Economics 703
Advanced Microeconomics

Problem Set 3

1. Consider an n-bidder second-price auction: the highest bidder wins the object but pays a price equal to the *second* highest bid. Let the bidders' valuations be independently and identically distributed according to $F(v_i)$ on support $[\underline{v},v]$. Find the symmetric equilibrium bidding strategy.

2. Consider the problem of locating a hazardous-waste dump in one of the n towns in a state. Let the town's *dis*utilities for taking the dump be independently and uniformly distributed on [0,1]. Suppose each town bids for the dump by stating the amount it would need to be compensated for taking the dump. The lowest bidder gets the dump and receives compensation equal to its bid; the n - 1 other towns pay the compensation in equal shares. Find the symmetric equilibrium bidding strategy.

3. A company solicits a new logo design via a crowdsourcing competition. Each participant submits a logo design, which the company ranks based on quality. The highest quality submission wins a reward r, and no other submissions win anything.

   The contest attracts n participants. Each participant i has a private skill level s_i, and must choose a level of effort e_i to put into their submission. Exerting effort e_i produces a submission of quality $q_i = s_i \cdot e_i$. Participant i's utility is

$$u_i = \begin{cases} r - e_i & \text{if } i \text{ wins the contest} \\ -e_i & \text{otherwise} \end{cases}$$

   Participants are risk-neutral utility maximizers, with skill levels drawn independently and uniformly from the interval [0,1].

   (a) Suppose there are only 2 bidders; find a symmetric Bayes-Nash equilibrium where quality is quadratic in skill level.

   (b) For general n, show that quality is monotonic in skill level.

   (c) For general n, find the symmetric Bayes-Nash equilibrium strategy for participants.

   (d) For general n, what is the expected quality of the best submission? How does this compare to the total (expected) effort expended by participants?

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4. Consider the Myerson-Satterthwaite [1983] framework in which both traders' valuations (s for the seller and b for the buyer) are uniformly distributed on [0,1].

   (a) Suppose an English (ascending-offer) auction is used to allocate the good between the seller and the buyer. The price rises continuously until one of the traders drops out. If the seller drops out first, the buyer gets

the good at the seller's droppout price. Show that the seller's optimal strategy is to drop out at the price 1/2 + s/2 and the buyer's optimal strategy is to drop out at the price b.

(b) Show that the extensive form in (a) implements the ex ante efficient mechanism that maximizes the seller's expected payoff (i.e., the welfare weights are 1 for the seller and 0 for the buyer).