Auctions and Market Design

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Auctions and Market Design

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Course logistics

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Requisites

• Integration and differentiation

• Probability and statistics

• Differential equations
Exercises

• **6 exercises**
  • 15:00-18:00 on Friday, Nov. 13-20-27 and Dec. 4-11-18

• **Structure**
  • Ex. 1-5: theory review + problem set + other exercises
  • Ex 6: practice exam:

<table>
<thead>
<tr>
<th>Reading Krishna</th>
<th>Ex. 1</th>
<th>Ex. 2</th>
<th>Ex. 3</th>
<th>Ex. 4</th>
<th>Ex. 5</th>
<th>Ex. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch. 1-2-3</td>
<td>Ch. 4-5</td>
<td>Ch. 6-7-8</td>
<td>Ch. 9-10</td>
<td>Ch. 12-13-14</td>
<td></td>
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</tr>
<tr>
<td>PS handed on</td>
<td>Oct. 23</td>
<td>Nov. 13</td>
<td>Nov. 20</td>
<td>Nov. 27</td>
<td>Dec. 4</td>
<td></td>
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<tr>
<td>Due by end of day</td>
<td>Nov. 12</td>
<td>Nov. 19</td>
<td>Nov. 26</td>
<td>Dec. 3</td>
<td>Dec. 10</td>
<td></td>
</tr>
</tbody>
</table>

**Estimated workload per exercise: 1 day reading + 1 day PS**
Exercises (continued)

Problem sets should be typed (Word, Latex, ...) and submitted in pdf format to Emmanuele:

ec.bobbio@gmail.com
Exam and grading

• **Final exam:**
  • Let’s find a time good for most

• **Grading:**
  • Problem sets, optional, but rewarded with bonus toward final
  • Can work in group, submit one solution for the group with names
  • Grade 0-100, total 100 x 5 = 500. Bonus point conversion table:

<table>
<thead>
<tr>
<th>PS points</th>
<th>401 or more</th>
<th>[400,301]</th>
<th>[0,300]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding bonus</td>
<td>0.7</td>
<td>0.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*Example:* PS points = 431, Exam = 2.0 → final grade = 2.0 - 0.7 = 1.3

• Bonus applies only if exam grade is 4.0 or better
• If no PS, but perfect exam still receive a grade of 1.0
Resources and contacts

Lecture slides, exercises, recordings, problem sets and solutions, on Ilias: https://www.ilias.uni-koeln.de/ilias/goto_uk_crs_3525655.html
Also see http://www.cramton.umd.edu/courses/market-design/

Contact: ec.bobbio@gmail.com
Market design

• Establishes rules of market interaction

• Economic engineering
  • Economics
  • Computer science
  • Engineering, operations research
Innovation in a few words

• Electricity
  • Open access

• Communications
  • Auction spectrum
  • Open access

• Transportation
  • Price congestion

• Climate policy
  • Price carbon

• Financial securities
  • Make time discrete

Resistance to market reforms is the norm:
Peter to regulator, "I can save you $10 billion."
Wall Street to regulator, "Hey, that’s my $10 billion."

*Distribution trumps efficiency*
Milton Friedman

There is enormous inertia—a tyranny of the status quo—in private and especially governmental arrangements. Only a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around. That, I believe, is our basic function [as economists]: to develop alternatives to existing policies, to keep them alive and available until the politically impossible becomes politically inevitable.
EKONOMIPRISET 2020
THE PRIZE IN ECONOMIC SCIENCES 2020

Paul R. Milgrom
Robert B. Wilson

"för förbättringar av auktionsteorin och uppfinningar av nya auktionsformat"
"for improvements to auction theory and inventions of new auction formats"

#nobelprize
1994  
JOHN C. HARSANYI, JOHN F. NASH and REINHARD SELTEN for their pioneering analysis of equilibria in the theory of non-cooperative games

2001  
GEORGE A. AKERLOF, A. MICHAEL SPENCE, and JOSEPH E. STIGLITZ, for their analyses of markets with asymmetric information

2005  
ROBERT J. AUMANN and THOMAS C. SCHELLING for having enhanced our understanding of conflict and cooperation through game-theory analysis

1996  
JAMES A. MIRRLEES and WILLIAM VICKREY for their fundamental contributions to the economic theory of incentives under asymmetric information

2007  
LEONID HURWICZ, ERIC S. MASKIN, and ROGER B. MYERSON for having laid the foundations of mechanism design theory

1978  
HERBERT A. SIMON for his pioneering research into the decision-making process within economic organizations

2014  
JEAN TIROLE for his analysis of market power and regulation

2002  
VERNON L. SMITH for having established laboratory experiments as a tool in empirical economic analysis, especially in the study of alternative market mechanisms, and DANIEL KAHNEMAN for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty

2017  
RICHARD THALER for contributions to behavioral economics

2016  
BENGT HOLMSTRÖM and OLIVER HART for contributions to contract theory

2009  
ELINOR OSTROM for her analysis of economic governance, especially the commons

2012  
ALVIN E. ROTH and LLOYD SHAPLEY for the theory of stable allocations and the practice of market design

2020  
PAUL MILGROM and ROBERT WILSON for improvements to auction theory and inventions of new auction formats
Objectives

• Efficiency
• Simplicity
• Transparency
• Fairness

*It is good to keep these goals in mind whenever designing a market*
Matching
One-sided matching

• Each of $n$ agents owns house and has strict preferences over all houses
• Can the agents benefit from swapping?
Top trading cycles

- Each owner $i$ points to her most preferred house (possibly $i$’s own)
- Each house points back to its owner
- Creates directed graph; identify cycles
  - Finite: cycles exist
  - Strict preference: each owner is in at most one cycle
- Give each owner in a cycle the house she points to and remove her from the market
- If unmatched owners/houses remain, iterate
Properties of top trading cycles

• Top-trading-cycle outcome is a core allocation
  • No subset of owners can make its members better off by an alternative assignment within the subset

• Top-trading-cycle outcome is unique

• Top-trading-cycle algorithm is strategy proof
  • No agent can benefit from lying about her preferences
Example

Owner’s ranking from best to worst

1: \((h_3, h_2, h_4, h_1)\)

2: \((h_4, h_1, h_2, h_2)\)

3: \((h_1, h_4, h_3, h_3)\)

4: \((h_3, h_2, h_1, h_4)\)
Two-sided matching

• Consider a set of $n$ women and $n$ men
• Each person has an ordered list of some members of the opposite sex as his or her preference list
• Let $\mu$ be a matching between women and men
• A pair $(m, w)$ is a blocking pair if both $m$ and $w$ prefer being together to their assignments under $\mu$. Also, $(x, x)$ is a blocking pair, if $x$ prefers being single to his/her assignment under $\mu$
• A matching is stable if it does not have any blocking pair
Example

Schroeder
Charlie

Charlie
Linus
Franklin

Charlie
Franklin
Linus
Schroeder

Linus
Franklin

Lucy
Peppermint

Peppermint

Marcie

Marcie

Sally

Charlie

Linus

Franklin

Schroeder

Lucy
Peppermint
Marcie

Marcie
Sally

Marcie
Lucy

Peppermint
Sally
Marcie

Stable!
Deferred Acceptance Algorithms

(Gale and Shapley, 1962)

• In each iteration, an unmarried man proposes to the first woman on his list that he hasn’t proposed to yet
• A woman who receives a proposal that she prefers to her current assignment accepts it and rejects her current assignment

This is called the men-proposing algorithm
Example

Stable!
Classical Results

• **Theorem 1.** The order of proposals does not affect the stable matching produced by the men-proposing algorithm

• **Theorem 2.** The matching produced by the men-proposing algorithm is the *best* stable matching for men and the *worst* stable matching for women
  
  This matching is called the *men-optimal* matching

• **Theorem 3.** In all stable matchings, the set of people who remain single is the same
Applications of stable matching

• Stable marriage algorithm has applications in the design of centralized two-sided markets
  • National Residency Matching Program (NRMP) since 1950’s
  • Dental residencies and medical specialties in the US, Canada, and parts of the UK
  • National university entrance exam in Iran
  • Placement of Canadian lawyers in Ontario and Alberta
  • Sorority rush
  • Matching of new reform rabbis to their first congregation
  • Assignment of students to high-schools in NYC
  • ...
Incentive Compatibility

• **Question:** Do participants have an incentive to announce their real preference lists?

• **Answer:** No!

   In the men-proposing algorithm, sometimes women have an incentive to be dishonest about their preferences.
Incentive Compatibility

• Next Question: Is there any truthful mechanism for the stable matching problem?

• Answer: No!

Roth (1982) proved that there is no mechanism for the stable marriage problem in which truth-telling is the dominant strategy for both men and women.
Auctions
Why auctions?

• Auctions solve complex problems ...
  • in infrastructure industries
    (communications, energy, transport)
  • in B2B markets
  • in internet markets
  • in financial markets

• ... in an economic way
  • high revenues
  • high efficiency
  • revelation of hidden information
Why auctions?

• Instead of guessing how much buyers are willing to pay, an auction lets buyers name their own price, revealing how much the item is worth.

• E.g., price = value of strongest competitor.

• If there are many buyers, this is almost as good as with complete information ideal.
Single-item auctions
(assuming that the pre-auction decisions, such as product design, have been made)
Auction formats

• **English (clock) auction**: the price rises continuously until there is only one bidder left willing to pay

• **Dutch (clock) auction**: the price drops continuously until the first bidder accepts

• **First price auction**: all bidders submit an offer unaware of what the other bidders are offering. The highest bid wins. The price equals the highest bid

• **Second price auction**: As first price auction, but the price equals the second highest bid
Bidding behavior: English auction

• Which auction format is the best one?

• To find out the ‘optimal’ auction, one has to analyze bidding behavior under the different formats

• Winner $i$ gets $v_i - p$, and all others get 0

• In the *English auction*, it is optimal to stay in the auction as long as the price is below one’s own value

• The bidder with the highest value will win at a price equal to the second highest value
Second-price auction

• The second price auction looks weird. Why should a seller be satisfied with the second highest bid?

• But, in fact, the second-price auction is similar to (the ‘sealed-bid equivalent’ of) the English auction

• Each bidder will bid her value

• As a consequence, the bidder with the highest value will win at a price equal to the second highest value
Second-price auction

Should I bid less than my value?

1) $b_i < v_i < \max b_{-i}$ (Zero-) payoff unchanged

2) $\max b_{-i} < b_i < v_i$ Payoff unchanged

3) $b_i < \max b_{-i} < v_i$ loss
Second-price auction

Should I bid more than my value?

1) \( v_i < b_i < \max b_{-i} \)  
   (Zero-) Payoff unchanged

2) \( \max b_{-i} < v_i < b_i \)  
   Payoff unchanged

3) \( v_i < \max b_{-i} < b_i \)  
   loss
Second-price auction

• Bidding one’s value never hurts and is sometimes rewarded

• So it is a dominant strategy to bid value

• The underlying idea is that bidders are price takers—a winner cannot influence one’s price, but only the probability of winning

• By bidding her value, she makes sure that she’ll win if and only if the price is below her value
First-price auction (‘pay-as-bid’) 

• Because the winner’s bid determines what she has to pay, the only way to make positive surplus is to bid less than one’s value

• Thus, bidders will shade their bids. But by how much?

• The winner must think about how much he needs to bid to win, which is difficult because of the risk-return trade-off she faces

• She wants to just outbid the bidder with the second-highest value

• So, under risk neutrality (an ex ante symmetry), she will bid the expected value of the second-highest value, conditional on winning (only then this is relevant)
First-price auction (‘pay-as-bid’)  

• Because the bidder with the highest value wins (in any equilibrium), the revenue equals the expected second-highest value

• Thus, expected revenue is the same in the first-price, second-price and English auction
Dutch auction

• The Dutch auction is strategically equivalent to the first price auction

• You must decide how much to bid at a time when you do not know how much the others bid

• So, all these auctions lead to the same expected revenue

• The winner pays the (expected) value of the strongest competitor
The revenue equivalence theorem

All auction protocols with the properties that
• the bidder with the highest value wins, and
• bidders with the lowest-possible value make zero
lead to the same expected revenue to the seller!
But strong assumptions are required

- **Single item**: one item is auctioned
- **Risk neutrality**: bidders are risk-neutral
- **Independence**: bidders’ values are private information and statistically independent
- **Symmetry**: values are drawn from the same probability distribution

*These assumptions never hold in practice, yet the revenue equivalence theorem is incredibly useful!*
William Vickrey

... received the Nobel prize 1996 for the revenue equivalence theorem!
Illustration with complete information

• An item is worth €15 to bidder A and €10 to bidder B

• In the English auction, B drops out at a price €10

• In the second price auction, both bidders bid their value, so that the price is €10

• In the first price auction, A bids just enough to win (€10 + epsilon), and B bids €10

• The Dutch auction is strategically equivalent to the first price auction
Remarks

• Standard single unit auction protocols fall under the revenue equivalence theorem
  • Bidders adjust their behavior in a way that does not allow the seller to get more revenue by some ‘sophisticated’ auction procedures

• However, when the underlying assumptions do not hold, (e.g. risk aversion, asymmetries, etc.) the choice of mechanism matters!
  • Still, the RET is useful

• Similarities to matching: if you ask me for my preferences, the critical question is what are you going to do with this information
Remarks:
The optimal auction and reserve prices

• The optimal auction has a reserve price that is equal to the optimal ‘take-it-or-leave-it’ price in case of only one bidder

• All four auction formats with this reserve price are optimal
Remarks: Interdependent values

• Here is a jar with cents
• Please chat to Emmanuele
  • Your estimate of number of cents: $E = \text{nnn}$
  • Your bid in 1\textsuperscript{st} price auction (in cents): $1^{\text{st}} = \text{nnn}$
  • Your bid in 2\textsuperscript{nd} price auction (in cents): $2^{\text{nd}} = \text{nnn}$
  • For example: “$E=525, 1^{\text{st}}=494, 2^{\text{nd}}=525$”
• The highest bid wins in each auction
Remarks: “The winner’s curse”

Average bid: 2,70€

Real value: 3,00€

Highest bid: 11,50€

Distribution of bids
Remarks: Interdependent values

You want to bid on this used car ...

