Economics 703
Advanced Microeconomics

Problem Set 1

Exercises (not graded):

A. Three players simultaneously pick a point on the interval $[0,1]$. The player closest to the average of the three points wins \$1. If there is a tie, then the dollar is split equally among the tied players. More formally, the players simultaneously choose strategies $s_i \in S_i = [0,1]$, and the average of their choices is computed as $\bar{s} = (s_1 + s_2 + s_3)/3$. Then, player i's payoff function is given by

$$U_i(s_1, s_2, s_3) = \begin{cases} 1/t & \text{if } i \in \text{argmin}_j \left| s_j - \bar{s} \right| \text{;and} \\ 0 & \text{otherwise,} \end{cases}$$

where t is the number of players who tie (their choices are equally close to the average).

(i) What are the pure-strategy equilibria of this game?

(ii) What are the mixed-strategy equilibria if the possible strategies are limited to playing 0 or 1, rather than [0,1]?

B. Change the payoff functions given above from argmin to argmax (each player tries to be the farthest from the mean). Describe the pure-strategy equilibria of the new game in terms of the following possibilities:

(i) all three players choose the same strategy,

(ii) exactly two of the three choose the same strategy, and

(iii) no two players choose the same strategy.

Note: The existence theorems for pure-strategy equilibria discussed in class do not apply to the games in exercises A and B. Nonetheless, pure-strategy equilibria exist.

Problems:

1. Consider the following variant of the game described in Exercises A and B: We still have three symmetric players, and each player i must select a strategy $s_i \in S_i = [0,1]$. Here, a player i can receive two kinds of payoffs:

   a. If player i chooses $s_i \notin \min_j |s_j - \bar{s}|$, they receive a payoff of 1.
   b. If player i chooses $s_i \notin \max_j |s_j - \bar{s}|$, they receive a payoff of 1.

   A player who satisfies both conditions above receive *both* payoffs, i.e. total payoff 2.

   Either describe the pure strategy equilibria of this game, or prove there are none.

2. Consider a road intersection coordination game in the context of correlated equilibria. Two players each approach an intersection, and must choose between (W)aiting and (P)roceeding with the following payoffs:

		2: W	2: P
1	W	-1, -1	0, 1
1	P	1, 0	-10, -10

   a. Describe the set of correlated equilibria possible in this game that avoid crashes, i.e. give the triples (a, b, c) of probabilities that a valid correlated equilibrium could place on the strategy profiles (W,W), (W,P), (P,W) respectively (with probability 0 placed on the profile (P,P))
   b. What if players 1 and 2 only appear (independently) with probabilities p_1, $p_2 \in [0,1]$, and players must commit to their strategies before discovering whether their opponent has shown up? Assume a player who does not appear always receives payoff 0; and facing a player who does not show up is identical to facing a player who always (W)aits. Assuming that $p_1, p_2 \geq 1/5$, describe the set of equilibria without crashes – given as triples as described in part (a) – as a function of p_1, p_2.
   c. Continuing from part (b) above, find the equilibrium (or set of equilibria) that maximize the social welfare (sum of expected payouts to players 1 and 2) in that game. In addition to assuming that $p_1, p_2 \geq 1/5$, you should also assume that the probability placed on the state

(W, W) is at least 5%. You can think of this constraint as requiring that stoplight changes require a transition period so vehicles may safely exit the intersection.

d. It is possible for a stoplight to adjust its schedule when cars are only present on one of the intersecting roads, to allow those cars to proceed immediately. Similarly, we could consider allowing a correlated equilibrium to condition on whether each of players 1 and 2 show up. What does the set of correlated equilibria look like? Which ones maximize social welfare? If a city has limited resources, which intersections should they focus on (in terms of p_1 and p_2)?

3. Consider a two-player game where all payoffs are distinct (a generic game). Such a game can be produced with probability one by drawing every entry in the payoff matrix uniformly from the distribution $U[0,1]$, where you are allowed to eliminate zero probability events (ties). The following questions relate to an arbitrary example of such a game.

   a. Assume each player has exactly two available pure strategies. Can you say anything about the number of Nash equilibria?

   b. How does your answer to part (a) change if one player has 2 available strategies, but the other has 3 available strategies?