Practical Auction Design

Peter Cramton
Introduction

• Auction design
  – Government perspective (design)
  – Bidder perspective (strategy)

• Based on my experience
  – Researching auctions
  – Advising governments (a dozen)
  – Advising bidders (more than two dozen)
Advising bidders
Advising bidders: Typical project

- Comment on auction design (regulatory)
- First strategy meeting
  - Objectives
  - Strategic issues
- Develop bid tracking tool
- Second strategy meeting
  - Development of strategy
  - Several mock auctions
- Daily auction advice during auction
  - Comment on auction strategy and end-of-day report
Fundamental strategic issues
Fundamental strategic issues

- Winner’s curse
- Demand reduction
- Exposure problem
- Asymmetries
- Collusion
- Complexity
## Auctioning a single good

Auctions typically take one of four simple forms:

<table>
<thead>
<tr>
<th>Dynamic</th>
<th>Sealed Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>English (↑ price)</td>
<td>2nd Price</td>
</tr>
<tr>
<td>Dutch (↓ price)</td>
<td>1st Price</td>
</tr>
</tbody>
</table>
Auction exercise

• Bid for single object
• Common value = $1 per bean
• On slip of paper write:
  – Name
  – Estimate (# of beans × $1)
  – Bid in first-price sealed-bid auction
  – Bid in second-price sealed-bid auction
Winner’s curse
Winner's curse

I won. Therefore, I overestimated the most. My bid only matters when I win, so I should condition my bid on winning (i.e., that I overestimated the most).

• Winning is bad news about my estimate of value. No one else was willing to bid as much.
Auction design pitfalls

• Auction design can force bidders to make guesses
  – In a simultaneous *sealed-bid* auction bidders must guess about the bids of others
  – In sequential auctions bidders must guess about future prices

• Bidder uncertainty
  – Increases likelihood of inefficient or low-value assignments
  – Can often be reduced
  – Makes bidding difficult, undermines confidence, and can lead to defaults
Exercise

• 2 bidders (L and R), 2 identical items
• L has a value of $100 for 1 and $200 for both
• R has a value of $90 for 1 and $180 for both
• Uniform-price auction
  – Submit bid for each item
  – Highest 2 bids get items
  – 3rd highest bid determines price paid
• Ascending clock auction
  – Price starts at 0 and increases in small increments
  – Bidders express how many they want at current price
  – Bidders can only lower quantity as price rises
  – Auction ends when no excess demand (i.e. just two demanded); winners pay clock price
Demand reduction
Inefficiency theorem

In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values.

- Winning bidder influences price with positive probability
- Creates incentive to shade bid
- Incentive to shade increases with additional units
- Differential shading implies inefficiency
Inefficiency theorem and bid shading

• Exceptions:
  – Pure common value
  – Bidders demand only a single unit
Inefficiency from differential shading

Large bidder makes room for smaller rival
What if private information?

- 2 bidders (L and S), 2 identical items
- L has constant marginal value u drawn U[0,1]
- S has constant marginal value v drawn U[0,1]
- Uniform-price auction
  - Submit bid for each item
  - Highest 2 bids get items
  - 3rd highest bid determines price paid
- Ascending clock auction
  - Price starts at 0 and increases in small increments
  - Bidders express how many they want at current price
  - Bidders can only lower quantity as price rises
  - Auction ends when no excess demand (i.e. just two demanded); winners pay clock price
Exercise

• Two bidders (L & R), two items (A & B)
  – L need both (value = 2 times birth month)
  – R needs one (value = birth month)
  – Simultaneous ascending auction
Exposure problem
Exposure problem

• With complements, bidding on individual lots is risky
  – Bidder must “go for it” or drop out early
  – Outcome is often inefficient
  – Experiments sometimes get high revenues

• Exposure problem eliminated with package bids
Exposure problem exercise: Optimal strategy in SAA

<table>
<thead>
<tr>
<th>L's Profit</th>
<th>-12</th>
<th>-10</th>
<th>-8</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>L's value</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>L's birth month</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Bidder R's birth month</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>Expected Cost</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cost to eliminate bidder R

Expected Cost
Exercise

• Two bidders bidding for sum of cash in wallets
• Auction cash prize to two bidders equal to sum of money in both:
  \[ t_i = \text{money in student i’s wallet} \]
  \[ v = t_1 + t_2 \]
• Ascending clock auction
Asymmetries
Wallet Game

• Symmetric equilibrium?
• Asymmetric equilibrium?
• What if bidder 1 has a small advantage?
  – Bidder 1 gets a $1 bonus if wins
Incumbency advantage