... but you don’t know the ‘true’ value
• You estimate the value somewhere between 0€ and 12.000€ (every value has the same probability)

• The seller knows the ‘true’ value, but he needs money and is willing to sell at a price of 2/3 of the value (otherwise he rejects your bid)

• How much do you bid?
Remarks: Interdependent values

The expected value is 6.000€

Assume you bid 4.000€

If you win at this price, is this a bargain?
Remarks: Interdependent values

Assume seller accepts your bid

Then you know that the value of the car cannot exceed 6,000€, because otherwise your bid would be rejected \((2/3 \times 6,000€ = 4,000€)\)

The (expected) value is thus only 3,000€

That is, independent of your bid, if you win you must always expect to make a loss!
Remarks: "The winner’s curse"

UMTS-auctions
MobilCom gave its license back for which it paid 8.5 billion Euros

Road construction and tender ...
Book manuscripts
Oil fields
Baseball players and Managers
eBay
....
Remarks: "The winner’s curse"

Bidders typically have different pieces of information about the true value.

Winning is bad news, because it means that others (other bidders or the seller) have more negative information about the value.

Thus, bids need to be substantially shaded: "what is the value of the item conditional on all opponents estimating this value to be lower?"

This bid shading reduces revenues.
Remarks: "The winner’s curse"

- What does this imply for auction design if bidders are rational? Reduce uncertainty!

- Each bidder would revise his value if he also had the information about other bidders

- This information might be inferred by the competitors’ bids in open (but not in sealed-bid) auctions

- Better information and less uncertainty allows bidders to bid more aggressively

- Thus, the English auction, theoretically, yields larger revenues
First some math

Calculating equilibrium behavior in games with incomplete information
Equilibrium in first-price auction

• Each of n bidder’s private value $v$ drawn from distribution $F$

• Bidder's expected profit:
$$\pi(v,b(v)) = (v - b(v))Pr(Win|b(v))$$

• By the envelope theorem,
$$\frac{d\pi}{dv} = \frac{\partial \pi}{\partial b} \frac{\partial b}{\partial v} + \frac{\partial \pi}{\partial v} = \frac{\partial \pi}{\partial v}$$

• But then $d\pi/dv = Pr(Win|b(v)) = Pr(highest bid)$
  $$= Pr(highest value) = F(v)^{n-1}$$
Equilibrium in first-price auction

• By the Fundamental Theorem of Calculus,

\[
\pi(v) = \pi(0) + \int_0^v F(u)^{n-1} du = \int_0^v F(u)^{n-1} du,
\]

• Substituting into \(\pi(v,b(v)) = (v - b(v))\Pr(\text{Win}|b(v))\) yields

\[
b(v) = v - \frac{\pi(v)}{\Pr(\text{Win})} = v - F(v)^{-(n-1)} \int_0^v F(u)^{n-1} du.
\]
Example

• $v \sim U$ on $[0,1]$
• Then $F(v) = v$, so
  \[ b(v) = v - \frac{v}{n} = \frac{v(n-1)}{n} \]
• The optimal bid converges to the value as $n \to \infty$, so in the limit the seller is able to extract the full surplus
• In equilibrium, the bidder bids the expected value of the second highest value given that the bidder has the highest value
Bargaining: Simultaneous offers  
(Chatterjee & Samuelson, *Operations Research* 1983)

- A seller and a buyer are engaged in the trade of a single item worth $s$ to the seller and $b$ to the buyer
- Valuations are known privately, as summarized below

<table>
<thead>
<tr>
<th>Traders</th>
<th>Distributed</th>
<th>Payoff</th>
<th>Private Info</th>
<th>Common Knowledge</th>
<th>Strategy (Offer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller</td>
<td>$s \sim F$ on $[\underline{s}, \overline{s}]$</td>
<td>$u = P - s$</td>
<td>$s$</td>
<td>F, G</td>
<td>$p(s)$</td>
</tr>
<tr>
<td>Buyer</td>
<td>$b \sim G$ on $[\underline{b}, \overline{b}]$</td>
<td>$v = b - P$</td>
<td>$b$</td>
<td>F, G</td>
<td>$q(b)$</td>
</tr>
</tbody>
</table>
Simultaneous Offers

• *Independent private value model*: s and b are independent random variables.

• *Ex post efficiency*: trade if and only if \( s < b \).

• *Game*: Each player simultaneously names a price; if \( p \leq q \) then trade occurs at the price \( P = (p + q)/2 \); if \( p > q \) then no trade (each player gets zero).
Simultaneous Offers

• Payoffs:
  • Seller
    \[ u(p, q, s, b) = \begin{cases} 
    P - s & \text{if } p \leq q \\
    0 & \text{if } p > q
    \end{cases} \]
  • Buyer
    \[ v(p, q, s, b) = \begin{cases} 
    b - P & \text{if } p \leq q \\
    0 & \text{if } p > q
    \end{cases} \]

where the trading price is \( P = (p + q)/2 \)
Example

• Let F and G be independent uniform distributions on [0,1]

• Equilibrium conditions:

(1) \[ \forall s \in [s, \bar{s}], p(s) \in \arg\max_{p} E_b \{u(p, q, s, b) | s, q(\cdot)\} \]

(2) \[ \forall b \in [b, \bar{b}], q(b) \in \arg\max_{q} E_s \{v(p, q, s, b) | b, p(\cdot)\} \]
Seller’s Problem

• Assume $p$ and $q$ are strictly increasing
• Let $x(\cdot) = p^{-1}(\cdot)$ and $y(\cdot) = q^{-1}(\cdot)$
• Optimization in (1) can be stated as

$$\max_{p} \int_{y(p)}^{1} \left[ \frac{(p + q(b))}{2} - s \right] db$$

• First-order condition

$$-y'(p)[p - s] + \left[ 1 - y(p) \right]/2 = 0,$$

since $q(y(p)) = p$
Buyer’s Problem

• Optimization in (2) can be stated as

\[
\max_q \int_0^{x(q)} \left[ b - \frac{p(s) + q}{2} \right] ds
\]

• First-order condition

\[
x'(q)(b - q) - \frac{x(q)}{2} = 0,
\]

since \( p(x(q)) = q \)
Equilibrium

• Equilibrium condition:
  
  \[ s = x(p) \text{ and } b = y(q) \]

• Equilibrium first-order conditions:

  (1') \[ -2y'(p)[p - x(p)] + [1 - y(p)] = 0, \]

  (2') \[ 2x'(q)[y(q) - q] - x(q) = 0 \]
Solution

• Solving (2') for \( y(q) \) and replacing \( q \) with \( p \) yields

\[
(2'') \quad y(p) = p + \frac{1}{2} \frac{x(p)}{x'(p)}, \quad \text{so} \quad y'(p) = \frac{3}{2} - \frac{1}{2} \frac{x(p)x''(p)}{[x'(p)]^2}
\]

• Substituting into (1') then yields

\[
(1') \quad [x(p)-p]\left[3 - \frac{x(p)x''(p)}{[x'(p)]^2}\right] + \left[1 - p - \frac{1}{2} \frac{x(p)}{x'(p)}\right] = 0
\]
Analytical Solution

• Linear Solution:
  
  \[ x(p) = \alpha p + \beta \]
  
  with \( \alpha = 3/2 \) and \( \beta = -3/8 \)

• Using (2") yields
  
  \[ y(q) = 3/2 q - 1/8 \]

• Inverting these functions results in
  
  \[ p(s) = 2/3 s + 1/4 \quad \text{and} \quad q(b) = 2/3 b + 1/12 \]
Split-the-difference trading: s and b $\sim U[0,1]$
Outcome

• Trade occurs if and only if

\[ p(s) \leq q(b), \text{ or } b - s \geq 1/4 \]

• The gains from trade must be at least 1/4 or no trade takes place

The outcome is inefficient
Mechanism design

General understanding of incentives in decision settings (bargaining, auctions, exchange, public goods, ...)

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Direct Revelation Game

Original Game \((\Gamma')\)

- Types: \(T\)
- Strategy: \(\sigma\)
- Actions: \(A\)
- Payoff: \(u\)
- Outcomes: \(\mathcal{R}^n\)
Direct Revelation Game

**Original Game** \((\Gamma)\)

- Types: \(T\)
- Strategy: \(\sigma\)
- Payoff: \(u\)
- Actions: \(A\)
- Outcomes: \(\mathbb{R}^n\)

**Direct Revelation Game** \((\Gamma')\)

- Types: \(T\)
- Identity: \(I\)
- Payoff: \(u' = u \circ \sigma\)
- Actions: \(A\)
- Outcomes: \(\mathbb{R}^n\)
A General Model  
(Myerson and Satterthwaite, *JET* 1983)

*Direct Revelation Game:*

- Bilateral Exchange with independent private value
- $s \sim F$ with positive pdf $f$ on $[S, \bar{S}]$
- $b \sim G$ with positive pdf $g$ on $[b, \bar{b}]$
- $F$ and $G$ are common knowledge

In the DRG, the traders report their valuations and then an outcome is selected. Given the reports $(s, b)$, an outcome specifies a probability of trade $(p)$ and the terms of trade $(x)$
Definition

Direct Mechanism

A direct mechanism is a pair of outcome functions \( \langle p, x \rangle \), where:
- \( p(s,b) \) is the probability of trade given the reports \( (s,b) \), and
- \( x(s,b) \) is the expected payment from the buyer to the seller.
Payoffs

*Ex post utilities*:

- Seller's ex post utility:
  \[ u(s,b) = x(s,b) - sp(s,b) \]

- Buyer's ex post utility:
  \[ v(s,b) = bp(s,b) - x(s,b) \]

Both traders are risk neutral (quasi-linear utility)
Payoffs

Define:

\[ X(s) = \int_{b}^{\bar{b}} x(s, b) g(b) \, db \quad Y(b) = \int_{s}^{\bar{s}} x(s, b) f(s) \, ds \]

\[ P(s) = \int_{b}^{\bar{b}} p(s, b) g(b) \, db \quad Q(b) = \int_{s}^{\bar{s}} p(s, b) f(s) \, ds. \]

\( X(s) \) is the seller's expected revenue given \( s \)
\( Y(b) \) is the buyer's expected payment given \( b \)
\( P(s) \) is the seller's probability of trade
\( Q(b) \) is the buyer's probability of trade
Payoffs

• Interim Utilities:
  \[ U(s) = X(s) - sP(s) \quad V(b) = bQ(b) - Y(b) \]

• The mechanism \( \langle p, x \rangle \) is *incentive compatible* if for all \( s, b, s', \) and \( b' \):
  \[ U(s) \geq X(s') - sP(s') \quad V(b) \geq bQ(b') - Y(b') \]

• The mechanism \( \langle p, x \rangle \) is *individually rational* if for all and
  \[ U(s) \geq 0 \quad V(b) \geq 0 \]
  \[ s \in [s, \bar{s}] \quad b \in [b, \bar{b}] \]
Lemma 1
(Mirrlees, Myerson)

The mechanism \( \langle p, x \rangle \) is IC if and only if \( P(\cdot) \) is decreasing, \( Q(\cdot) \) is increasing, and

\[
\text{(IC')} \quad U(s) = U(\bar{s}) + \int_s^{\bar{s}} P(t)dt
\]

\[
V(b) = V(b) + \int_b^b Q(t)dt
\]
Lemma 1: Proof

*Only if:*

- By definition, $U(s) = X(s) - sP(s)$ and $U(s') = X(s') - s'P(s')$. This and (IC) imply

  $$U(s) \geq X(s') - sP(s') = U(s') + (s' - s)P(s'), \text{ and}$$

  $$U(s') \geq X(s) - s'P(s) = U(s) + (s - s')P(s)$$

- Putting these inequalities together yields

  $$(s' - s)P(s) \geq U(s) - U(s') \geq (s' - s)P(s')$$
Lemma 1: Proof

• Taking $s' > s$ implies that $P(\cdot)$ is decreasing
• Dividing by $(s' - s)$ and letting $s' \to s$, then yields $dU(s)/ds = -P(s)$
• Integrating produces (IC')
• The same is true for the buyer
Lemma 1: Proof

If:

• To prove (IC) for the seller, note that it suffices to show that

\[ s[P(s) - P(s')] + [X(s') - X(s)] \leq 0 \text{ for all } s, s' \in [s, \bar{s}] \]

• Substituting for \( X(s') \) and \( X(s) \) using (IC') and the definition of \( U(s) \) yields

\[ X(s) = sP(s) + U(\bar{s}) + \int_{s}^{\bar{s}} P(t)dt . \]
Lemma 1: Proof

If:

• Then it suffices to show for every $s, s' \in [\underline{s}, \bar{s}]$ that

\[
0 \geq s[P(s) - P(s')] + s'P(s') + \int_{s'}^{\bar{s}} P(t)dt - sP(s) - \int_{\underline{s}}^{s} P(t)dt
\]

\[
= (s' - s)P(s') + \int_{s'}^{\bar{s}} P(t)dt = \int_{s'}^{\underline{s}} [P(t) - P(s')]dt ,
\]

which holds because $P(\cdot)$ is decreasing

• The proof for the buyer is similar
Lemma 2

An incentive compatible mechanism $\langle p, x \rangle$ is individually rational if and only if

$$(\text{IR}') \quad U(s) \geq 0 \quad \text{and} \quad V(\cdot) \geq 0$$

**Proof**

- Clearly, (IR') is necessary for $\langle p, x \rangle$ to be IR

- By Lemma 1, $U(\cdot)$ is decreasing; hence, (IR') is sufficient as well
Theorem: Characterization of IC & IR mechanisms

• An incentive-compatible, individually rational mechanism \( \langle p, x \rangle \) satisfies

\[
(*) \quad U(s) + V(b) = \int_{b}^{\bar{b}} \int_{s}^{\bar{s}} \left[ b - \frac{1 - G(b)}{g(b)} - s - \frac{F(s)}{f(s)} \right] p(s, b)f(s)g(b) ds db \geq 0.
\]
Theorem: Characterization of IC & IR mechanisms

