Auctioning Many Similar Items

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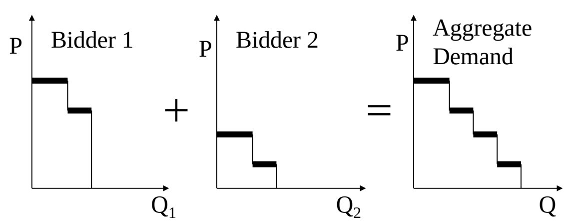
Examples of auctioning similar items

- Treasury bills
- Stock repurchases and IPOs
- Telecommunications spectrum
- Electric power
- Emissions permits

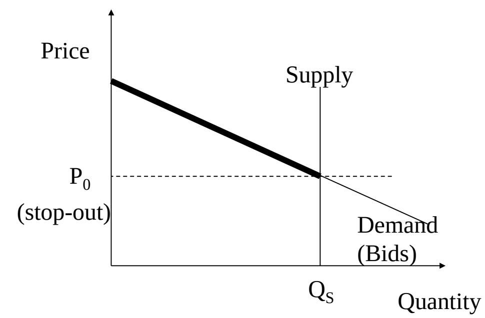
Ways to auction many similar items

- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)



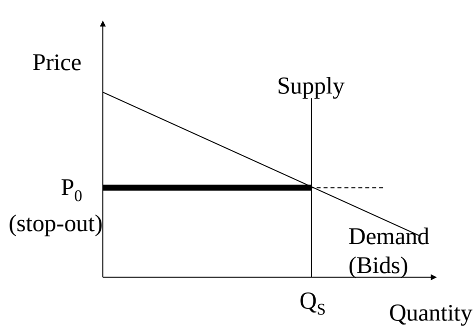
Pay-as-bid Auction:

All bids above P_0 win and pay bid



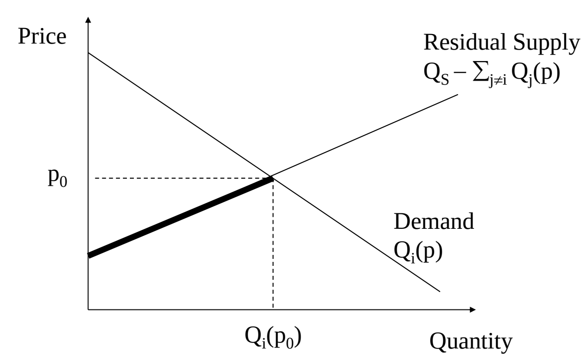
Uniform-Price Auction:

All bids above P_0 win and pay P_0

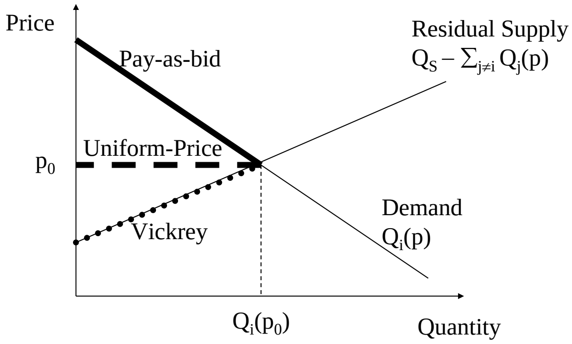


Vickrey Auction:

