Problem Set 4

1. The class notes on games with incomplete information claim that the unique symmetric equilibrium for the two-bidder first-price auction with valuations independently and uniformly distributed on \([0,1]\) is \(b(v_i) = v_i/2\). Prove it.

2. Consider an \(n\)-bidder second-price auction: the highest bidder wins the object but pays a price equal to the second highest bid. Let the bidders' valuations be independently and identically distributed according to \(F(v_i)\) on support \([v,v]\). Find the symmetric equilibrium bidding strategy.

3. Consider the problem of locating a hazardous-waste dump in one of the \(n\) towns in a state. Let the town's disutilities for taking the dump be independently and uniformly distributed on \([0,1]\). Suppose each town bids for the dump by stating the amount it would need to be compensated for taking the dump. The lowest bidder gets the dump and receives compensation equal to its bid; the \(n-1\) other towns pay the compensation in equal shares. Find the symmetric equilibrium bidding strategy.

4. Prove that a Bayesian equilibrium exists for the Bayesian game

\[ \Gamma = \{S_1,\ldots,S_n; T_1,\ldots,T_n; p_1,\ldots,p_n; u_1,\ldots,u_n\}, \]

where the type spaces \(T_i\) and pure-strategy spaces \(S_i\) are finite.