Proof

• Using (IC') and the definition of \( U(s) \) yields

\[
X(s) = sP(s) + U(\overline{s}) + \int_{s}^{\overline{s}} P(t)dt.
\]
Theorem: Characterization of IC & IR mechanisms

• Taking the expectation with respect to $s$ (and substituting in the definitions of $X(s)$ and $P(s)$) shows that

$$\int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\bar{s}} x(s,b)f(s)g(b)dsdb =$$

$$U(\bar{s}) + \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\bar{s}} sp(s,b)f(s)g(b)dsdb$$

$$+ \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\bar{s}} p(s,b)F(s)g(b)dsdb.$$
Theorem:
Characterization of IC & IR mechanisms

- The third term in the right hand side follows, since

\[
\int_{s}^{\bar{s}} \int_{s}^{\bar{s}} p(t, b)f(s)dt ds \\
= \int_{s}^{\bar{s}} \int_{s}^{t} p(t, b)f(s)ds dt = \int_{s}^{\bar{s}} p(s, b)F(s)ds.
\]
Theorem:
Characterization of IC & IR mechanisms

• Preceding analogously for the buyer yields

\[
\int_{\bar{b}}^{b} \int_{\bar{s}}^{s} x(s, b)f(s)g(b)dsdb
= -V(b) + \int_{\bar{b}}^{b} \int_{\bar{s}}^{s} bp(s, b)f(s)g(b)dsdb
- \int_{\bar{b}}^{b} \int_{\bar{s}}^{s} p(s, b)f(s)[1 - G(b)]dsdb.
\]

• Equating the right-hand sides of the last two equations and applying (IR') completes the proof
Corollary:
Impossibility of efficient trade

If it is not common knowledge that gains exist (the supports of the traders' valuations have non-empty intersection), then no incentive-compatible, individually rational trading mechanism can be ex-post efficient.
A mechanism is ex-post efficient if and only if trade occurs whenever $s \leq b$:

$$p(s, b) = \begin{cases} 
1 & \text{if } s \leq b \\
0 & \text{if } s > b.
\end{cases}$$
Proof

• To prove that ex-post efficiency cannot be attained, it suffices to show that the inequality (*) in the Corollary fails when evaluated at this $p(s,b)$. Hence,

$$\int_{\bar{b}}^{\bar{b}} \int_{s}^{\min\{b,\bar{s}\}} \left[ b - \frac{1 - G(b)}{g(b)} - s - \frac{F(s)}{f(s)} \right] f(s)g(b)dsdb$$
Proof

\[
\begin{align*}
&= \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\min\{\underline{b}, \bar{s}\}} [bg(b)+G(b) - 1]f(s)dsdb - \int_{\underline{b}}^{\bar{b}} \int_{\underline{s}}^{\min\{\underline{b}, \bar{s}\}} [sf(s)+F(s)]dsg(b)db \\
&= \int_{\underline{b}}^{\bar{b}} [bg(b)+G(b) - 1]F(b)db - \int_{\underline{b}}^{\bar{b}} \min\{bF(b), \bar{s}\}g(b)db \quad \text{(by parts)} \\
&= -\int_{\underline{b}}^{\bar{b}} [1 - G(b)]F(b)db + \int_{\underline{s}}^{\bar{b}} (b - \bar{s})g(b)db \\
&= -\int_{\underline{b}}^{\bar{b}} [1 - G(b)]F(b)db + \int_{\underline{s}}^{\bar{b}} [1 - G(b)]db \quad \text{(by parts)} \\
&= -\int_{\underline{s}}^{\bar{s}} [1 - G(t)]F(t)dt < 0, \quad \text{since } \underline{b} < \bar{s}.
\end{align*}
\]
Proof

• The second term in the second line follows, since by integrating by parts

\[ \int_{s}^{x} [sf(s) + F(s)]ds = xF(x). \]

Since ex-post efficiency is unattainable, we need a weaker efficiency criterion with which to measure a mechanism's performance.
What IC & IR mechanism maximizes expected gains from trade?

• Ex ante efficient mechanism maximizes the expected gains from trade:

\[ \int_s^{\bar{s}} U(s)f(s)ds + \int_b^{\bar{b}} V(b)g(b)db \]

subject to IC & IR
What IC & IR mechanism maximizes expected gains from trade?

• Ex ante efficient mechanism maximizes the expected gains from trade subject to (*)

\[
\max_{p(s,b)} \int_{b}^{\bar{b}} \int_{s}^{\bar{s}} (b - s) p(s,b) f(s) g(b) dsdb \\
\int_{b}^{\bar{b}} \int_{s}^{\bar{s}} \left( b - \frac{G(b)}{g(b)} - (s + \frac{1 - F(s)}{f(s)}) \right) p(s,b) f(s) g(b) dsdb \geq 0
\]
\[
\max_{p(s,b)} \int_b^\bar{b} \int_s^\bar{s} (b - s) p(s,b) f(s) g(b) ds \, db \\
\int_b^\bar{b} \int_s^\bar{s} \left( b - \frac{G(b)}{g(b)} - \left( s + \frac{1 - F(s)}{f(s)} \right) \right) p(s,b) f(s) g(b) ds \, db \geq 0
\]

\[
L(p, \lambda) = \int_b^\bar{b} \int_s^\bar{s} \left( (b - s)(1 + \lambda) - \lambda \left( \frac{G(b)}{g(b)} + \frac{1 - F(s)}{f(s)} \right) \right) p(s,b) f(s) g(b) ds \, db
\]

\[
\int_b^\bar{b} \int_s^\bar{s} (b - s - \alpha \left( \frac{G(b)}{g(b)} + \frac{1 - F(s)}{f(s)} \right)) p(s,b) f(s) g(b) ds \, db \quad \text{for} \quad \alpha = \frac{\lambda}{1 + \lambda}
\]

\(\alpha=0\) implies max objective; \(\alpha=1\) implies max constraint
Optimal trading game: pointwise optimization

• Find $p(s,b)$ to maximize pointwise

$$
\int_{\bar{b}}^{b} \int_{s}^{\bar{s}} \left( b - s - \alpha \left( \frac{G(b)}{g(b)} + \frac{1-F(s)}{f(s)} \right) \right) p(s,b) f(s) g(b) ds db \quad \text{for} \quad \alpha = \frac{\lambda}{1+\lambda}
$$

$$
p^\alpha (s,b) = \begin{cases} 
1 & \text{if } b - \alpha \frac{G(b)}{g(b)} \geq s + \alpha \frac{1-F(s)}{f(s)} \\
0 & \text{otherwise}
\end{cases}
$$

• Find $\alpha$ so that constraint binds
Trade with probability 1 or 0

• The ex ante efficient trading rule has the property that, given the reports, trade either occurs with probability one or not at all

• Analysis also applies to dynamic trading mechanism, $t(s,b) =$ time of trade with discount rate r, then $p(s,b) = e^{-rt(s,b)}$

• Optimal mechanism involves immediate trade or no trade

• Optimal mechanism violates sequential rationality
Example

Valuations are uniformly distributed on [0,1]

• Ex ante efficient mechanism: linear equilibrium in which trade occurs if and only if the gains from trade are at least 1/4 (Chatterjee & Samuelson)

• If the traders cannot commit to walking away from gains from trade, then they would be unable to implement this mechanism

• So long as it is not common knowledge that gains exist, the traders will, with positive probability, make incompatible demands in situations where gains from trade exist
Accomplishments

• Characterization of the set of all BE of all bargaining games in which the players' strategies map their private valuations into a probability of trade and a payment from buyer to seller

• Proof that ex post efficiency is unattainable if it is uncertain that gains from trade exist

• Determination of the set of ex ante efficient mechanisms

• Proof that ex ante efficiency is incompatible with sequential rationality
Dissolving a Partnership
(Cramton, Gibbons, and Klemperer, 1987)

• n traders. Each trader $i \in \{1, \ldots, n\}$ owns a share $r_i \geq 0$ of the asset, where $r_1 + \ldots + r_n = 1$

• As in MS, player i's valuation for the entire good is $v_i$

• The utility from owning a share $r_i$ is $r_i v_i$

• Private values, $v_i$’s are iid $\sim F(\cdot)$ on $[\underline{v}, \overline{v}]$

• A partnership $(r,F)$ is fully described by the vector of ownership rights $r = \{r_1, \ldots, r_n\}$ and the traders' beliefs $F$ about valuations

*Why is this important?* Challenges of private information are the worst when ownership is extreme (e.g. buyer-seller). More balanced ownership mitigates the adverse impact of private information. This is a general point that applies to all trade.
Dissolving a Partnership

MS Case:

• $n = 2$ and $r = \{1,0\}$

• There does not exist a BE $\sigma$ of the trading game such that:
  (1) $\sigma$ is (interim) individually rational and
  (2) $\sigma$ is ex post efficient

CGK Case:

• If the ownership shares are not too unequally distributed, then it is possible to satisfy both (1) and (2), (satisfying IC, IR, EE and BB)
Dissolving a Partnership

A partnership \((r,F)\) can be \textit{dissolved efficiently} if there exists a Bayesian Equilibrium \(\sigma\) of a Bayesian trading game such that \(\sigma\) is interim individually rational and ex post efficient.
Theorem

- The partnership \((r,F)\) can be dissolved efficiently if and only if

\[
(*) \quad \sum_{i=1}^{n} \left[ \int_{v_i^*}^{V_i} [1 - F(u)]u dG(u) - \int_{V_i^*}^{V_i} F(u)u dG(u) \right] \geq 0
\]

where \(v_i^*\) solves \(F(v_i)^{n-1} = r_i\) and \(G(u) = F(u)^{n-1}\)

Worst-off type \(v_i^*\) just as likely to be buyer as seller:

Expected purchase = Expected sales

\[
(1 - r_i)r_i = r_i(1 - r_i)
\]
Examples $F(v_i) = v_i$

- $n=2$, then (*) $\sum_{i=1}^{2} r_i^2 \leq \frac{2}{3}$

- $n=3$, then

(*) $\sum_{i=1}^{3} r_i^{3/2} \leq 3/4$
Proposition

For any distribution $F$, the one-owner partnership $r = \{1,0,0,\ldots,0\}$ cannot be dissolved efficiently

- The one-owner partnership can be interpreted as an auction
- Ex post efficiency is unattainable because the seller's value $v_1$ is private information: the seller finds it in her best interest to set a reserve price above her value $v_1$
- An optimal auction maximizes the seller's expected revenue over the set of feasible (ex post inefficient) mechanisms
Theorem

• If a partnership \((r,F)\) can be dissolved efficiently, then the unique symmetric equilibrium of the following bidding game is interim individually rational and achieves ex-post efficiency: given an arbitrary minimum bid \(b\),
• each player receives a side-payment, independent of the bidding,
\[
c_i(r_1, \ldots, r_n) = \int_{v_i}^{v_i^*} udG(u) - \frac{1}{n} \sum_{j=1}^{n} \int_{v_j}^{v_j^*} udG(u). \]
• the players choose bids \(b_i \in [b, \infty)\)
• the good goes to the highest bidder
• each bidder \(i\) pays
\[
p_i(b_1, \ldots, b_n) = b_i - \frac{1}{n-1} \sum_{j \neq i}^{n} b_j
\]
Auctioning many similar items

Lawrence M. Ausubel, Peter Cramton, Marek Pycia, Marzena Rostek, and Marek Weretka (2014) "Demand Reduction and Inefficiency in Multi-Unit Auctions," Review of Economic Studies, 81:4, 1366-1400
Examples of auctioning similar items

• Treasury bills
• Stock repurchases and IPOs
• Telecommunications spectrum
• Electric power
• Emission allowances
• ...

Ways to auction many similar items

• Sealed-bid: bidders submit demand schedules
  • Pay-as-bid auction (traditional Treasury practice)
  • Uniform-price auction (Milton Friedman 1959)
  • Vickrey auction (William Vickrey 1961)
Pay-as-bid Auction:
All bids above $P_0$ win and pay bid
Uniform-Price Auction:  
All bids above $P_0$ win and pay $P_0$
Vickrey Auction:
All bids above $P_0$ win and pay opportunity cost

\[ Q_s - \sum_{j \neq i} Q_j(p) \quad \text{(Residual Supply)} \]

\[ Q_i(p) \quad \text{(Demand)} \]

\[ p_0 \quad \text{(Price)} \]

\[ Q_i(p_0) \quad \text{(Quantity)} \]
Vickrey Auction: \( m \) Discrete Items

- Allocate \( m \) items efficiently: \( m \) highest marginal values
- Winning bidder pays \( k^{th} \) highest losing bid of others on \( k^{th} \) item won
- Payment = social opportunity cost of items won

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<td>2(^{nd})</td>
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<td>3(^{rd})</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
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Payment rule affects behavior

\[ Q_s = \sum_{j \neq i} Q_j(p) \]

**Price**

**Quantity**

- **Pay-as-bid**
- **Uniform-Price**
- **Vickrey**

Demand: \( Q_i(p) \)

Residual Supply: \( Q_s - \sum_{j \neq i} Q_j(p) \)
Optimal bids by payment rule

- Uniform-Price
- Vickrey
- Pay-as-bid

Price

Quantity

$p_0$

True demand $Q_i(p)$
More ways to auction many similar items

• Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  • Clock auction (single price)
  • Clock auction with Vickrey prices (Ausubel 1997)
Ascending clock:
All bids at $P_0$ win and pay $P_0$
Ascending clock with Vickrey prices
All bids at $P_0$ win and pay price at which clinched

\[
Q_s - \sum_{j \neq i} Q_j(p) = Q_i(p)
\]

Excess Demand

Residual Supply

Demand

Price

Clock

Quantity

$Q_i(p_0)$
More ways to auction many similar items

• Ascending-bid
  • Simultaneous ascending auction (FCC spectrum)

• Sequential
  • Sequence of English auctions (auction house)
  • Sequence of Dutch auctions (fish, flowers)

• Optimal auction
  • Maskin & Riley 1989
Research Program
How do standard auctions compare?