• Toe hold
  – UK 3G auction
  – US PCS auction (Los Angeles and PacTel)
Incumbency disadvantage

• Hold up
  – Vodafone and BT in UK 3G auction
  – Cingular and Verizon in US PCS auction
Collusion
Anti-Collusion

• Can’t talk about bidding strategy with another bidder between application and end of auction
• Immediately report to Regulator any inappropriate communication made by another
Explicit collusion
Tacit collusion
Complexity
Auction complexity vs. strategic complexity
Objectives
Government goals

- Competition in post-auction market
- Efficiency
- Revenue
Bidder preferences
Bidder preferences

- Substitutes
- Complements
- Heterogeneity
- Externalities
Simultaneous ascending auction
Auction rules

- **Simultaneous**
  - All lots at the same time

- **Ascending**
  - Can raise bid on any lot

- **Stopping rule**
  - All lots open until no bids on any lot

- **Activity rule**
  - Must be activity to maintain eligibility
Strategy in SAA
Strategy in SAA

- Translating valuations into strategy
- Auction as a negotiation
- Elements of a good strategy
- Retaliation
- Managing eligibility
- Auction opening
- Stage transitions
- Auction closing
Auction as a Negotiation

- Learn what competitors need
- Learn how costly it is to ask for more
- Ask for more at the right time in the right place
- Manage eligibility
Elements of a Good Strategy

• Conservative (don’t leave money on table)
• Flexible (responsive to others)
• Clear
• Nice, but firm
  – Begin with a cooperative position
  – “Quick to punish, quick to forgive”
Retaliation
Retaliatory Bidding

• Stake a claim
  – Bid early on licenses of primary interest
  – Consistently retake license whenever bumped

• Punish intruders
  – Punish by bumping intruder from a license it holds
  – Works well against rivals with primary interests that overlap with your secondary interests
## Example of Code Bidding

<table>
<thead>
<tr>
<th>Round</th>
<th>Marshalltown, IA</th>
<th>Rochester, MN</th>
<th>Waterloo, IA</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>283 E</td>
<td>378 D</td>
<td>452 E</td>
</tr>
<tr>
<td>24</td>
<td>McLeod</td>
<td>USWest</td>
<td>AT&amp;T</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>287,000</td>
</tr>
<tr>
<td>46</td>
<td>56,000</td>
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<td>568,000</td>
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<td>52</td>
<td></td>
<td>689,000</td>
<td></td>
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<td>55</td>
<td></td>
<td></td>
<td>723,000</td>
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<tr>
<td>58</td>
<td></td>
<td>795,000</td>
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<td>59</td>
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<td>875,000</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td>313,378</td>
</tr>
<tr>
<td>62</td>
<td></td>
<td></td>
<td>345,000</td>
</tr>
<tr>
<td>64</td>
<td></td>
<td>62,378</td>
<td>1,059,000</td>
</tr>
<tr>
<td>65</td>
<td>69,000</td>
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<tr>
<td>68</td>
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<td></td>
<td>371,000</td>
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</table>
Critical times
Critical times

- Auction opening
- Stage transitions
- Auction closing
Bidder
decision making
Bidder decision making

- Valuation model
- Competitor analysis
- Bid tracking tool
- Residual supply
- Communication with investors
- Deviations from rational bidding
Valuation model

• Typically complex spreadsheet, calculating cost and revenues over 10 year horizon (NPV)

• Most sophisticated models reconfigure network in response to extra spectrum
Bid tracking tool
Bid tracking tool

- Import round (download results)
- Market analysis
- Bidder analysis
- Bid form (upload bids)
- Map
- Rounds summary
- Market pivot table
- Bidder pivot table
Round results

<table>
<thead>
<tr>
<th>R</th>
<th>Minimum bid</th>
<th>Must bid</th>
<th>Tim</th>
<th>Omnitel</th>
<th>Blu</th>
<th>Wind</th>
<th>Ipse</th>
<th>Andala</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>400</td>
<td>Andala</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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<td>422</td>
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<td>420</td>
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<tr>
<td>2</td>
<td>420</td>
<td>Blu</td>
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<td>421</td>
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<td>422</td>
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</tr>
<tr>
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<td>425</td>
<td>422</td>
<td>422</td>
<td>422</td>
<td>420</td>
</tr>
<tr>
<td>4</td>
<td>443</td>
<td>Andala</td>
<td>422</td>
<td>425</td>
<td>444</td>
<td>422</td>
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<tr>
<td>5</td>
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<tr>
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<td>7</td>
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<td>447</td>
<td>445</td>
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<td>8</td>
<td>468</td>
<td>Tim</td>
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<td>9</td>
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<td>10</td>
<td>472</td>
<td>Omnitel</td>
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<td>474</td>
<td>449</td>
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<td>473</td>
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<tr>
<td>11</td>
<td>492</td>
<td>Blu</td>
<td>468</td>
<td>449</td>
<td>470</td>
<td>473</td>
<td>470</td>
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Ranking

