Three Theorems About Package Bidding

Based largely on
"Ascending Auctions with Package Bidding"
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June 2002

Outline

- Introduction: Complements and the need for package bidding.
- Understanding the laboratory successes of complex auction designs:
  - Theorem 1: proxy auction outcomes are in the (NTU) core with respect to reported preferences.
- Equilibrium in the TU proxy auction.
  - Theorem 2: Equilibrium in semi-sincere strategies (like in matching theory).
- Reasons to reject the Vickrey auction.
  - Theorem 3: "Good performance" of the Vickrey auction (various criteria) is guaranteed if and only if goods are substitutes.

Complements and the Need for Package Bidding

Exposure Problem in the Netherlands

- Variant of SAA completed February 18, 1998 after 137 rounds.
- Raised NLG 1.84 billion.
- Prices per band in millions of NLG
  - Lot A: 8.0
  - Lot B: 7.3
  - Lots 1-16: 2.9-3.6

Prices: Substitutes & Complements

- Theorem: If all items are **mutual substitutes** then (despite indivisibilities), a competitive equilibrium exists.
- Theorem (Milgrom, Gul-Stacchetti). If the set of possible valuations strictly includes the ones for which items are substitutes, then it includes a profile for which no CE exists.

	Item A	Item B	Package AB
Bidder 1	a	b	a+b+c
Bidder 2	$a+\alpha c$	$b+\alpha c$	a+b

- Market clearing prices do not exist if $.5<\alpha<1$.

Understanding the lab successes of complex auction designs

FCC-Cybernomics Experiment

Complementarity Condition:	None	Low	Medium	High
Efficiency				
SAA (No packages)	97%	90%	82%	79%
SAAPB ("OR" bids)	99%	96%	98%	96%
Revenues				
SAA (No packages)	4631	8538	5333	5687
SAAPB ("OR" bids)	4205	8059	4603	4874
Rounds				
SAA (No packages)	8.3	10	10.5	9.5
SAAPB ("OR" bids)	25.9	28	32.5	31.8

Scheduling Trains in Sweden

- Paul Brewer and Charles Plott
- Lab environment
  - Additive values for trains
  - Single N-S track
  - Complex "no crashing" constraint
- Ascending offer process
- Efficient outcomes

The General Proxy Model

- Each bidder *l* has
  - a finite set of feasible offers X_l and
  - a strict ordering over them represented by u_l.
- Auctioneer has
  - a feasible set $X \subset X_1 \times \ldots \times X_L$.
  - a strict ordering over X represented by u_0.
- Proxy auction rules
  - Auction proceeds in a sequence of rounds
  - Provisional winning bidders make no new bid
  - Others add "most preferred" remaining bid, unless "no trade" is preferred to that bid.
  - Auctioneer takes at most one bid per bidder to maximize u_0.

Proxy Auction Analysis

- Generalized Proxy Auction
  - By round *t*, proxy has proposed null bid and all packages for bidder satisfying a minimum profit constraint: $u_l(x_l) \geq \pi_l^t$
- At round *t*, the auctioneer tentatively accepts the feasible bid profile that maximizes $u_0(x^t)$.
  - Therefore, utility vector π^t is unblocked by any coalition *S*.
- Bidders not selected reduce their target utilities to include one new offer, but do not reduce below "zero" (the value of no trade).
  - Therefore, when the auction ends, the utility allocation is feasible.

Proxy Auctions & the Core

- Theorem 1. The generalized proxy auction terminates at a (non-transferable-utility) core allocation relative to reported preferences.
- Proof. The payoff vector is unblocked at every round, and the allocation is feasible when the auction ends. **QED**

The Quasi-linear (TU) Case

- Seller's revenue at round t is given by:

$$\begin{aligned}
\pi_0^t &= \max_{x \in X} \sum\nolimits_{l \neq 0} B_l^t(x_l) \\
&= \max_{x \in X} \sum\nolimits_{l \neq 0} \max\left(0, v_l(x_l) - \pi_l^t\right) \\
&= \max_{x \in X} \left[\max_{S \subset L} \sum\nolimits_{l \in S \setminus 0} v_l(x_l) - \pi_l^t \right] \\
&= \max_{S \subset L} \left[\max_{x \in X} \sum\nolimits_{l \in S \setminus 0} v_l(x_l) - \pi_l^t \right] \\
&= \max_{S \subset L} \left[w(S) - \sum\nolimits_{l \in S \setminus 0} \pi_l^t \right] \\
&\therefore (\forall S) w(S) \leq \sum\nolimits_{l \in S} \pi_l^t
\end{aligned}$$

- Payoffs are unblocked at every round
- "Coalitional second price auction"

Applications (w/o Proxies!?!)

- Train Schedules (Brewer-Plott)
  - Bidders report additive values for each train
  - Auctioneer maximizes total bid at a round, respecting scheduling constraints (to avoid crashes).
- FCC package auctions
  - Bidders report valuations of packages
  - Final outcome is a "core allocation" (for the reported preferences).
- Package Auctions with Budget Constraints
  - Bidders report valuations and a budget limit.
  - Final outcome is a "core allocation."

A Novel "Matching" Procedure

- Uniquely among deferred acceptance algorithms:
  - Offers are multidimensional and/or package offers
  - Feasible sets may be arbitrarily complex
  - The algorithm is ***not*** monotonic over "held offers": it may backtrack to take previously rejected offers
  - The analysis does ***not*** employ a "substitutes" condition.
  - The outcome may ***not*** be a bidder-Pareto-optimal point in the core.
- Unique in matching theory analysis
  - Equilibrium will be characterized with complex offers.