• Efficiency
  • FCC: those with highest values win

• Revenue maximization
  • Treasury: sell debt at least cost
Efficiency
(not pure common value; capacities differ)

- Uniform-price and standard ascending-bid
  - Inefficient due to demand reduction
- Pay-as-bid
  - Inefficient due to different shading
- Vickrey
  - Efficient in private value setting
  - Strategically simple: dominant strategy to bid true demand
  - Inefficient with affiliated information
- Dynamic Vickrey (Ausubel 1997)
  - Same as Vickrey with private values
  - Efficient with affiliated information
Inefficiency Theorem

In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values

• Winning bidder influences price with positive probability
• Creates incentive to shade bid
• Incentive to shade increases with additional units
• Differential shading implies inefficiency
Inefficiency from differential shading

Large Bidder

Small Bidder

Large bidder makes room for smaller rival
Vickrey inefficient with affiliation

• Winner’s Curse in single-item auctions
  • Winning is bad news about value

• Winner’s Curse in multi-unit auctions
  • Winning more is worse news about value
  • Must bid less for larger quantity
  • Differential shading creates inefficiency in Vickrey
What about seller revenues?

\[
Q_i(p) - \sum_{j \neq i} Q_j(p)
\]

Price

\[ p_0 \]

Pay-as-bid

Uniform-Price

Vickrey

Residual Supply

Demand

Quantity
Exercise

• 2 bidders (L and R), 2 identical items
• L has a value of $100 for 1 and $200 for both
• R has a value of $90 for 1 and $180 for both
• Uniform-price auction
  • Submit bid for each item
  • Highest 2 bids get items
  • 3rd highest bid determines price paid
• Ascending clock auction
  • Price starts at 0 and increases in small increments
  • Bidders express how many they want at current price
  • Bidders can only lower quantity as price rises
  • Auction ends when no excess demand (i.e. just two demanded); winners pay clock price
Uniform price may perform poorly

- Independent private values uniform on [0,1]
- 2 bidders, 2 units; L wants 2; S wants 1
- Uniform-price: unique equilibrium
  - S bids value
  - L bids value for first and 0 for second
  - Zero revenue; poor efficiency
- Vickrey
  - price = v_{(2)} on one unit, zero on other
Standard ascending-bid may be worse

• 2 bidders, 2 units; L wants 2; S wants 2

• Uniform-price: two equilibria
  • Poor equilibrium: both L and S bid value for 1
    • Zero revenue; poor efficiency
  • Good equilibrium: both L and S bid value for 2
    • Get $v_{(2)}$ for each (max revenue) and efficient

• Standard ascending-bid: unique equilibrium
  • Both L and S bid value for 1
    • S’s demand reduction forces L to reduce demand
    • Zero revenue; poor efficiency
Efficient auctions tend to yield high revenues

**Theorem.** *With flat demands drawn independently from the same regular distribution, seller’s revenue is maximized by awarding good to those with highest values*

Generalizes to non-private-value model with independent signals:

\[ v_i = u(s_i, s_{-i}) \]

Award good to those with highest signals if downward sloping MR and symmetry
Downward-sloping demand:
\[ p_i(q_i) = v_i - g_i(q_i) \]

**Theorem.** If intercept drawn independently from the same distribution, seller’s revenue is maximized by

- awarding good to those with highest values if constant hazard rate
- shifting quantity toward high demanders if increasing hazard rate

- Note: uniform-price shifts quantity toward low demanders
But uniform price has advantages

• Participation
  • Encourages participation by small bidders
    (since quantity is shifted toward them and less strategy)
  • May stimulate competition

• Post-bid competition
  • More diverse set of winners may stimulate competition in post-auction market
Bidding behavior in electricity markets

- Marginal cost bidding is a useful benchmark, but not a norm of behavior
- Profit maximization is an appropriate norm of behavior in markets
- Profit maximization should be expected and encouraged
- Market rules should be based on this norm
Uniform-price auction:
All bids below $p_0$ win and get paid $p_0$
Residual demand removes supply of other bidders
Residual demand curve

\[ D_i(p) = D(p) - \sum_{j \neq i} S_j(p) \]

As-bid supply

\[ S_i(p) \]

Price

Quantity

\[ q_i \]

\[ p_0 \]
Bidding strategy with perfect competition

As-bid supply

\[ S_i = MC_i \]

Residual demand

\[ D_i \]

Price

\[ p_0 \]

Quantity

\[ q_i \]

Loss
Incentive to bid above marginal cost: tradeoff higher price with reduced quantity
Optimal bid balances marginal gain and loss

Optimal bid balances marginal gain and loss
Still bid above marginal cost when others bid marginal cost
Residual demand response reduces incentive to inflate bids

\[ p \]
\[ q \]

As-bid supply
\[ S_i \]
\[ MC_i \]

Loss
Gain

Residual demand
\[ D_i \]

\[ p_0 \]
Residual demand is steeper for large bidders
Large bidder makes room for its smaller rivals
Economic vs. Physical Withholding
Forward contracts mitigate incentive to bid above marginal cost
Revenue equivalence and optimal auction in general model

Identical items model

• Seller has quantity 1 of divisible good (value = 0)
• $n$ bidders; $i$ can consume $q_i \in [0, \lambda_i]$
  $q = (q_1, ..., q_n) \in Q = \{q \mid q_i \in [0, \lambda_i] \& \Sigma q_i \leq 1\}$
• $t_i$ is $i$’s type; $t = (t_1, ..., t_n)$; $t_i \sim F_i$ w/ pos. density $f_i$
• Types are independent
• Marginal value $v_i(t, q_i)$
• $i$’s payoff if gets $q_i$ and pays $x_i$:

$$\int_0^{q_i} v_i(t, y) \, dy - x_i$$
Identical items model (cont.)

Marginal value $v_i(t,q_i)$ satisfies:

• **Value monotonicity**
  - non-negative
  - increasing in $t_i$
  - weakly increasing in $t_j$
  - weakly decreasing in $q_i$

• **Value regularity**: for all $i$, $j$, $q_i$, $q_j$, $t_{−i}$, $t_i' > t_i$,
  $$v_i(t_i,t_{−i},q_i) > v_j(t_i,t_{−i},q_j) \Rightarrow v_i(t_i',t_{−i},q_i) > v_j(t_i',t_{−i},q_j)$$
Identical items model (cont.)

- Bidder $i$'s marginal revenue: marginal revenue seller gets from awarding additional quantity to bidder $i$

$$MR_i(t,q_i) = v_i(t,q_i) - \frac{1 - F_i(t_i)}{f_i(t_i)} \frac{\partial v_i(t,q_i)}{\partial t_i}$$
Revenue Equivalence

**Theorem 1.** In any equilibrium of any auction game in which the lowest-type bidders receive an expected payoff of zero, the seller’s expected revenue equals

\[ E_t \left[ \sum_{i=1}^{n} \int_{0}^{q_i(t)} MR_i(t, y) \, dy \right] \]
Optimal Auction

• MR monotonicity
  • increasing in $t_i$
  • weakly increasing in $t_j$
  • weakly decreasing in $q_i$

• MR regularity: for all $i, j, q_i, q_j, t_{-i}, t'_i > t_i$,

$$MR_i(t, q_i) > MR_j(t, q_j) \Rightarrow MR_i(t'_i, t_{-i}, q_i) > MR_j(t'_i, t_{-i}, q_j)$$

**Theorem 2.** Suppose MR is monotone and regular. Seller’s revenue is maximized by awarding the good to those with the highest marginal revenues, until the good is exhausted or marginal revenue becomes negative.
Optimal Auction is Inefficient

• Assign goods to wrong parties
  • High MR does not mean high value

• Assign too little of the good
  • MR turns negative before values do
Three Seller Programs

1. Unconstrained optimal auction

(standard auction literature)
Select assignment rule and pricing rule to

\[
\max \ E[\text{Seller Revenue}]
\]
\[
s.t. \quad \text{Incentive Compatibility}
\]
\[
\text{Individual Rationality}
\]
Three Seller Programs

2. Resale-constrained optimal auction

(Coase Theorem critique)
Select assignment rule and pricing rule to

\[
\text{max } E[\text{Seller Revenue}]
\]

s.t. Incentive Compatibility

Individual Rationality

Efficient resale among bidders
Three Seller Programs

3. Efficiency-constrained optimal auction

(Coase Conjecture critique)
Select assignment rule and pricing rule to

\[
\text{max} \quad E[\text{Seller Revenue}]
\]

s.t. Incentive Compatibility

Individual Rationality

Efficient resale among bidders and seller
1. Unconstrained optimal auction

Select assignment rule \( q(t) \) to

\[
\max_{q(t) \in Q} \mathbb{E}_t \left[ \sum_{i=1}^{n} \int_{0}^{q_i(t)} MR_i(t, y) \, dy \right]
\]

\( Q = \{ \text{All feasible assignment rules.} \} \)
1. Unconstrained optimal auction (two bidders)
3. Efficiency-constrained optimal auction

Select assignment rule $q^R(t)$ to

$$\max_{q(t) \in \mathcal{Q}^R} E_t \left[ \sum_{i=1}^{n} \int_0^{q_i(t)} MR_i(t, y) \, dy \right]$$

$\mathcal{Q}^R = \{ \text{Ex post efficient assignment rules.} \}$
3. Efficiency-constrained optimal auction (two bidders)
2. Resale-constrained optimal auction

Select assignment rule $q^R(t)$ to

$$\max_{q(t) \in Q^R} \mathbb{E}_t \left[ \sum_{i=1}^{n} \int_0^{q_i(t)} MR_i(t, y) \, dy \right]$$

$Q^R = \{\text{Resale-efficient assignment rules.}\}$
2. Resale-constrained optimal auction (two bidders)
Theorem. In the two-stage game (auction followed by perfect resale), the seller can do no better than the resale-constrained optimal auction.

Proof. Let $a(t)$ denote the probability measure on allocations at end of resale round, given reports $t$. Observe that, viewed as a static mechanism, $a(t)$ must satisfy IC & IR. In addition, $a(t)$ must be resale-efficient. □
Can we obtain the upper bound on revenue?

The resale process is *coalitionally-rational against individual bidders* if bidder $i$ obtains no more surplus $s_i$ than $i$ brings to the table:

$$s_i \leq v(N \mid q,t) - v(N \sim i \mid q,t).$$

That is, each bidder receives no more than 100% of the gains from trade it brings to the table.
Vickrey auction with reserve pricing

Seller sets monotonic aggregate quantity that will be assigned to the bidders, an efficient assignment $q^*(t)$ of this aggregate quantity, and the payments $x^*(t)$ to be made to the seller as a function of the reports $t$ where

$$x_i^*(t) = \int_0^{q_i^*(t)} v_i(\hat{t}_i(t_{-i}, y), t_{-i}, y) \, dy,$$

where

$$\hat{t}_i(t_{-i}, y) = \inf_{t_i} \{ t_i \mid q_i^*(t_i, t_{-i}) \geq y \}.$$ 

Bidders simultaneously and independently report their types $t$ to the seller.
Can we attain the upper bound on revenue?

Theorem 5 (Ausubel and Cramton 1999). Consider the two-stage game consisting of the Vickrey auction with reserve pricing followed by a resale process that is coalitionally-rational against individual bidders. Given any monotonic aggregate assignment rule, sincere bidding followed by no resale is an ex post equilibrium of the two-stage game.
Applications

Spectrum auction design
Mobile communications
Electricity market design
Climate policy
Transportation
Financial markets
Spectrum
Spectrum auctions

• Many items, heterogeneous but similar
• Competing technologies and business plans
• Complex structure of substitutes and complements

• Government objective: Efficiency
  • Make best use of scarce spectrum
  • Address competition issues in downstream market
Key design issues

• Establish term to promote investment
• Enhance substitution
  • Product design
  • Auction design
• Encourage price discovery
  • Dynamic price process to focus valuation efforts
• Encourage truthful bidding
  • Pricing rule
  • Activity rule
Simultaneous ascending auction
Prepare
Italy 4G Auction, September 2010
470 rounds, 22 days, €3.95B

• Auction conducted on-site with pen and paper
• Auction procedures failed in first day
• No activity rule
Thailand 3G Auction, October 2012

• 3 incumbents bid
• 3 nearly identical licenses
• Auction ends at reserve price + 2.8%
Conflict and cooperation in Germany’s spectrum auctions

In Germany (1999) 10 spectrum blocks were simultaneously sold:

- New bid for a particular block must exceed prior bid by at least 10 percent
- Auction ends when no bidder is willing to bid higher on any block

What happened?
The auction begins ...

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The “offer” of Mannesmann (=M)

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</table>
T-Mobile (=T) “accepted”

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Perfect cooperation

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</table>
Combinatorial Clock Auction
Combinatorial clock auction

- Auctioneer names prices; bidder names package
  - Price increased if excess demand
  - Process repeated until no excess demand
- Supplementary bids
  - Improve clock bids
  - Bid on other relevant packages
- Optimization to determine assignment/prices
- No exposure problem (package auction)
- Second pricing to encourage truthful bidding
- Activity rule to promote price discovery
Pricing rule
Bidder-optimal core pricing

• Minimize payments subject to core constraints
• Core = assignment and payments such that
  • Efficient: Value maximizing assignment
  • Unblocked: No subset of bidders offered seller a better deal
Optimization

• Core point that minimizes payments readily calculated
  • Solve Winner Determination Problem
  • Find Vickrey prices
  • Constraint generation method
    (Day and Raghavan 2007)
      • Find most violated core constraint and add it
      • Continue until no violation

• Tie-breaking rule for prices is important
  • Minimize distance from Vickrey prices
5 bidder example with bids on \{A,B\}

- \(b_1\{A\} = 28\)  
- \(b_2\{B\} = 20\)  
- \(b_3\{AB\} = 32\)  
- \(b_4\{A\} = 14\)  
- \(b_5\{B\} = 12\)  

Winners

Vickrey prices:

- \(p_1 = 14\)
- \(p_2 = 12\)
The Core

Efficient outcome

Bidder 1 Payment

Bidder 2 Payment

The Core

b₁{A} = 28
b₂{B} = 20
b₃{AB} = 32
b₄{A} = 14
b₅{B} = 12
Vickrey prices: How much can each winner’s bid be reduced holding others fixed?

