Auctioning Many Similar Items

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Examples of auctioning similar items
- Treasury bills
- Stock repurchases and IPOs
- Telecommunications spectrum
- Electric power
- Emissions permits

Ways to auction many similar items
- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)

Pay-as-bid Auction:
All bids above $P_0$ win and pay bid

Uniform-Price Auction:
All bids above $P_0$ win and pay $P_0$

Vickrey Auction:
All bids above $P_0$ win and pay opportunity cost
Payment rule affects behavior

More ways to auction many similar items

- Pay-as-bid
- Uniform-price
- Vickrey

Demand

More ways to auction many similar items

- Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  - Standard ascending-bid
  - Ausubel ascending-bid (Ausubel 1997)

Standard Ascending-Bid Auction:
All bids at $P_0$ win and pay $P_0$

Ausubel Ascending-Bid:
All bids at $P_0$ win and pay price at which clinched

More ways to auction many similar items

- Ascending-bid
  - Simultaneous ascending auction (FCC spectrum)
- Sequential
  - Sequence of English auctions (auction house)
  - Sequence of Dutch auctions (fish, flowers)
- Optimal auction
  - Maskin & Riley 1989

Research Program
How do standard auctions compare?

- Efficiency
  - FCC: those with highest values win
- Revenue maximization
  - Treasury: sell debt at least cost
**Efficiency**

*(not pure common value; capacities differ)*

- Uniform-price and standard ascending-bid
  - Inefficient due to demand reduction
- Pay-as-bid
  - Inefficient due to different shading
- Vickrey
  - Efficient in private value setting
  - Strategically simple: dominant strategy to bid true demand
  - Inefficient with affiliated information
- Ausubel ascending-bid
  - Same as Vickrey with private values
  - Efficient with affiliated information

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**Inefficiency Theorem**

*In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values.*

- Winning bidder influences price with positive probability
- Creates incentive to shade bid
- Incentive to shade increases with additional units
- Differential shading implies inefficiency

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**Inefficiency from differential shading**

- Large Bidder
- Small Bidder

Large bidder makes room for smaller rival

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**Vickrey inefficient with affiliation**

- Winner’s Curse in single-item auctions
  - Winning is bad news about value
- Winner’s Curse in multi-unit auctions
  - Winning more is worse news about value
  - Must bid less for larger quantity
  - Differential shading creates inefficiency in Vickrey

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**What about seller revenues?**

- Price
- Pay-as-bid
- Uniform-Price
- Vickrey
- Residual Supply $Q_5 - \sum_{j \neq i} Q_j(p)$
- Demand $Q(p)$
- Quantity

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**Uniform price may perform poorly**

- Independent private values uniform on $[0,1]$
- 2 bidders, 2 units; L wants 2; S wants 1
- Uniform-price: unique equilibrium
  - S bids value
  - L bids value for first and 0 for second
  - Zero revenue; poor efficiency
- Vickrey
  - price $= v(2)$ on one unit, zero on other
Standard ascending-bid may be worse
• 2 bidders, 2 units; L wants 2; S wants 2
• Uniform-price: two equilibria
  – Poor equilibrium: both L and S bid value for 1
    • Zero revenue; poor efficiency
  – Good equilibrium: both L and S bid value for 2
    • Get $v_1$ for each (max revenue) and efficient
• Standard ascending-bid: unique equilibrium
  – Both L and S bid value for 1
    • S’s demand reduction forces L to reduce demand
    • Zero revenue; poor efficiency

Efficient auctions tend to yield high revenues

Theorem. With flat demands drawn independently from the same regular distribution, seller’s revenue is maximized by awarding good to those with highest values.

Generalizes to non-private-value model with independent signals:

$$v_i = u(s_i, s)$$

Award good to those with highest signals if downward sloping MR and symmetry.

Downward-sloping demand:

$$p_i(q_i) = v_i - g_i(q_i)$$

Theorem. If intercept drawn independently from the same distribution, seller’s revenue is maximized by

– awarding good to those with highest values if constant hazard rate
– shifting quantity toward high demanders if increasing hazard rate

• Note: uniform-price shifts quantity toward low demanders

But uniform price has advantages

• Participation
  – Encourages participation by small bidders (since quantity is shifted toward them)
  – May stimulate competition
• Post-bid competition
  – More diverse set of winners may stimulate competition in post-auction market

Auctioning Securities

A pure common-value model with affiliation

• $n$ risk-neutral symmetric bidders
• Each bidder has pure common value $V$ for security and can purchase any quantity (flat demand curve w/o capacity)

Models

• Common uncertainty
  – Bidders have no private information
• Affiliated private signals
  – Bidder $i$ gets signal $S_i$
  – Random variables $V$, $S_1$, $S_2$, $S_n$ are affiliated
Results: Common Uncertainty

**Proposition.** (Wilson ‘79; Maxwell ‘83; Back & Zender ‘93)
- Wide range of prices can be supported as equilibrium in uniform-price auction, even if supply is stochastic; highest yields EV

**Proposition.** (Wang & Zender ‘96)
- Many equilibria in pay-as-bid auction, even if supply is stochastic; highest yields EV
- Indeterminacy avoided if set reserve price (even 0)

Results: Affiliated Private Signals

- With affiliated signals, each auction format has a “simple equilibrium” where bidders submit flat demand curves
- Conjecture: These simple equilibria provide upper bounds on revenues from each format
- Std. ascending-bid > Uniform > Pay-as-bid

Vickrey and Ausubel ascending-bid eliminate bottom end of revenue indeterminacy:

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Conclusion

- Efficient auctions should be favored
- Treasury should try Ausubel ascending-bid
- IPOs should be auctioned