<table>
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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Ipse 400</td>
<td>Omnitel 400</td>
<td>Tim 400</td>
<td>Blu 400</td>
<td>Wind 400</td>
<td>Andala 400</td>
</tr>
<tr>
<td>Andala 420</td>
<td>Wind 420</td>
<td>Ipse 400</td>
<td>Omnitel 400</td>
<td>Tim 400</td>
<td>Blu 400</td>
</tr>
<tr>
<td>Omnitel 423</td>
<td>Wind 422</td>
<td>Blu 421</td>
<td>Andala 420</td>
<td>Ipse 400</td>
<td>Tim 400</td>
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<tr>
<td>Omnitel 425</td>
<td>Wind 422</td>
<td>Tim 422</td>
<td>Ipse 422</td>
<td>Blu 421</td>
<td>Andala 420</td>
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<tr>
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<td>Wind 422</td>
<td>Tim 422</td>
<td>Ipse 422</td>
<td>Blu 421</td>
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<td>Andala 443</td>
<td>Omnitel 425</td>
<td>Wind 422</td>
<td>Tim 422</td>
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<tr>
<td>Ipse 445</td>
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<td>Andala 446</td>
<td>Ipse 445</td>
<td>Tim 444</td>
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<tr>
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<td>Ipse 468</td>
<td>Blu 449</td>
<td>Omnitel 448</td>
<td>Wind 447</td>
<td>Andala 446</td>
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<tr>
<td>Wind 470</td>
<td>Andala 470</td>
<td>Tim 468</td>
<td>Ipse 468</td>
<td>Blu 449</td>
<td>Omnitel 448</td>
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<td>Ipse 473</td>
<td>Wind 470</td>
<td>Andala 470</td>
<td>Tim 468</td>
<td>Blu 449</td>
</tr>
</tbody>
</table>

**STRATEGY ROUND 11**

Basic/Enhanced relates to E2

**BLU**

= lowest (Blu, Ipse, Andala)

**E1**

Andala = 2nd lowest (Blu, Ipse, Andala)

**Minimum Bid**

492

<table>
<thead>
<tr>
<th>OPI position</th>
<th>Highest. Above Ipse, Andala and Blu.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rationale (Enhanced default strategy)</td>
<td>No bid. Case 11: Do not have to bid, above E1.</td>
</tr>
<tr>
<td>Epsilon</td>
<td>0</td>
</tr>
<tr>
<td>Basic bid + epsilon</td>
<td>No bid</td>
</tr>
<tr>
<td>Net value of extra bid</td>
<td>N/A</td>
</tr>
<tr>
<td>Extra bid</td>
<td>0</td>
</tr>
<tr>
<td>Enhanced bid</td>
<td>No bid</td>
</tr>
<tr>
<td>Enhanced bid + epsilon</td>
<td>No bid</td>
</tr>
<tr>
<td>Pr(Andala quits before Ipse)</td>
<td>0.50</td>
</tr>
<tr>
<td>Pr(Blu quits before Ipse)</td>
<td>0.50</td>
</tr>
<tr>
<td>Pr(Blu quits before Andala)</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**END GAME STRATEGY (suggestion)**

Suggest high pr(Blu quits before Ipse) and high pr(Blu quits before Andala)

BLU failed to move above Andala for 1 round. BLU failed to move above Ipse for 2 rounds. BLU inactive for 3 rounds.