Bidder 2 Payment

Bidder 1 Payment

\[ b_3\{AB\} = 32 \]
\[ b_4\{A\} = 14 \]
\[ b_1\{A\} = 28 \]
\[ b_2\{B\} = 20 \]
\[ b_5\{B\} = 12 \]

The Core

Problem: Bidder 3 can offer seller more (32 > 26)!
Bidder-optimal core prices: *Jointly* reduce winning bids as much as possible

**Problem:** bidder-optimal core prices are not unique!
Core point closest to Vickrey prices

Each pays equal share above Vickrey
Activity rule
Clock stage performs well

Proposal: With revealed preference w.r.t. final round
If clock stage ends with no excess supply,
final assignment = clock assignment

Supplementary bids can’t change assignment;
but can change prices

• May destroy incentive for truthful bidding in supplementary round
• Supplementary round still needed to determine competitive prices
• Possible solutions
  • Do not reveal demand at end of clock stage; possibility of excess supply motivates more truthful bidding (Canada 700 MHz)
  • Do not impose final price cap (UK 4G)
Summary:
CCA is an important tool

• Eliminates exposure
• Reduces gaming
• Enhances substitution
• Allows auction to determine band plan, technology
• Readily customized to a variety of settings
• Many other applications
Broadcast Incentive Auction
29 March 2016
Motivation

<table>
<thead>
<tr>
<th>Year</th>
<th>Value per MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>Value of over-the-air broadcast TV</td>
</tr>
<tr>
<td>1990</td>
<td>TV signal received via cable and satellite</td>
</tr>
<tr>
<td>1995</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Explosion in use of smartphones and tablets</td>
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<tr>
<td>2005</td>
<td>Gains from trade</td>
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<tr>
<td>2010</td>
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<td>2015</td>
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</table>
What is the Incentive Auction?

• The world’s most complicated auction!

Goal: clear broadcast spectrum, and repurpose for wireless use

• Consumer demand for broadcast TV has waned, while demand for mobile broadband keeps increasing

• Spectrum in use for broadcast television is very desirable
  • Excellent propagation characteristics

• Convert a set of 6MHz channels into 5+5MHz paired wireless blocks
  • Ex: 21 channels = 126MHz of spectrum = 10 paired blocks + guard bands
What is the Incentive Auction?

• Auction has been in the works for a long time
  • Part of the FCC’s National Broadband Plan released in 2010
  • Initiated by Spectrum Act legislation passed in 2012
  • Initial design released in 2014; final design released June 2015
  • Began 29 March 2016; ends late 2016 or early 2017
  • Stage 1, reverse auction ended 29 June 2016; revenue requirement $88 billion

• Auction involves many novel challenges
  • Combines both a reverse and a forward auction into a single process
  • Clearing television spectrum is a hard problem
  • Interplay between forward and reverse auctions introduces additional complexity
Auction summary
29 March 2016 to 10 February 2017

• 16 Oct: FCC announces opening price for each station
• 29 Mar: Station decides whether to accept opening price (participate)
• 29 Apr: FCC optimization (min impairment) to determine clearing target
  • Largest target with impairment < equivalent of 1 nationwide block (10% at 126MHz)
• Stage 1: initial clearing target (126MHz, 10 blocks)
  • 31 May-29 Jun: Reverse auction to determine clearing cost ($88 billion)
  • 16-30 Aug: Forward auction to determine auction proceeds ($22 billion)
• Stage 2: reduce clearing target (114MHz, 9 blocks)
  • 10 Sep-13 Oct: Reverse continues with lower target ($57 billion)
  • 19 Oct-19 Oct: Forward continues with fewer blocks ($21 billion)
• Stage 3: reduce clearing target (108MHz, 8 blocks)
  • -: Reverse continues with lower target ($42 billion)
  • -5 Dec: Forward continues with fewer blocks ($19 billion)
• Stage 4: reduce clearing target (84MHz, 7 blocks)
  • 13 Dec-13 Jan: Reverse continues with lower target ($12 billion)
  • 18 Jan-10 Feb: Forward continues with fewer blocks ($18 billion)
• End: FCC optimization to determine channel assignments
Auction progress
29 March 2016 to 10 February 2017

Price (billion $/block)

Reverse clearing cost
88 billion $

Forward auction proceeds
18 billion $

Stage 1
Stage 2
Stage 3
Stage 4

Done!
Keys features of good design

• Simple expression of supply and demand
  • Station: at this lower price are you still willing to clear?
  • Carrier: at this higher price how many blocks do you want?

• Monotonicity
  • Reverse: prices only go down (excess supply)
  • Forward: prices only go up (excess demand)

• Activity rule
  • Reverse: station exit is irreversible
  • Forward: 95% activity requirement
Reverse auction
(broadcasters: supply side)

• Identifies prices and stations to be cleared to achieve target

• There are many different ways for broadcasters to participate
  • Relinquish their broadcast license entirely
  • Move to a lower band (change from UHF to VHF)
  • Share a channel and facilities with another broadcaster

• Decision to participate is voluntary
  • Any broadcaster that does not participate can continue broadcasting with their current coverage, but channel may change
Forward auction
(carriers: demand side)

• Ascending clock, rather than simultaneous multiple round

• Issue: participation in reverse auction determines licenses on offer
  • Nonparticipating broadcasters *must* be assigned a channel
  • FCC needs to impair some licenses if too many nonparticipants

• Issue: closing requires forward auction to generate enough revenue
  • Must pay for reverse auction payments + relocation costs + FCC costs
  • If closing conditions are not met, FCC holds an extended round to increase bidding in top-40 markets, or continues process with a lower clearing target
What is “Repurposing broadcast spectrum”?
Possible band plans

<table>
<thead>
<tr>
<th>Number of Paired Blocks</th>
<th>Broadcast Spectrum Cleared (MHz)</th>
<th>Current Broadcast Television Band</th>
<th>Repacked Broadcasters</th>
<th>Proposed Mobile Wireless Services</th>
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Notes:
1. Numbers in the table represent current broadcast television channels.
2. Letters in the table represent paired blocks for mobile wireless services.
3. Grey blocks represent guard bands and their width (in MHz).
4. Impaired markets will have a band plan from the table, but not at the nationwide clearing target.
5. Band plan above stops at UHF channel 21 (does not show UHF channels 14 to 20).
Why is this complicated?

• Have a supply side (broadcasters) and demand side (wireless carriers), so why not just determine where supply meets demand?

• Issue: the supply side is extremely complicated
  • “Buy stations until # stations = # channels” only works at local scale
  • Two stations can only use the same channel if they do not cause interference

• Interference between stations creates a complex set of constraints
  • Depends on physical factors: distance, transmitting power, terrain, antenna…
  • If stations interfere strongly enough, cannot even be on adjacent channels
  • Large markets can interfere with each other, e.g. New York and Philadelphia
Nationwide interference constraints
Finding channel assignments

- Finding a *nationwide* channel assignment is extremely complicated
  - Hundreds of thousands of interference constraints between stations
  - Border regions must respect treaties with Canada and Mexico
  - Some channels reserved for emergency communications in many markets

- Checking if a set of stations can be repacked is computationally hard
  - Equivalent to classic Satisfiability (SAT) problem from computer science
  - No generally efficient solution known; widely believed not to exist
  - The auction needs to make hundreds of thousands of these checks
  - Fortunately, this can be made (mostly) efficient in practice
Design of the reverse auction

The reverse auction is a scored descending clock auction:
• Each television station is assigned a score (starting price)
• A single clock ticks down from 100% to 0%; at each time step:
  • Every station is offered a price of \((\text{score}) \times (\text{current } \%)\)
  • If a station rejects this offer, it is assigned a channel
  • Then if any station no longer can be assigned a channel, that station is bought

Note: this ignores many factors that make the auction more complex
Note: immediately buying stations with no available channels ensures that active stations always have a feasible channel assignment
What is the “right” scoring rule?

- Choice of scoring rule has a huge impact on auction outcomes
  - Determines opening prices, and hence participation levels
  - Determines exit order and payments

- Exit order will have a lasting impact on broadcast television
  - Nearby markets often have to “share” channels due to interference
  - If stations in Philadelphia all exit before stations in New York, then post-auction Philadelphia will have more broadcast television than New York
  - This phenomenon is repeated across the country, both between pairs of large markets, and between large markets and nearby small markets.
What is the “right” scoring rule?

FCC scoring rule:
Geometric mean of population served and # of interference constraints

• Population served gives a proxy for geographic area
  • Interference propagates further than service; stations in markets near major markets often serve small populations but interfere with large ones

• # of interference constraints gives a proxy for packing compatibility
  • Not all interference constraints are equal: some never bind in the auction
  • Many other potential measures suggested by graph theory
  • May be better to use simulation to determine repackability
Methodology

• Simulate reverse auction
  • To test implications of alternative auction rules
  • To evaluate alternative bidding strategies

• Key assumptions
  • Station participation decisions
  • Station reservation values
  • Straightforward bidding (initially)
Input 1:
• Participation model
• Bidder valuations

Clearing Target Optimizer
- Impairments calculated using full ISIX implementation
- Optimization stages match FCC
- Modified program specification for reduced processing time
- Extensive optimizer tuning

Optimizer

Clearing target
Initial channels

Input 2:
• Auction rules

Auction Simulator
- Runs at scale in the cloud
- Uses customized feasibility checker
- Incorporates advanced presolve routines
- Supports a variety of bidder strategies

Simulator

Revise and iterate
Station outcomes
Freeze prices

Output:
• Repacked station
• Freeze prices

Analysis Tools
- Assess auction effectiveness
- Identify impact of scoring rule
- Find new candidate scoring rules to test in simulator
- Evaluate robustness of scoring rule across scenarios

Analysis Tools
Example: scoring rule

• Trade-off between reverse auction efficiency and clearing cost
  • Efficiency: Emphasize a station’s value in clearing (how much it gets in the way)
  • Clearing cost: Price discriminate by offering stations with low broadcast value less
  • Broadcast population (broadcast value) vs. precluded population (clearing value)

• But to clear maximum spectrum, FCC must be concerned about clearing cost
Conclusions

• The incentive auction is hugely complicated and hugely important
  • First of its kind, with many novel complications
  • Won’t be the last of its kind: every country faces this spectrum issue

• Critical to analyze and evaluate the design decisions

• Simulation is a useful way to assess design and robustness to strategic behavior
Important lessons

• No auction design is perfect
• Design must be customized for setting
  • Simultaneous ascending clock
    • Simple settings (upcoming UK)
  • Combinatorial clock
    • Packaging is essential (UK 4G, Canada 700 MHz)
  • Two-sided clock
    • Incentive auction in US
• Never ignore essentials
  • Encourage participation
  • Demand performance
  • Avoid collusion and corruption
Mobile Communications
An Open Access Wireless Market

Supporting Public Safety, Universal Service, and Competition

Peter Cramton
Linda Doyle
From monopoly to vibrant competition

- **Monopoly**
  - Original "Ma Bell" telecommunications

- **Oligopoly**
  - Spectrum auctions
  - Mobile communications

- **Competition**
  - Open access wireless market
  - Internet ecosystem of innovation
Mobile networks

Wholesale market

Open access network (ISO)

Proprietary network 1 (MNO₁)

Proprietary network 2 (MNO₂)

Service providers

Mobile virtual network operator (MVNO)

MNO₁

MNO₂

Users/devices

A B C D E F G

Retail market

Same market model as electricity successfully operating for two decades
Product design

• Product should be directly valued by consumer
  • Network throughput at particular location and time interval
  • A market for throughput not spectrum
  • Energy (MWh) in an electricity market

• Aggregation in both time and location to simplify and improve liquidity
  • Example: Particular cell over one hour (GB/h)
All markets use single-price auction

Price ($/GB)

Quantity (GB/h)

Winning buyers

Winning sellers

Demand

Supply

Clearing price $p^*$

Quantity traded $Q^*$

Winning buyers

Winning sellers

All markets use single-price auction
Multiple opportunities to trade: Yearly, monthly, hourly

Yearly aggregates monthly aggregates hourly in time and location
Service provider estimates demand and stages purchase in three markets.