Equilibrium in a TU Proxy Auction

Formulation

- Assume that all payoffs are quasi-linear
  - For bidders: value received less money paid.
  - For seller: value of allocation plus money received.
- Consider limiting process as the size of the bid increments goes to zero.
  - Focus shifts to transferable utility core.
  - Call this the “TU-proxy auction.”

The Substitutes Case

- Theorem. In the TU-proxy auction, suppose that the set of possible bidder values **V** includes all the purely additive values. Then these three statements are equivalent:
  - The set **V** includes only values for which goods are substitutes.
  - For every profile of bidder valuations drawn from **V**, sincere bidding is an ex post Nash equilibrium of the proxy auction.
  - For every profile of bidder valuations drawn from **V**, sincere bidding results in the Vickrey allocation and payments for all bidders.

"Semi-Sincere" Bidding

- Definitions. A strategy in a direct revelation trading mechanism is "semi-sincere" if it can be obtained from sincere reporting by changing the utility of the "no trade" outcome.
- Theorem. In the TU-proxy auction, fix any pure strategy profile of other bidders and let π_l be bidder l's maximum profit. Then, bidder l has a semi-sincere best reply, which is report to its proxy that its values are given by $v_l(x) - \pi_l$.
  - An anti-collusion property.

Selected Equilibria

- Selection criterion ☹
  - All bidders play semi-sincere strategies
  - Losers play sincere strategies
- Theorem 2. Let π be a bidder-Pareto-optimal point in *Core*(*L*,*w*) with respect to actual preferences. Then in the TU-proxy auction, semi-sincere strategies with values reduced by π constitute a (full-information) Nash equilibrium. Moreover, for any equilibrium satisfying the selection criterion, the payoff vector has bidder profits in *Core*(*L*,*w*).

Vickrey auctions for complements?

Vickrey Auction Rules

- Bids and allocations
  - One or more goods of one or more kinds
  - Each bidder i makes bids $b_i(x)$ on all bundles
  - Auctioneer chooses the feasible allocation $x^* \in X$ that maximizes the total bid accepted
- Vickrey ("pivot") payments for each bidder i are:

$$p_i = \max_{x \in X} \sum_{j \neq i} b_j(x_j) - \sum_{j \neq i} b_j(x_j^*)$$

- *Vickrey auction advantages are well known, but there are also important disadvantages.*

Direct vs. Indirect Mechanisms

- The Vickrey auction is a direct mechanism, requiring the bidder to evaluate 2^N packages to make its bids.
- Indirect mechanisms may be favored (CRA Report to FCC: Milgrom, et al) to economize on valuation efforts.

Vickrey: Substitutes & Other

- Theorem 3. Suppose that the set of possible bidder values **V** includes all the purely additive values. Then these six statements are all equivalent:
  - The set **V** includes only values for which goods are substitutes.
  - For every profile of bidder valuations drawn from **V**, Vickrey auction revenue is isotone in the set of bidders.
  - For every profile… **V**, Vickrey payoffs are in the core.
  - For every profile… **V**, there is no profitable shill ("false name") bidding strategy in the Vickrey auction.
  - For every profile… **V**, there is no profitable joint deviation by losing bidders in the Vickrey auction.

Monotonicity and Revenue Problems

- Vickrey Auction and the Core
  - Two identical spectrum bands for sale
  - Bidders 1 wants the pair only and will pay up to $2 billion.
  - Bidders 2 and 3 want single license and will pay up to $2B.
  - Outcome:
    - Bidders 2 and 3 acquire the licenses.
    - Price is zero.
- Problems in this example:
  - Adding bidder 3 reduces revenue from $2B to zero.
  - The Vickrey outcome lies outside the core.
- Conclusions change if 1 will pay up to $1B each.
  - Substitutes condition is the key.

The Shills Problem

- Yokoo, Sakurai and Matsubara (2000) emphasize "false name bids" in Vickrey Internet auctions.
- Example: two identical spectrum bands for sale
  - Bidder 1 wants only the pair, will pay up to $2B.
  - Bidder 2 is willing to pay $0.5B each, $1B for the pair
  - By bidding $2B for each license using two names, bidder 2 can win both licenses at a price of zero.
- The Vickrey auction is vulnerable to shill bidders.
- Conclusion changes if 1 will pay up to $1B each.
  - Substitutes condition is the key.

Loser Collusion

- Example: two identical spectrum bands for sale
  - Bidder 1 wants only the pair, will pay up to $2B.
  - Bidder 2 is willing to pay $0.5B for one
  - Bidder 3 is willing to pay $0.5B for one
  - Losing bidders 2 and 3 have a profitable joint deviation, bidding $2B each, winning both licenses at a price of zero.
- The Vickrey auction is unique in its vulnerability to collusion even among losing bidders
- Conclusion changes if 1 will pay up to $1B each.
  - Substitutes condition is the key.

Vickrey's "Efficiency Problem"

- Example: 2 licenses, East and West
  - Bidder 1 has value $1.2B for the pair
  - Bidder 2 has value of $1B for East
  - Bidder 3 has value $1B for West
  - *Merged bidders 2 & 3 have value $2.5 for E-W package*
- Vickrey price and profit effects of a merger
  - Unmerged firms total price is $400 million, profit of $1.6B.
  - Merged firm's price is $1.2 billion, profit $1.3B
- Incentive is not to merge; value is not maximized.
  - Result reverses if 1's value is $0.6B per license.

Comparing Auctions

- + means "has the property generally"
- * means "has the property when goods are substitutes"

Property	Vickrey Auction	Proxy Auction
Sincere bidding is a Nash equilibrium.	+	*
Equilibrium outcomes are in the core.	*	+
No profitable shill bids	*	+
No profitable joint deviations for losers	*	+
Competing technologies property	No	+
Fully adaptable to limited budgets	No	+

The End