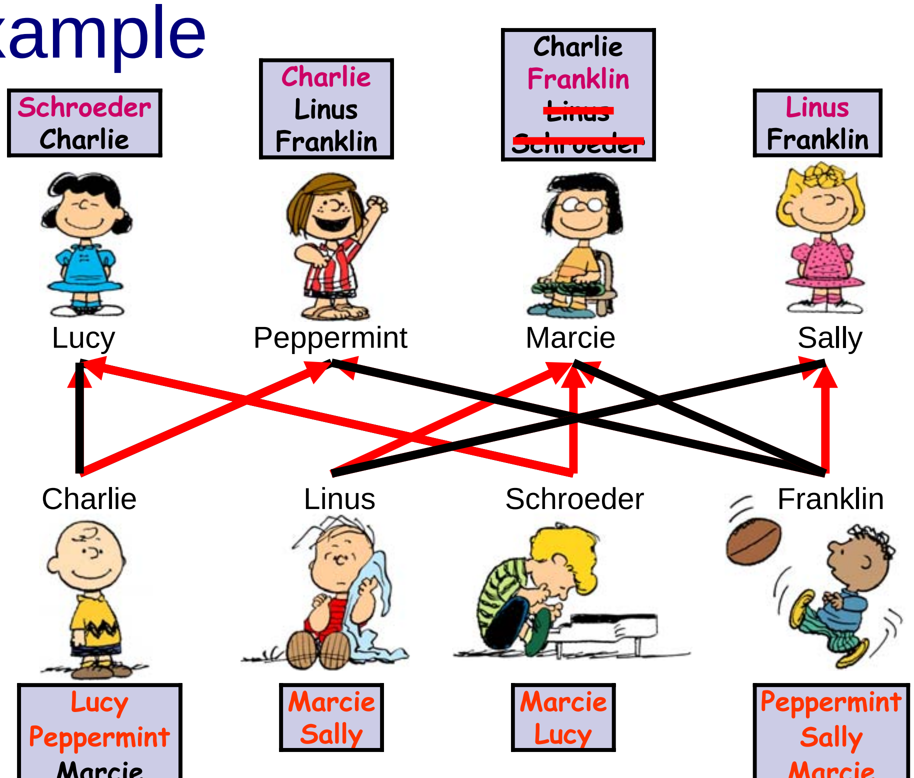
Incentive Compatibility

- Question: Do participants have an incentive to announce a list other than their real preference lists?

- Answer: Yes!

  In the men-proposing algorithm, sometimes women have an incentive to be dishonest about their preferences.

Example



Stable!

Incentive Compatibility

- Next Question: Is there any truthful mechanism for the stable matching problem?

- Answer: No!

  Roth (1982) proved that there is no mechanism for the stable marriage problem in which truth-telling is the dominant strategy for *both* men and women.

However, data from NRMP show that the chance that a participant can benefit from lying is slim.

	1993	1994	1995	1996
# applicants	20916	22353	22937	24749
# positions	22737	22801	22806	22578
# applicants who could lie	16	20	14	21

Number of applicants who could lie can be computed using the following theorem.

Theorem. The best match a woman can receive from a stable mechanism is her optimal stable husband with respect to her true preference list and others' announced preference lists.

In particular, a woman can benefit from lying only if she has more than one stable husband.

Explanations

(Roth and Peranson, 1999)

The following limit the number of stable husbands of women:

- Preference lists are correlated.

  Applicants agree on which hospitals are most prestigious; hospitals agree on which applicants are most promising.

  If all men have the same preference list, then everybody has a unique stable partner, whereas if preference lists are independent random permutations almost every person has more than one stable partner. (Knuth et al., 1990)

- Preference lists are short.

  Applicants typically list around 15 hospitals.

A Probabilistic Model

- Men choose preference lists uniformly at random from lists of at most *k* women.
- Women randomly rank men that list them.

Conjecture (Roth and Peranson, 1999): Holding *k* constant as *n* tends to infinity, the fraction of women who have more than one stable husband tends to zero.

Our Results

- **Theorem.** Even allowing women *arbitrary* preference lists in the probabilistic model, the expected fraction of women who have more than one stable husband tends to zero.

Economic Implications

- **Corollary 1.** When other players are truthful, almost surely a given player's best strategy is to tell the truth.
- **Corollary 2.** The stable marriage game has an equilibrium in which in expectation a (1-o(1)) fraction of the players are truthful.
- **Corollary 3.** In stable marriage game with incomplete information there is a (1+o(1))-approximate Bayesian Nash equilibrium in which everybody tells the truth.

Structure of proof

- Step 1: An algorithm that counts the number of stable husbands of a given woman.
- Step 2: Bounding the probability of having more than one stable husband in terms of the number of singles
- Step 3: Bounding the number of singles by the solution of the occupancy problem.