Yearly auction = buy 165 GB per hour, for every peak hour in the year, in the yearly area.
Refine estimate and make adjustment in monthly market

Yearly auction = buy 165 GB per hour, for every peak hour in the year, in the yearly area

Monthly auction = sell 8 GB per hour, for every peak hour in the month, in the red area

Monthly auction = buy 15 GB per hour, for every peak hour in the month, in the green area

Monthly auction = buy 20 GB per hour, for every peak hour in the month, for the blue area
Finalize estimate one hour ahead and make final adjustment to demand

Hourly area H
Peak hour product

<table>
<thead>
<tr>
<th>Hour</th>
<th>Change</th>
<th>Net</th>
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<tr>
<td>07</td>
<td>Buy 3 GB</td>
<td>11 GB</td>
</tr>
<tr>
<td>08</td>
<td>Sell 2 GB</td>
<td>6 GB</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>17</td>
<td>Buy 1 GB</td>
<td>9 GB</td>
</tr>
</tbody>
</table>

8 GB
Prototype auction platform

• To illustrate market
• Demonstrate proof of concept
Auction design

• Uniform-price auction for each product
• Preferences expressed as piecewise-linear strictly-decreasing demand curves
  • Consistent with underlying preferences
  • Unique clearing prices and quantities
• Yearly and monthly auctions: simultaneous ascending clock
• Hourly auction: sealed bid
Current Published Supply Curve for Manhattan (Peak)

Select a product to view its supply curve calculated from the relative supply curve currently in the system.
Sample demand for bidder

Manhattan (peak) monthly

<table>
<thead>
<tr>
<th>Manhattan (Peak)</th>
<th>Price</th>
<th>Quantity</th>
<th>Change in Quantity</th>
<th>Commitment (180 hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future Rounds</td>
<td>$9.00/GB</td>
<td>-1,000,000 GB/h</td>
<td>-3,000,000 GB/h</td>
<td>-$4,860,000</td>
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<tr>
<td></td>
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<td>-200,000 GB/h</td>
<td>-2,200,000 GB/h</td>
<td>-$2,772,000</td>
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<td>$6.00/GB</td>
<td>500,000 GB/h</td>
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<td>2,666.667 GB/h</td>
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<td>$240,000</td>
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<tr>
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<td>3,000,000 GB/h</td>
<td>1,000,000 GB/h</td>
<td>$180,000</td>
</tr>
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</table>
Possible implementations

• Mexico (competition)
  • 90 MHz of 700 MHz set aside for open access
  • RFP to select implementer in 28 Sep 2016

• United States (public safety)
  • 20 MHz of 700 MHz set aside + $7 billion
  • RFP to select implementer in November 2016

• European Union and United States (merger remedy)
  e.g. UK: Three & O2; T-Mobile & Sprint
  • Proposed mergers leading from 4 to 3 carriers
  • Merged entity allocates portion of network to open access
Electricity
Electricity Markets in Transition

A forty-year model of entry and exit

Peter Cramton, Emmanuele Bobbio, David Malec and Pat Sujarittanonta

19 October 2020

We are grateful to PJM Interconnection for funding and expert help. Funding also from Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC 2126/1–390838866 and by the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program under grant agreement No 741409.
THE FUTURE IS NOW
Goal of electricity markets: *Reliable electricity at least cost*

**Short-run efficiency**
Least-cost operation of existing resources

**Long-run efficiency**
Right quantity and mix of resources
Challenges of electricity markets

• Must balance supply and demand at every instant
• Physical constraints of network and resources
• Shocks in supply
  – Transmission line or generator outage
  – Intermittent resources: wind and solar
• Absence of demand response
• Climate policy
A successful market design

- Get the spot market right
  - Day ahead
    - Scheduling and unit commitment
  - Real time
    - Bid-based security constrained economic dispatch
- Forward trade to manage risk and support long-run investment
Day-ahead market

Unit commitment and scheduling
- Energy and ancillary services each hour of day
- Prices for energy and reserves; financially binding

Three-part offers from fossil resources
- Startup cost
- Minimum-energy cost
- Energy offer curve

Virtual offers and bids
- Arbitrage day-ahead and real-time markets

Objective: maximize social welfare s.t. transmission and resource constraints
- Co-optimized energy and reserves
- Competitive equilibrium with locational marginal prices (marginal value of energy at each location)
Day-ahead market

Handling non-convexities, such as startup and minimum energy costs

- If total cost of unit not covered by energy & reserve revenue, then unit gets make-whole payment for shortfall
- Make-whole payments small in practice
- LMPs are approximate supporting prices

Procompetitive

- Allows small generators to optimally schedule
- Allows small participants to hedge real-time risk
Real-time market

Security constrained economic dispatch

Determines optimal dispatch and prices every five minutes

Financially and physically binding

Allows efficient settlement from forward positions
- **52 inches** of rainfall in southeast Texas
- Harvey made landfall **multiple times**
  - **Category 4** near Port Aransas, Texas
  - **Tropical storm** in Cameron, Louisiana
- More than **42,000** lightning strikes
- Record number of tornado warnings in southeast Texas

STP-to-Whitepoint 345-kV transmission structures

Transmission Damage

Tatton Substation
Shortage pricing

- Reserves have value in avoiding load shedding
- Marginal value of reserves depends on
  - Value of Lost Load, e.g. $9000/MWh
  - Probability of Lost Load, e.g. 1 when start shedding load
- Load’s implicit preference for reliability given by operating reserve demand curve
Scarcity pricing to reflect marginal value of reserves: Operating Reserve Demand Curve

- In near-shortage conditions, set administrative reserve price equal to

  \[ \text{Loss of Load Probability} \times \text{Value of Lost Load} = 1 \text{ at min reserve of 2GW} = 9000/\text{MWh} \]

- Captures system-wide benefits of reserves and energy in real time co-optimization
Loss of Load Probability and Value of Lost Load

• Decreases quickly from 1 as reserves are added beyond minimum

• Depends on
  – Errors in forecast demand
  – Forced outages
  – Errors in forecast of intermittent renewable production
Changes to Operating Reserve Demand Curve

![Graph showing changes in operating reserve demand curve with price of reserves in $/MWh and available reserves in MW for 2018, 2019, and 2020.]
Load Patterns – 13:00-20:00 on 8/12-8/16
Forward contracts

- Forward contracts are essential to manage risk
  - California energy crisis 2000-2001
  - Forward provides hedge for load
  - Generator + fuel contract provides physical hedge for supply
- Shortage pricing motivates forward contracts
- Forward contracting improves bidding incentives
Figure 24: Monthly Load Exposure

Source: Potomac Economics
Investment
Projected Planning Reserve Margins

Source: ERCOT Capacity, Demand and Reserves Report, December 2018 with Gibbons Creek capacity removed
Capacity market

• ERCOT is “energy only”; many others have a capacity market (PJM, ISO-NE, ...)
• Good capacity markets rely on shortage pricing, just like energy-only market
• Buy enough in advance
  – Conducted several years in advance, so new entry can compete before costs are sunk
  – Product is ability to deliver energy during shortage
  – Strong performance obligation
    • Financial obligation to provide energy during shortage
    • Provides hedge to load from shortage prices
  – Coordinated investment to ensure adequate resources
Solution looking forward

- But what if >80% renewable
- Core design still works well
- More flexibility needed
  - Demand response
    (smart homes)
  - Battery storage

Need to encourage technology-neutral solutions!
Greater need for flexibility ⇒ efficient price signals increasingly important

• Nodal pricing
  – Price reflects scarcity at time and location
  – Pretending no congestion does not work
    • German redispatch cost of €1.5 billion in 2018
    • Wrong price signal; poor location incentives

• Shortage pricing
  – Motivate those to provide flexibility
Electricity market design matters
Texas (ERCOT): $10/month plus wholesale cost of 9 cents/kWh

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
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</thead>
<tbody>
<tr>
<td>real time price</td>
<td>3.8</td>
</tr>
<tr>
<td>delivery</td>
<td>3.7</td>
</tr>
<tr>
<td>taxes &amp; fees</td>
<td>1.4</td>
</tr>
<tr>
<td>wholesale cost</td>
<td>8.9</td>
</tr>
</tbody>
</table>
California ISO:
$16/month + about 36 cents/kWh

400% more than Texas!

EV-TOU-5, a plan for your home and electric vehicle: This new plan is similar to EV-TOU-2 but the On-Peak and Off-Peak pricing is reduced by one cent kW/h and the Super Off-Peak rate is reduced to just 9¢ kW/h when you pay a Basic Monthly Service Fee of $16. Super Off-Peak hours are midnight to 6 am weekdays, and midnight to 2 pm on weekends and holidays.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Peak</th>
<th>Off peak</th>
<th>Super off peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>26</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
<td>Summer</td>
<td>50</td>
<td>29</td>
<td>9</td>
</tr>
</tbody>
</table>

SDGE EV Time of Use Plans (cents/kWh)
Rich free-ride on poor thanks to net metering

ROLLING OUTAGES may appear to be a symptom of climate change. Extreme heat and intermittent renewables certainly challenge electricity markets. But these challenges can be met with good market design. The California market has flaws that make California electricity more expensive and less reliable than it should be. Fixing these flaws should be a priority.

This spring I moved to California. With delight, I replaced our two gas cars with a single electric vehicle. I researched electricity rates for our home in San Diego and found E-V-TOU-5 was our best plan: $18 a month plus a per-kilowatt-hour charge that depends on the time of use — 6 cents from 4 to 9 p.m., 16 cents from midnight to 6 a.m., and 29 cents at other times. This is best because I can schedule our car to charge after midnight. We pay about 9 cents per kilowatt-hour for our electricity, ignoring the car.

Suppose instead we had moved to Texas — as many Californians are doing. Texas has retail choice so I would see a long list of choices. My best plan would be an innovative plan offered by two providers: $16/month plus the wholesale cost, which averages less than 9 cents per kilowatt-hour. I would charge my car at night when the cost is below 9 cents, but for the rest of my electricity I would pay 9 cents.

My monthly bill in Texas would be $250. In California, it is $1,000. This is a remarkable difference. Is this because the California energy is green, and the Texas energy is dirty? Just the opposite: the 9 cents price in Texas is 100% green energy; the 9 cents price in California is fueled partly from gas. The main source of the difference are two poor decisions of a well-intentioned regulator.

First, the substantial fixed cost of delivering reliable electricity is allocated through a per-kilowatt-hour charge. To make matters worse, California follows a net-metering policy. If you put solar panels on your roof, all the electricity you produce is subtracted before the per-kilowatt-hour charge is applied to your consumption. Thanks to abundant sunshine, this means that homes with solar panels can largely avoid the fixed cost of providing electricity services. Unfortunately, these costs must be paid, so the utility raises the per-kilowatt-hour price. The solar customers free ride on everyone else. This would not be so bad if solar-panel owners were primarily poor. However, the opposite is true. It is the rich who have an advantage in installing high-fixed-cost solar panels.

Two crucial flaws fuel our unsustainable energy market

BY PETER CRAMTON

The seemingly green policy transfers wealth from the poor to the rich. Further it harms the incentives for electrification, which is an essential element of the climate solution.

The first flaw is the absence of retail choice. Innovative and competitive rate plans are unavailable to consumers. In San Diego, my choices are limited to those offered by the monopoly utility.

Fortunately, San Diego Gas & Electric offers one plan that works reasonably well, but the plan in Texas is superior: $30 a month plus the wholesale cost without markup. This plan exposes me to the real-time price swings needed to balance supply and demand.

That is a good thing. It means I am motivated to take advantage of these swings and purchase readily available devices, such as smart thermostats. I can use a free AI-powered app to manage both my spend and my comfort. The price variation provides an opportunity for me to save money during heat waves. I am rewarded for providing the flexibility the system needs.

These two flaws create an electricity market that is unsustainable. Each problem reinforces the other and both get worse as the transition to green energy progresses. This gives me confidence the regulator will fix these problems.

The first flaw is easy to fix. Allocate the fixed cost of providing reliable electricity based on home or rental value (Zillow has an accurate model). Home value is highly correlated with the ability and willingness to pay. This is an efficient and just way to allocate fixed costs.

The second flaw is addressed by requiring the monopoly utility to offer a “$10/month + wholesale cost” plan.

Who could object to adding one new plan to the menu? Consumers still select the plan they prefer. Why not give them an option of a plan well-suited to the energy transition?

California illustrates that good intentions do not necessarily produce good policy. Good policy is designed from what we know about markets and human behavior. Good policy is the only way to provide reliable electricity at least cost.

Cramton is a professor of economics at the University of Cologne and the University of Maryland and an independent board member of ERCOT, the electricity operator in Texas. The views here are his own and not those of ERCOT or ERCOT’s board. He lives in Del Mar.
Load and Generation Changes

**Load**
Load growth (2020-2025) from 2019 Regional Transmission Plan (RTP) cases

- 8.8 GW

**Generation**
Existing thermal units older than 40 years

- 18.0 GW

Planned generators that meet Planning Guide Section 6.9 (1)

- 19.7 GW

Existing thermal unit capacities are from the Final Summer 2020 SARA report and planned generation capacities are from the June 2020 GIS report. [http://www.ercot.com/gridinfo/resource](http://www.ercot.com/gridinfo/resource)
Climate policy matters

Global energy related CO2 emissions, 1990-2019
Estimated cost of new entry
ERCOT Interconnection Queue (August 2020)

Technology

- Solar
- Wind
- Storage
- Gas
- Coal
- Nuclear

Notes:
- Total MW capacity for which a Full Interconnect Study (FIS) has been started or completed, and an Interconnection Agreement has been executed.
- Total MW capacity for which an FIS is either pending or completed and reviewed by ERCOT.
- Total MW capacity for which a Screening Study is underway or completed.
2020 Planned (Summer Capacity MW) EIA, Mar 2020

Wind
22,241

Solar
12,114

Natural Gas
4,097
## 2020 Retiring (Summer Capacity MW) EIA, Mar 2020

<table>
<thead>
<tr>
<th>Energy Source</th>
<th>United States</th>
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<tr>
<td>Coal</td>
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<tr>
<td>Natural Gas</td>
<td>1,022</td>
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<tr>
<td>Other</td>
<td>232</td>
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<tr>
<td>Wind</td>
<td>123</td>
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</table>
Summer Capacity MW, EIA, Mar 2020

<table>
<thead>
<tr>
<th>Energy Source</th>
<th>Capacity (MW)</th>
<th>Contribution (%)</th>
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<tbody>
<tr>
<td>Natural Gas</td>
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<tr>
<td>Coal</td>
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<td>Wind</td>
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<tr>
<td>Nuclear</td>
<td>98,119</td>
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<td>Hydro</td>
<td>79,788</td>
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<td>Solar</td>
<td>39,197</td>
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<td>Other</td>
<td>51,833</td>
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<tr>
<td>Storage</td>
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Total Capacity: 1,102,084 MW

Planned Retiring: 39,034 MW, 3.5%
Change: 43,350 MW, 3.9%
CLIMATE

FERC takes 'landmark action' on carbon pricing

Jeremy Dillon and Arianna Skibell, E&E News reporters • Published: Thursday, October 15, 2020
How does transition depend on market rules and policy?