All bids above P_0 win and pay opportunity cost



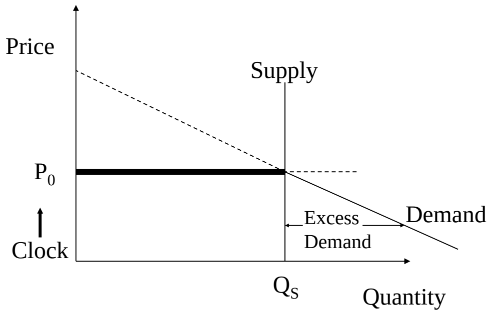
Payment rule affects behavior



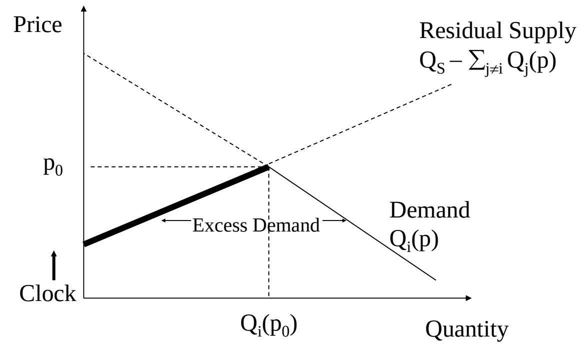
More ways to auction many similar items

- Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  – Standard ascending-bid
  – Ausubel ascending-bid (Ausubel 1997)

Standard Ascending-Bid Auction:
All bids at P_0 win and pay P_0



Ausubel Ascending-Bid:
All bids at P_0 win and pay price at which clinched



More ways to auction many similar items

- Ascending-bid
  – Simultaneous ascending auction (FCC spectrum)
- Sequential
  – Sequence of English auctions (auction house)
  – Sequence of Dutch auctions (fish, flowers)
- Optimal auction
  – Maskin & Riley 1989

Research Program
How do standard auctions compare?

- Efficiency
  – FCC: those with highest values win
- Revenue maximization
  – Treasury: sell debt at least cost

Efficiency
(not pure common value; capacities differ)

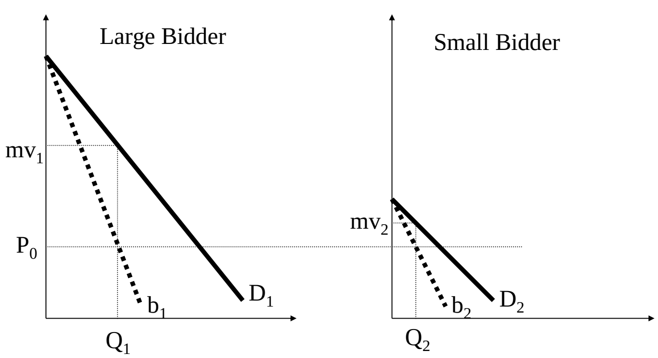
- Uniform-price and standard ascending-bid
  - Inefficient due to demand reduction
- Pay-as-bid
  - Inefficient due to different shading
- Vickrey
  - Efficient in private value setting
  - Strategically simple: dominant strategy to bid true demand
  - Inefficient with affiliated information
- Ausubel ascending-bid
  - Same as Vickrey with private values
  - Efficient with affiliated information

Inefficiency Theorem

In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values.

- Winning bidder influences price with positive probability
- Creates incentive to shade bid
- Incentive to shade increases with additional units
- Differential shading implies inefficiency

Inefficiency from differential shading

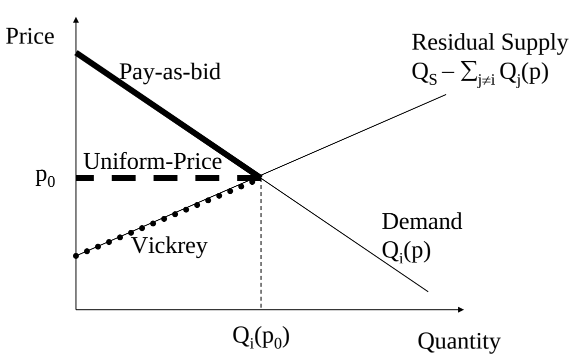


Large bidder makes room for smaller rival

Vickrey inefficient with affiliation

- Winner's Curse in single-item auctions
  - Winning is bad news about value
- Winner's Curse in multi-unit auctions
  - Winning more is worse news about value
  - Must bid less for larger quantity
  - Differential shading creates inefficiency in Vickrey

What about seller revenues?



