Economics 703
Advanced Microeconomics

Problem Set 1

Exercises (not graded):

A. Three players simultaneously pick a point on the interval $[0,1]$. The player closest to the average of the three points wins $1. If there is a tie, then the dollar is split equally among the tied players. More formally, the players simultaneously choose strategies $s_i \in S_i = [0,1]$, and the average of their choices is computed as $\bar{s} = (s_1 + s_2 + s_3)/3$. Then, player i's payoff function is given by

$$U_i(s_1, s_2, s_3) = \begin{cases} 1/t & \text{if } i \in \operatorname{argmin}_j \left| s_j - \bar{s} \right| ;\text{and} \\ 0 & \text{otherwise,} \end{cases}$$

where t is the number of players who tie (their choices are equally close to the average).

(i) What are the pure-strategy equilibria of this game?

(ii) What are the mixed-strategy equilibria if the possible strategies are limited to playing 0 or 1, rather than [0,1]?

B. Change the payoff functions given above from argmin to argmax (each player tries to be the farthest from the mean). Describe the pure-strategy equilibria of the new game in terms of the following possibilities:

(i) all three players choose the same strategy,

(ii) exactly two of the three choose the same strategy, and

(iii) no two players choose the same strategy.

Note: The existence theorems for pure-strategy equilibria discussed in class do not apply to the games in exercises A and B. Nonetheless, pure-strategy equilibria exist.]

Problems:

1. Consider the following variant of the game described in Exercises A and B: We still have three symmetric players, and each player i must select a strategy $s_i \in S_i = [0,1]$. Here, a player i can receive two kinds of payoffs:

   a. If player i chooses s_i such that $i \notin \text{argmin}_j |s_j - \bar{s}|$, they receive a payoff of 1.
   b. If player i chooses s_i such that $i \notin \text{argmax}_j |s_j - \bar{s}|$, they receive a payoff of 1.

   A player who satisfies both conditions above receive *both* payoffs, i.e. total payoff 2.

   Either describe the pure strategy equilibria of this game, or prove there are none.

2. Consider the road intersection/stoplight game described in class in the context of correlated equilibria. Two players each approach an intersection, and must choose between (W)aiting and (P)roceeding with the following payoffs:

		2: W	2: P
1	W	-1, -1	0, 1
1	P	1, 0	-10, -10

   a. Describe the set of correlated equilibria possible in this game that avoid crashes, i.e. give the triples (a, b, c) of probabilities that a valid correlated equilibrium could place on the strategy profiles (W,W), (W,P), (P,W) respectively (with probability 0 placed on the profile (P,P))
   b. What if players 1 and 2 only appear (independently) with probabilities p_1, $p_2 \in [0,1]$, and players must commit to their strategies before discovering whether or not their opponent has shown up? Assume a player who does not appear always receives payoff 0; and facing a player who does not show up is identical to facing a player who always (W)aits. Assuming that $p_1, p_2 \geq 1/5$, describe the set of equilibria without crashes – given as triples as described in part (a) – as a function of p_1, p_2.
   c. Continuing from part (b) above, find the equilibrium (or set of equilibria) that maximize the social welfare (sum of expected payouts to players 1 and 2) in that game. In addition to assuming that $p_1, p_2 \geq 1/5$, you should also assume that the probability placed on the state

(W, W) is at least 5%. You can think of this constraint as requiring that stoplight changes require a transition period so vehicles may safely exit the intersection.

d. It is possible for a stoplight to adjust its schedule when cars are only present on one of the intersecting roads, to allow those cars to proceed immediately. Similarly, we could consider allowing a correlated equilibrium to condition on whether each of players 1 and 2 show up. What does the set of correlated equilibria look like? Which ones maximize social welfare? If a city has limited resources, which intersections should they focus on (in terms of p_1 and p_2)?

3. Consider a two-player game where all payoffs are distinct. For example, such a game could be produced with probability 1 by drawing every entry in the payoff matrix uniformly from the distribution $U[0,1]$. The following questions relate to an arbitrary example of such a game.
   a. Assume each player has exactly two available strategies. How many pure-strategy Nash equilibria are there? How many mixed-strategy equilibria are there? How are these related?
   b. How does your answer to part (a) change if one player has 2 available strategies, but the other has 3 available strategies?
   c. How (if at all) would your answer to part (b) change if we disallowed any equilibria that place mass on weakly dominated strategies?