Long run model

Not steady state

Must model energy market
The model

Entry and exit based on forward looking, rational investors

Each resource has anticipated cash flows for life of plant

   Capacity payments (if any)

   Energy and reserve revenues

   Less fixed and variable costs

Most uneconomic resources exit; most economic resources enter

Approximate equilibrium found where expectations are consistent
Must model energy market

Energy revenues depend critically on resource structure

Some resources are substitutes, some are complements

Even with a fixed resource structure, energy rents are hard to compute

- Day-ahead market is a large mixed-integer program (MIP)
  - Determines schedule and prices (financially binding)
- Intraday is done every hour to reschedule (for planning)
- Real time is economic dispatch every 5 minutes

Many days in the year
Entry and exit is a long run decision

Life of plant is 15 to 40 years or more

State space is infinite dimensional

- Resource structure
- Market rules and parameters
- Climate policy
- Extent of price responsive demand
- Evolution of technologies
- Fuel prices
Simplifying assumptions

Transmission constraints do not bind

Bids and offers for energy and capacity are competitive

Exception: hockey-stick offers when resource is near its upper limit
Climate policy

Investors anticipate carbon price path over life of plant
Forecast \textit{net load} hourly

Load (traditional) minus production from non-responsive resources:
Solar
Onshore wind
Offshore wind

Forecasts for 36.5, 35.5,...,0.5 hours ahead

Realized net load in five-minute intervals
For a *fixed resource structure*, get net load forecast

Run energy market for 120 days (one week per month)

Energy and reserves co-optimized both day ahead and real time

Day ahead unit commitment for scheduling and day-ahead pricing

Intraday MIP to adjust schedule

Real time economic dispatch for real time prices and quantities

Settlement

Key output: Energy rents (energy + reserves profits) for each type of resource
(a point in our “truth table”)

Energy Market Model
Energy Market Model

For a fixed resource structure, get net load forecast
Run energy market for 120 days (one day per month)
Energy and reserves co-optimized both day ahead and real time
Day ahead unit commitment for scheduling and day-ahead pricing
Intraday MIP to adjust schedule
Real time economic dispatch for real time prices and quantities
Settlement

Key output: Energy rents (energy + reserves profits) for each type of resource
(a point in our “truth table”)
Scenario: Increase renewable by 15 GW/year starting in 2035

Daily net load quickly goes negative

July 2060

But hourly peak is still 100 GW with 450 GW of renewable capacity
Scenario: Increase renewable by 15 GW/year starting in 2035

Daily net load quickly goes negative

April 2060

But net load is always substantially negative in shoulder months
Storage

Batteries are fundamentally different

Marginal cost (benefit) is opportunity cost (benefit)

Opportunity cost depends on price expectations and capabilities

Approach

Day ahead: directly model battery characteristics and schedule optimally

Real time: optimally dispatch based on linear program
Price responsive demand

Portion of load is traditional

Portion of load is price responsive

  Constant elasticity (a 1% increase in price, decreases quantity by 0.1%)

Demand curve for price responsive demand explicitly modeled in MIPs and LPs
Energy Market Proxy

Reduce list of units to almost sufficient statistics to describe resource structure
From truth table (actual energy market rents), econometrically estimate
  Energy rents for each resource type
  Energy rents for each unit

Periodically call energy market model to compute exact energy rents
Update parameter estimates using expanded truth table
Simulate for years 2019 to 2113

Form expectations up to 50 years ahead

Run capacity market (if any)

Find capacity price where supply and demand intersect

Alternate exit (most uneconomic) and entry (most economic)

Run energy market for delivery year (to expand truth table on equil path)

Update expectations (continue until expectations are consistent)
Resource types (15)

Coal, combustion turbine, combined cycle (with and without carbon capture)

Nuclear, Next-Gen Nuclear (2030 on)

Hydro

Onshore Wind, Offshore Wind

Solar

Battery Storage (1, 2, 4, 8-hour duration)
Other calibration factors

Initial list of units and characteristics

Operating costs, variable costs, fixed costs, fuel prices, dispatch characteristics

Efficiency of renewables

Initial capacity values (updated with exponential smoothing based on performance)

Financial parameters (discount rate)

Demand parameters

Fuel prices and carbon price
Scenarios

Market rules (capacity market with and without minimum offer price floor, energy-only market, other)

Rate of technical progress (CT, CC, next-gen nuclear renewables)

Fuel prices (low, medium, high)

Carbon price (none, low-end 2°C, low-end 1.5°C, twice that)

Price response demand (none, annual growth 1 percentage point, twice that)
Detailed evidence of impact of market rules and policies on:

- Pace of transition
- Market efficiency
- Cost to load
- Reliability
Climate policy
Donald J. Trump @realDonaldTrump

The concept of global warming was created by and for the Chinese in order to make U.S. manufacturing non-competitive.

Peter Cramton @pcramton · 39s

The concept of Donald J. Trump was created by and for the Russians in order to destabilize the West.
Global Carbon Pricing

The Path to Climate Cooperation

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Climate change is a cooperation problem
Narrow self-interest?

IPCC (2014): “International cooperation is necessary to significantly mitigate climate change because of the global nature of the problem” … but:

Christine Figueres (2015): “Frankly, none of them are doing it [agreeing to their pledges] to save the planet. Let us be very clear. They’re doing it for what I think is a much more powerful political driving force, which is for the benefit of their own economy”

Nicholas Stern (2015): "most of what is necessary for emissions reductions over the next two decades is in the self-interest of the individual nations”

Fergus Green (2015): “the default assumption in social science scholarship should be that actions to reduce emissions are nationally net-beneficial”
Narrow self-interest?

Limited carbon supply?
  • Carbon supply >> Carbon budget at 2°C

Technological miracle?
  • Very risky bet

Addressing local pollution?
  • Global and domestic damages are additive

*No need for an agreement to act in self-interest*
Can cooperation be designed?
Reciprocity!

Virtually all cooperation research agrees that reciprocity is the key to cooperation.

Nations exist mainly to achieve cooperation on public goods through enforced reciprocity.

Reciprocity can sometimes be designed, even absent a central authority.
Individual commitments cannot promote cooperation

• 10 players; individual endowment = $10

• Each $ pledged will be doubled and distributed evenly to all players

• Result: *Zero* cooperation, all pledge $0

![Diagram showing pledge range from $0 to $10 with unique equilibrium at zero cooperation](image-url)
Dynamics of individual commitments: “Upward spiral of ambition”?

- Japan, Russia, Canada, and New Zealand left the Kyoto agreement

- Ostrom (2010), based on hundreds of field studies: insufficient reciprocity leads to a “downward cascade”

- Supported by theory and laboratory experiments
“I will if you will” promotes cooperation

- 10 players; Individual endowment = $10
- Each $ pledged will be doubled and distributed evenly to all players
- *Pledge is commitment to reciprocally match the minimum pledge of others*
- Result: *Full cooperation, all pledge $10*
Reciprocal common commitment in climate negotiations
Consensus Aspiration: 2°C goal

1.5°

How to bridge gulf between goal and intentions?

IPCC, SYR Figure SPM.10
Intended Nationally Determined Contributions fall short

Edenhofer et al. 2016
Global carbon price
• Direct
• Efficient
• Transparent
• Promotes international cooperation
Price is focal
common commitment
(and Kyoto proved quantity is not)
Consensus that price should be uniform reduces dimensionality problem:

\[ P_{\text{country}} = P_{\text{global}} \]  

(No such consensus exists for quantity commitment)

Consistent with tax or cap & trade  
(flexible at country level)

Direct, efficient, common intensity of effort
Price commitment reduces risk
(compared to quantity commitment)

Stochastic shocks and other uncertainties destabilize cooperation, both in theory and empirics (e.g., Ambrus and Greiner 2012)

With carbon pricing, countries keep carbon revenues, which mitigates the risk of needing to buy carbon credits

Otherwise, huge windfall gains and losses possible
Price commitment reduces risk

- Quantity
- Price

Japan → Rest of World

Japan → Rest of World
Asymmetries

Asymmetries hamper cooperation
  • Establish coalition of the willing, omitting some
  • Last resort enforcement with trade sanctions

Use Green Fund to maximize abatement
Cramton and Stoft 2012; Cramton et al. 2015a, Kornek, Edenhofer 2015; Roofls et al. 2015

As before, reduce dimensionality
  • Carbon price = intensity of cooperation
  • “Generosity parameter” = intensity of Green Fund
National solution: carbon dividends

THE CONSENSUS CLIMATE SOLUTION

The Founding Members of the Climate Leadership Council believe that America needs a consensus climate solution that bridges partisan divides, strengthens our economy and protects our shared environment.

The Council’s carbon dividends solution embodies the conservative principles of free markets and limited government. It also offers an equitable, popular and politically-viable way forward, paving the way for a much-needed bipartisan climate breakthrough.

Our carbon dividends program is based on four interdependent pillars:

1. A gradually rising and revenue-neutral carbon tax;
2. Carbon dividend payments to all Americans, funded by 100% of the revenue;
3. The rollback of carbon regulations that are no longer necessary; and
4. Border carbon adjustments to level the playing field and promote American competitiveness.

This plan would achieve significantly greater emissions reductions than all current and prior climate regulations, while helping America’s businesses and workers get ahead. In fact, the bottom 70% of Americans would be financially better off.

Our carbon dividends solution is: Pro-Environment, Pro-Growth, Pro-Jobs, Pro-Competitiveness, Pro-Business and Pro-National Security.

Working with a range of constituencies, the Climate Leadership Council will develop and promote a consensus climate solution based on these pillars.
Legislative proposal: Carbon Fee and Dividend

Findings:
1. **Causation:** Whereas the weight of scientific evidence indicates that greenhouse gas emissions from human activities including the burning of fossil fuels and other sources are causing rising global temperatures.

2. **Mitigation** (Return to 350 ppm or below): Whereas the weight of scientific evidence also indicates that a return to the current concentration of more than 400 parts per million (“ppm”) of carbon dioxide (“CO₂”) to the atmosphere to 350 ppm CO₂ or less is necessary to slow or stop the rise in global temperatures.

3. **Endangerment:** Whereas further increases in global temperatures pose imminent and substantial dangers to human health, the natural environment, the economy, national security, and an unacceptable risk of catastrophic impacts to human civilization.

4. **Co-Benefits:** Whereas the measures proposed in this legislation will benefit the economy, human health, the environment, and national security, even without consideration of global temperatures, as a result of correcting market distortions, reductions in non-greenhouse-gas pollutants, reducing the outflow of dollars to oil-producing countries and improvements in the energy security of the United States.

5. **Benefits of Carbon Fees:** Whereas phased-in carbon fees on greenhouse gas emissions (1) are the most efficient, transparent, and enforceable mechanism to drive an effective and fair transition to a domestic-energy economy, (2) will stimulate investment in alternative-energy technologies, and (3) give all businesses powerful incentives to increase their energy-efficiency and reduce their carbon footprints in order to remain competitive.

6. **Equal Monthly Per-Person Dividends:** Whereas equal monthly dividends (or “rebates”) from carbon fees paid to every American household can help ensure that families and individuals can afford the energy they need during the transition to a greenhouse gas-free economy and the dividends will stimulate the economy.

Therefore the following legislation is hereby enacted:

1. **Collection of Carbon Fees:** Carbon Fee Trust Fund. Upon enactment, impose a carbon fee on all fossil fuels and other greenhouse gases at the point where they first enter the economy. The fee shall be collected by the Treasury Department. The fee on that date shall be $15 per ton of CO₂-equivalent emissions and result in equal charges for each ton of CO₂-equivalent emissions potential in each type of fuel or greenhouse gas. The Department of Energy shall propose and promulgate regulations setting forth CO₂-equivalent fees for other greenhouse gases including at a minimum methane, nitrous oxide, sulfur hexafluoride, hydrofluorocarbons (HFCs), perfluorocarbons, and nitrogen trifluoride. The Treasury shall also collect the fees imposed on the other greenhouse gases. All fees are to be placed in the Carbon Fees Trust Fund and rebated to American households as outlined in #3 below.

2. **Emissions Reduction Targets:** To align US emissions with the physical constraints identified by the Intergovernmental Panel on Climate Change (IPCC) to avoid irreversible climate change, the yearly increase in carbon fees including other greenhouse gases, shall be at least $10 per ton of CO₂-equivalent each year. Anually, the Department of Energy shall determine whether an increase larger than $10 per ton per year is needed to achieve program goals. Yearly price increases of at least $10 per year shall continue until total U.S. CO₂-equivalent emissions have been reduced to 10% of U.S. CO₂-equivalent emissions in 1990.

3. **Equal Per-Person Monthly Dividend Payments:** Equal monthly per-person dividend payments shall be made to all American households ($5 payment per child under 18 years old, with a limit of 2 children per family) each month. The total value of all monthly dividend payments shall represent 100% of the net carbon fees collected per month.

