Economics 703
Advanced Microeconomics
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Problem Set 3

1. In Rubinstein's bargaining model, the time between offers is arbitrary. Let it be \( t > 0 \), and let player \( i \)'s impatience be measured by \( \Delta_i > 0 \), where \( \delta_i = e^{-\Delta_i t} \). What happens to the equilibrium payoffs as \( t \) goes to zero?

2. This problem considers cartel maintenance over the business cycle by varying the parameter \( a \), the intercept in Porter's linear demand curve. Following Proposition 3.2, Porter shows that

\[
\frac{dq^*}{da} = \frac{2N + 1 + \eta^*}{2NB(N + 1 + \eta^*)},
\]

where \( q^* \) is the quantity produced by each firm under the optimal trigger-price strategy. In contrast,

\[
\frac{dr}{da} = \frac{1}{2NB},
\]

where \( r \) is the quantity produced by each firm under maximal symmetric collusion. [Both of these assume \( \mu = 1 \).] Recall from Proposition 3.4 that \( q^* > r \), so \( q^* - r \) is a measure of firms' (in)ability to collude. What happens to this measure during a boom (i.e., if \( a \) increases), and why?

3. Consider the both-pay auction discussed in class. A prize of $10 is auctioned to the highest of two bidders. The players alternate bidding. At each stage, the bidding player must decide either to raise the bid by $1 or to quit. The game ends when one of the two bidders quits in which case the other bidder gets the $10 prize, and both bidders pay the auctioneer their most recent bids. The first player begins with an initial bid of $1 (unless she decides to quit immediately, with the result that the $10 prize goes to player 2 and both pay nothing to the auctioneer). If both players play the raise-forever strategy, then there payoffs are \((-\infty, -\infty)\).

(a) What are the two pure-strategy equilibria?

(b) Consider the following mixed-strategy equilibrium. Player 1 bids $1 for sure and then each player quits with a constant probability in every subsequent round (i.e., player 2 quits with probability \( q \) and bids $2 with probability \( 1 - q \); if player 2 raised to $2, player 1 quits with probability \( p \) and raises to $3 with probability \( 1 - p \); etc.). For what values of \( p \) and \( q \) is this a subgame-perfect equilibrium? [Hint: A player must be indifferent among the pure strategies he is randomizing over.]

(c) In the equilibrium in (b), what is the most that player 1 would be willing to pay for the right to make the first bid?

(d) Are there any other equilibria, besides those found in (a) and (b)?