Economics 703: Advanced Micro

Problem Set 2

1. The normal-form game described below is played twice; the players' preferences are represented by the average of their stage-game payoffs. The variable x is greater than 4, so that (4,4) is not an equilibrium payoff in the stage game. How large can x be if the following strategy is to be subgame-perfect equilibrium behavior for the two players? Play (s_2,t_2) in the first stage. If no one deviates or if both deviate then play (s_1,t_1) in the second stage. If only player 1 deviates then play (s_3,t_3) in the second stage, and if only player 2 deviates then play (s_4,t_4) in the second stage.

1 \ 2	t_1	t_2	t_3	t_4
s_1	2, 2	x, 0	-1, 0	0, 0
s_2	0, x	4, 4	-1, 0	0, 0
s_3	0, 0	0, 0	0, 2	0, 0
s_4	0, -1	0,-1	-1,-1	2, 0

2. The normal-form game described below is repeated infinitely. Both players discount payoff streams using the discount factor $\delta = .9$. Determine the length of the punishment period described in the strategies in Theorem 1 in Fudenberg and Maskin that is necessary to support (4,4) as the payoff in every stage of a subgame-perfect equilibrium. What punishment length is necessary to support (3/4, 3/4) in every stage of a subgame-perfect equilibrium? Note that the latter does not Pareto dominate (1,1), the payoff to the pure-strategy Nash equilibrium of the stage game.

1 \ 2	t_1	t_2	t_3
s_1	1, 1	5, 0	0, 0
s_2	0, 5	4, 4	0, 1
s_3	0, 0	1, 0	-1,-1

3. Consider complete-information Rubenstein bargaining between two agents to decide the apportionment of a finitely-divisible good. Agents alternate making offers, and face a common discount factor δ. As usual, each offer takes one round, and receiving a portion p of the item in round t is worth $\delta^t \cdot p$ to an agent. The item consists of $\mathrm{m} \in \mathbb{Z}_+$ equally-sized, indivisible components, and any offer that requires dividing one of these components is invalid. That is, only offers $\left(\frac{k}{\mathrm{m}}, 1 - \frac{k}{\mathrm{m}}\right)$, with $k \in \{0,1,\ldots,\mathrm{K}\}$, may be proposed by either agent.

(a) Say the item can only be divided into fourths and agents have discount factor of 85%, i.e. $\mathrm{K} = 4$ and $\delta = 0.85$. Find the subgame perfect equilibria when the game lasts for $T = 4$ rounds.

(b) Now, consider the infinite-horizon version of this bargaining game. Keeping $\mathrm{K} = 4$ and $\delta = 0.85$, find all divisions supported by subgame perfect equilibria.

(c) In the case of an infinitely-divisible item, the subgame perfect equilibrium in the finite-horizon case converges to that in the infinite-horizon case as the number of rounds $T \to \infty$. Based on your answers to (a) and (b), is that still the case here? Give some intuition for why we should expect this answer. (Hint: consider the case $T = 6$)