4. **Border Adjustments:** In order to ensure there is no domestic or international incentive to relocate production of goods or services to regimes more permissive of greenhouse gas emissions, and thus encourage lower global emissions, Carbon Fee-Equivalent Tariffs shall be charged for goods entering the U.S. from countries without comparable Carbon Fees-Carbon Pricing. Carbon Fee- Equivalent Rebates shall be used to reduce the price of exports to such countries. The State Department will determine rebate amounts and exemptions if any.
Conclusion

Cooperation can be engineered

Key to a strong climate treaty

• “I will if you will” (common commitment)

• Simplify: two parameters
  • Carbon price (common intensity of effort)
  • Green fund intensity (addresses asymmetries)
Price Carbon

I will if you will
Transportation
Markets for Road Use

Eliminating Congestion through Scheduling, Routing, and Real-time Pricing

Peter Cramton
R. Richard Geddes
Axel Ockenfels
• Global congestion costs $1 trillion/year
• Los Angeles congestion costs $23 billion/year
Transport market

• Open access
• Scheduled/routed transport
• Efficient congestion pricing

No congestion
The time is right

• Advances in mobile communications enable
  • Precise (to 1 cubic meter) location of vehicles
  • Easy communication of preferences, prices, schedules

• Advances in computers and markets enable efficient
  scheduling/routing and pricing of transport

And case for innovation gets stronger each year
Autonomous vehicles are here
Market objectives

- Efficiency
- Transparency
- Simplicity
- Fairness

*Draw on best practice from existing time and locational markets*
Key market principle: open access

• Transport network is open to all
• Nondiscriminatory terms
• Network capacity cannot be withheld
  ⇒ Efficient congestion pricing

• Basis for restructured electricity markets in US, Europe, ...
Open access transport network (Independent System Operator)

Users/vehicles

Integrated wholesale and retail market

Singapore market model
Open access transport network
(Independent System Operator)

Service providers

SP_1
SP_2
SP_3

Users/vehicles
A B C D E F G

Wholesale market
Retail market

Same market model as electricity successfully operating for two decades
Players

• Independent System Operator (ISO)
  • Runs market

• ISO or service providers
  • Develop user app for expression of demand
  • Aggregate user demand
  • Guide user (scheduling/routing)
  • Establish user plans and settle payment

• Users
  • Provide fundamental demand for road use
Product design

• Slot on congested road segment at particular time (e.g. 10 minute time interval)
Important features of setting

• Limited number of congested road segments
  • Bridges, tunnels, and other bottlenecks
• Congested segments are highly predictable
• Demand *does* respond to price even close to real time
  • Time shifters: shift transport to less congested time
  • Route shifters: shift route to less congested route
  • Mode shifters: take train, bus, bike or work-at-home
How today’s apps would change
Value of time $75/hour  Edit
Vehicle: MD 012 ABC  Edit

via I-495 N  31 min  18.4 miles $3.42↑
Fastest route, the usual traffic
DETAILS
Tip: Prices typically increase in late afternoon.

via I-495 E  38 min  15.2 miles $3.78↑

via MD-410 E  42 min  13.1 miles $4.65↑
GOOGLE TRAVEL TIMES

Home
Dahlonega Rd
1 min

Work
I-495 N
30 min
$3.42↑

JUST NOW

Route options

Avoid highways

Avoid tolls

Avoid ferries
Value of time $75/hour
Vehicle: MD 012 ABC

Remember settings
Complex consumer trade-off today

• Leaving early results in less delay and lower variance but imposes high social cost on other commuters
• By contrast road pricing results in no delay and much higher throughput, but commuters pay a higher price to travel at the most popular times
Commute to Tysons
Must arrive by 9am
(a) Depart at 7:45am
Arrive about 9am
Delay of 14 to 49 min.
Commute to Tysons
Must arrive by 9am
(b) Depart at 6:45am
Arrive about 7:40am
Delay of 4 to 29 min.
Commute to Tysons
Must arrive by 9am
(c) Depart at 5:45am
Arrive about 6:25am
Delay of 0 to 14 min.
Multiple opportunities to trade

• Real time (10 minutes)
  • Apps redirect traffic given preferences and prices

• Daily
  • Service providers estimate demand for each congested road segment in each 10 minute interval

• Monthly
  • Service providers estimate demand for month

Forward markets are financial; real time is physical
Sequence of auctions

- Multiple opportunities to trade
  - Reduces risk of service provider
  - Facilitates planning of service provider
  - Provides price transparency
  - Mitigates market power
Step 1: Vehicle measurement of road use

• Real time kinematic
  • 2 cm accuracy
  • Retail cost about $50 with scale
Step 2: Congestion pricing

- addresses congestion externality (Vickrey 1963)
- addresses environmental externality
- encourages drivers to explore travel alternatives during peak times
- simplifies consumer decision-making
- improves safety
- allows joint optimization of all transport (roads and transit)
- provides essential information to direct scarce investment resources
- generates the funds that underlie that investment with non-distortionary taxes
- adopts basic fairness principles
- allows scarce road space to be allocated to those who value it most highly
- incentivizes technological innovations that reduce demand on scarce capacities
Incremental implementation

1. Identify issues with current system
2. Introduce measurement; simulate network
3. Introduce initial time-and-location prices (fixed!)
4. Re-estimate prices and introduce refined prices
5. Repeat until learn behavioral response
6. Introduce real-time pricing (e.g. adapts to shocks in supply)
7. Introduce forward purchase (establish sensible default plan)
8. Study long-term impact
Conclusion

• Assures transport network is used efficiently
• Eliminates congestion through scheduling/routing and congestion pricing
• Transparent pricing motivates network investment and provides much needed funds
Financial Markets
Budish, Cramton and Shim (2015) • Speed race is built into current market design • Speed race is costly • Costs are borne by fundamental investors • Frequent batch auctions eliminate speed race • Transforms competition on speed to competition on price • Computational benefits for traders, exchanges, and regulators
The HFT Arms Race

- In 2010, Spread Networks invests $300mm to dig a high-speed fiber optic cable from NYC to Chicago
- Shaves round-trip data transmission time... from 16ms to 13ms
- Industry observers: 3ms is an “eternity”
- Joke at the time: next innovation will be to dig a tunnel, “avoiding the planet’s pesky curvature”
- Joke isn’t that funny... Spread’s cable quickly obsolete!

- Question: how could such tiny speed advantages be worth so much money?
  - 3 milliseconds too short to be about fundamentals
  - Economists intrinsically skeptical about technical trading
The HFT Arms Race: Market Design Perspective

- We approached the HFT arms race from the perspective of market design.
  - We assume that HFT’s are optimizing with respect to market rules as presently given
  - But, ask whether these are the right rules (avoids “is HFT good or evil?”).
  - Focus on the precise institutional details of the markets in question. Al Roth: “Economist as Engineer”
  - Milton Friedman: “rules of the game”.

- Indeed, we find a subtle flaw in the design of modern financial exchanges.
  - Flaw: exchanges treat time as a continuous variable and process requests to trade serially
The HFT Arms Race: Market Design Perspective

- Continuous-time + serial processing $\rightarrow$ riskless arbitrage profits from symmetric public information
  - (info either technical or fundamental)

- That is... a violation of the weak-form and semi-strong form EMH, built directly into the market design.

- These riskless arbitrage profits
  1. Are not supposed to exist in a well-functioning market
  2. Harm liquidity
  3. Induce a never-ending arms race for speed

- Market design solution: put time into units ("discrete time") and process requests to trade in batch, using auctions.
  1. Transforms competition on speed into competition on price.
  2. Fixes the violation of EMH.
  3. Improves liquidity and stops the arms race.
Brief Description of the Continuous Limit Order Book

- Basic building block: limit order
  - Specifies a price, quantity, and buy/sell (bid/ask)
  - “Buy 100 shares of XYZ at $100.00”
- Traders may submit limit orders to the market any time during the trading day
  - Also may cancel or modify outstanding limit orders at any time
  - Orders and cancelations are processed by the exchange one-at-a-time in order of receipt (serial process)
- Set of outstanding orders is known as the limit order book
- Trade occurs whenever a new limit order is submitted that is either (i) bid $\geq$ lowest ask; (ii) ask $\leq$ highest bid
  - New limit order is interpreted as accepting (fully or partially) one or more outstanding orders

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<tr>
<td>62.31</td>
<td>5,600</td>
</tr>
</tbody>
</table>
Market Correlations Break Down at High Frequency
ES vs. SPY: 1 Day
Market Correlations Break Down at High Frequency

ES vs. SPY: 1 hour

[Graph showing the correlation between ES and SPY indices over time, with peaks and troughs indicating volatility and correlation changes.]
Market Correlations Break Down at High Frequency
ES vs. SPY: 1 minute
Market Correlations Break Down at High Frequency

ES vs. SPY: 250 milliseconds
Arb Durations over Time: 2005-2011

Median over time

Distribution by year
Arb Per-Unit Profits over Time: 2005-2011

- Median over time
- Distribution by year
Correlation Breakdown Over Time: 2005-2011
Latency Arb and Arms Race are “Constants”

To summarize:

- Competition **does** increase the speed requirements for capturing arbs (“raises the bar”)
- Competition **does not** reduce the size or frequency of arb opportunities
- Suggests we should think of latency arbitrage and the resulting arms race as a “constant” of the current market design
The Case for Frequent Batch Auctions

A simple idea: discrete-time trading.

1. Empirical Facts: continuous market violates basic asset pricing principles at HFT time horizons.
   - Market correlations completely break down.
   - Frequent mechanical arbitrage opportunities.
   - Mechanical arbs $\rightarrow$ arms race. Arms race does not compete away the arbs, looks like a “constant”.

2. Theory: root flaw is continuous-time serial-process trading
   - Mechanical arbs are “built in” to market design. Sniping.
   - Harms liquidity.
   - Induces never-ending, wasteful, arms race for speed.

3. Solution: frequent batch auctions
   - Competition on speed $\rightarrow$ competition on price.
   - Enhances liquidity and stops the arms race.
   - Simplifies the market computationally.
Model: Preliminaries

- Descendant of the famous Glosten Milgrom (1985) model
- Security $x$ that trades on a continuous limit-order book market
- Publicly observable signal $y$ of the value of security $x$
- Purposefully strong assumption:
  - Fundamental value of $x$ is *perfectly* correlated to the public signal $y$
  - $x$ can always be costlessly liquidated at this fundamental value
  - Goal: “best case” scenario for price discovery and liquidity provision
- The public signal $y$ evolves as a compound Poisson jump process, symmetric with mean zero
  - Arrival rate $\lambda_{\text{jump}}$
  - “Jump size” distribution $J$ (dist. of absolute value of jumps)
"Sniping"

Fundamental value and bid-ask spread
“Sniping”

Fundamental value jumps

$Y_1$  
$Y_2$

ASK  
BID
TFs providing liquidity send messages to cancel old quotes and add new quotes.
“Sniping”

TFs providing liquidity send messages to cancel old quotes and add new quotes
At the same time, other TFs send messages to “snipe” the stale quotes
Because the market design processes messages in serial, liquidity providers get sniped with probability $\frac{N-1}{N}$ . . . even though the information was public and all TFs have the exact same technology.
First Chicago-NYC Microwave Network
The HFT Arms Race: Continued
Active Microwave Networks in the Chicago-NYC-DC Region as of 2010-01-01
Active Microwave Networks in the Chicago-NYC-DC Region as of 2016-12-01
The Case for Frequent Batch Auctions

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   - Competition on speed $\rightarrow$ competition on price.
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   - Simplifies the market computationally.
Frequent Batch Auctions: 3 Cases

Case 1: Nothing happens during the batch interval

- Very common case: most instruments, most 1ms periods, there is zero activity
- All outstanding orders carry forward to next interval
- Analogous to displayed liquidity in a LOB market
Frequent Batch Auctions: 3 Cases

Case 2: **Small amount of trade**
- Example: an investor arrives wanting to buy a small amount at market
- Demand will cross supply at the bottom of the supply curve
- Analogous to trading at the ask in a LOB market

![Graph showing price and quantity relationship in Case 2: Investor Buys q^*](image-url)
Frequent Batch Auctions: 3 Cases

Case 3: Burst of activity in the interval

- Example: there is public news (jump in $y$) and many algos respond
- In this case, FBA and LOB are importantly different
Why FBA Solves the Problem

\[ \tau - \delta_{slow} \quad \tau - \delta_{fast} \]

0 \[ \tau \]

**Reason 1: Discrete time reduces the economic relevance of tiny speed advantages**

- Most public information arrives at a time such that all market participants see it equally.
  - 0 \( \rightarrow \) \( \tau - \delta_{slow} \) everybody sees it
  - \( \tau - \delta_{fast} \rightarrow \tau \) nobody sees it
  - \( \tau - \delta_{slow} \rightarrow \tau - \delta_{fast} \) speed advantage relevant. Proportion \( \frac{\delta}{\tau} \)

- If the public information is information from past prices... proportion zero.

- Whereas: in the continuous market, the speed advantage is relevant for *ALL* public information.
Reason 2: Auction changes the nature of competition. From competition on speed to competition on price

- Suppose:
  - Public information arrives in the critical window
  - There are some slow traders with stale quotes in the book
  - There are some fast traders who see the new information
- Continuous market: competition on speed, to snipe the stale quotes
- Batch auction market: competition on price!
Computational Benefits of Discrete Time

- Overall claim/conjecture: discrete-time trading has significant computational simplicity benefits for markets relative to continuous-time trading.

- Main intuition
  - Continuous-time markets implicitly assume that computers and communications technology are infinitely fast.
  - Computers and communications are fast but not infinitely so.
  - Discrete time respects the limits of computers and communications.

- Let me give some specific examples for:
  1. Exchanges
  2. Algorithmic Traders
  3. Regulators / Market observers

- Caveat: argument is qualitative / informal. Would benefit from more formal treatment.
Flow Trading

Eric Budish, Peter Cramton, Pete Kyle, Mina Lee, and David Malec
Key elements

- Periodic clearing
- Sophisticated expression of preferences
- Optimization of gains from trade
- Improved outcome discovery, transparency, trust, and privacy
“At $1,730.22 I want to buy 10.1234 shares of Amazon per minute.”
Maximize gains from trade (social welfare) s.t. linear constraints

\[
\text{maximize } \sum_{o \in O} V_o(x_o) \quad \text{subject to}
\]

\[
\sum_{o \in O} P_o(a) \cdot x_o = 0 \quad \forall a \in A : \mu_a
\]

\[
x_o \leq \bar{q}_o \quad \forall o \in O : \bar{\beta}_o
\]

\[
x_o \geq q_o \quad \forall o \in O : \beta_o
\]
Competitive equilibrium quantities and prices exist and are unique when it matters (positive quantity traded and not perfect substitutes)

Outcome maximizes as-bid social welfare s.t. constraints

Incentives for truthful bidding are good (and excellent for most liquid products)

Outcome is as-bid envy free (given prices, everyone gets their favorite bundle)

Scales to large number of products and participants

Compact expression of preference make this especially attractive for blockchain implementation
Market design innovation

• Electricity
  ✓ Open access
  ✓ Pay for performance

• Communications
  ✓ Auction spectrum
  • Open access

• Transportation
  • Price congestion

• Climate policy
  • Price carbon

• Financial securities
  • Make time discrete
  • Improve preference expression

Plenty of work ahead!