Uniform price may perform poorly

- Independent private values uniform on [0,1]
- 2 bidders, 2 units; L wants 2; S wants 1
- Uniform-price: unique equilibrium
  - S bids value
  - L bids value for first and 0 for second
  - Zero revenue; poor efficiency
- Vickrey
  - price = $v_{(2)}$ on one unit, zero on other

Standard ascending-bid may be worse

- 2 bidders, 2 units; L wants 2; S wants 2
- Uniform-price: two equilibria
  - Poor equilibrium: both L and S bid value for 1
    - Zero revenue; poor efficiency
  - Good equilibrium: both L and S bid value for 2
    - Get $v_{(2)}$ for each (max revenue) and efficient
- Standard ascending-bid: unique equilibrium
  - Both L and S bid value for 1
    - S's demand reduction forces L to reduce demand
    - Zero revenue; poor efficiency

Efficient auctions tend to yield high revenues

Theorem. *With flat demands drawn independently from the same regular distribution, seller's revenue is maximized by awarding good to those with highest values.*

Generalizes to non-private-value model with independent signals:

$v_i = u(s_i, s_{-i})$

Award good to those with highest signals if downward sloping MR and symmetry.

Downward-sloping demand: $p_i(q_i) = v_i - g_i(q_i)$

Theorem. *If intercept drawn independently from the same distribution, seller's revenue is maximized by*

- *awarding good to those with highest values if constant hazard rate*
- *shifting quantity toward high demanders if increasing hazard rate*

- Note: uniform-price shifts quantity toward *low* demanders

But uniform price has advantages

- Participation
  - Encourages participation by small bidders (since quantity is shifted toward them)
  - May stimulate competition
- Post-bid competition
  - More diverse set of winners may stimulate competition in post-auction market

Auctioning Securities

A pure common-value model with affiliation

- n risk-neutral symmetric bidders
- Each bidder has pure common value V for security and can purchase any quantity (flat demand curve w/o capacity)

Models

- Common uncertainty
  - Bidders have no private information
- Affiliated private signals
  - Bidder i gets signal S_i
  - Random variables $V, S_1, \dots, S_n$ are affiliated

Results: Common Uncertainty

Proposition. (Wilson '79; Maxwell '83; Back & Zender '93)

- *Wide range of prices can be supported as equilibrium in uniform-price auction, even if supply is stochastic; highest yields EV*

Proposition. (Wang & Zender '96)

- *Many equilibria in pay-as-bid auction, even if supply is stochastic; highest yields EV*
- *Indeterminacy avoided if set reserve price (even 0)*

Results: Common Uncertainty

Theorem.

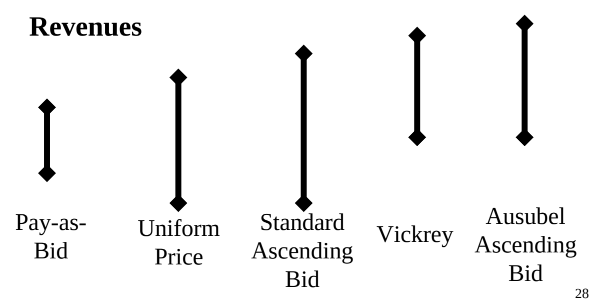
- *Vickrey auction has a unique equilibrium that survives elimination of weakly-dominated strategies*
- *Vickrey auction has a unique symmetric equilibrium consistent with stochastic supply*
- *This equilibrium revenue-dominates all equilibria of **all** auction formats consistent with voluntary bidder participation*

Results: Affiliated Private Signals

- With affiliated signals, each auction format has a "simple equilibrium" where bidders submit flat demand curves
- Conjecture: These simple equilibria provide upper bounds on revenues from each format
- Alt. ascending-bid > Vickrey > Pay-as-bid
- Std. ascending-bid > Uniform > Pay-as-bid

Results: Affiliated Private Signals

Vickrey and Ausubel ascending-bid eliminate bottom end of revenue indeterminacy:



Conclusion

- Efficient auctions should be favored
- Treasury should try Ausubel ascending-bid
- IPOs should be auctioned