Economics 703
Advanced Microeconomics
Prof. Peter Cramton

Problem Set 8

1. Show that the unique stable component of the game below is (T,L) to (T, 1/2 L); that is, it must include randomizations for 2 that put weight of up to 1/2 on the strategy L.

\[
\begin{array}{c|c|c}
 & L & R \\
\hline
B & 1 & 0 \\
M & 2 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
 & L & R & & \\
\hline
T & 2,0 & 2,0 & & \\
M & 1,0 & 3,1 & & \\
B & 1,2 & 3,1 & & \\
\end{array}
\]

2. Prove the following proposition from the notes.

**Proposition.** A strategy profile \( \pi \) is a forward induction equilibrium if and only if \( \pi \) is a sequential equilibrium satisfying the extended intuitive criterion.

3. Consider using the Crawford and Sobel model to describe wages attached to jobs. The sender S is a firm that employs workers of different productivities. The receiver R is the rest of the market. The firm privately observes each worker's productivity, \( m \). Each worker has firm-specific human capital in the amount \( b \). There are \( N \) jobs to which each worker can be assigned. The jobs differ only in job title: each job uses exactly the same technology. [Think of assistant and associate professors, or the various grades in the Civil Service.]

The market observes each worker's job assignment \( n \in N \) and offers a wage \( y(n) \). Under perfect information, competition among firms in the market would bid up this wage to the productivity \( m \). This is modeled as

\[ U^R(y,m) = -(y - m)^2. \]

If the firm wants to retain the worker, it must match the wage offer \( y(n) \). Thus, \( U^S(y,m,b) = m(1 + b) - y. \)

Describe and interpret the equilibrium job assignments. In particular, show that all workers must receive the same wage.

4. Prove Crawford and Sobel's claim that if \( U^S_{12} \) and \( U^R_{12} \) both are one-signed but have opposite signs then only one action can be induced in equilibrium.