**JUMP WARNINGS**

round bidder jump

**INACTIVITY WARNINGS**

TIM for 2 rounds

BLU for 3 rounds

**NEW ENTRANTS & BLU COMPARISON**

BLU not moved above Andala for 1 round

BLU not moved above Ipse for 2 rounds

**Suggested bid**

BLU+1

No bid

**Rationale (Enhanced default strategy)**

BLU for 3 rounds

BLU not moved above Ipse for 2 rounds

**Epsilon**

0

**Basic bid + epsilon**

No bid

**Net value of extra bid**

N/A

**Extra bid**

0

**Enhanced bid**

No bid

**Enhanced bid + epsilon**

No bid

**Pr(Andala quits before Ipse)**

0.50

**Pr(Blu quits before Ipse)**

0.50

**Pr(Blu quits before Andala)**

0.50

**END GAME STRATEGY (suggestion)**

Suggest high pr(Blu quits before Ipse) and high pr(Blu quits before Andala)

BLU failed to move above Andala for 1 round. BLU failed to move above Ipse for 2 rounds. BLU inactive for 3 rounds.

**Suggested bid**

BLU+1

No bid

**BLU failed to move above Andala for 1 round. BLU failed to move above Ipse for 2 rounds. BLU inactive for 3 rounds.**
<table>
<thead>
<tr>
<th>Rnd</th>
<th>Block 1 Bidder</th>
<th>Block 2 Bidder</th>
<th>Block 3 Bidder</th>
<th>Block 4 Bidder</th>
<th>Block 5 Bidder</th>
<th>Block 6 Bidder</th>
<th>Block 7 Bidder</th>
<th>Block 8 Bidder</th>
<th>Block 9 Bidder</th>
<th>Block 10 Bidder</th>
<th>Block 11 Bidder</th>
<th>Block 12 Bidder</th>
</tr>
</thead>
</table>

**German 3G (Debitel out; Mobilcom to 2)**
| Rnd | Bid  | Block 1 Bidder | Bid  | Block 2 Bidder | Bid  | Block 3 Bidder | Bid  | Block 4 Bidder | Bid  | Block 5 Bidder | Bid  | Block 6 Bidder | Bid  | Block 7 Bidder | Bid  | Block 8 Bidder | Bid  | Block 9 Bidder | Bid  | Block 10 Bidder | Bid  | Block 11 Bidder | Bid  | Block 12 Bidder | Bid  |
|-----|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|

**Still to**

<table>
<thead>
<tr>
<th>Round</th>
<th>Revenue</th>
<th>go</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>32.2</td>
<td>18.3</td>
<td>Debitel drops out (down to 6 bidders)</td>
</tr>
<tr>
<td>146</td>
<td>42.4</td>
<td>8.1</td>
<td>Common knowledge that MobilCom dropped to 2</td>
</tr>
<tr>
<td>173</td>
<td>50.5</td>
<td>0.0</td>
<td>End of auction</td>
</tr>
</tbody>
</table>
Deviations from rational bidding

- Too little demand reduction (too greedy)
- Lack of clarity (this is bridge not poker)
- Don’t recognize winner’s curse
- Retaliation either too weak or too strong
- Failure to recognize other options
  - Secondary market
  - Subsequent offerings
Variations of SAA
Variations of SAA

• Anonymous bidding
• Generic lots vs. specific lots
Simultaneous ascending auction

• **Strengths**
  – Simple price discovery process
  – Allows arbitrage across substitutes
  – Piece together desirable packages
  – Reduces winner’s curse

• **Weaknesses**
  – Demand reduction
  – Tacit collusion
  – Parking
  – Exposure
  – Hold up
  – Complex bidding strategies
Combinatorial auctions

• Threshold problem
• Reducing exposure and mitigating threshold problem
• Alternative methods
  – SAA with augmented switching
  – SAA with package bids
  – Clock auctions
Threshold problem

• In SAA with package bids, bidders on individual lots may find it difficult to top a large package bid
  – Each individual bidder hopes others will raise
  – Result may be that too few raise to top package even though individual bids are higher
Reducing exposure while mitigating the threshold problem

- Limit packages to lots where complementaries are strong
- Eliminate jump bids
- Use clock auction to resolve negotiation among bidders on individual lots
Alternative methods

- SAA with augmented switching
- SAA with package bids
- Clock auctions
SAA with augmented switching
SAA with package bids

- Highly complex if all packages allowed
- Threshold problem can be severe
- Prices don’t exist or are made up
A consistent family of auctions

- Clock auction
- Many variations allow customization to particular auction
  - Information policy
  - Activity rule
  - Final round
Advantages to a consistent family

• Design is easier
  – Pick and choose from a menu

• Implementation is easier
  – All designs built on same stable platform

• Bidder participation costs are reduced
  – Bidders understand auction and variations
Clock-Proxy Auction
Clock auction

• Auctioneer names prices; bidders name only quantities
  – Price adjusted according to excess demand
  – Process repeated until market clears
• No exposure problem (package auction)
Proxy auction

- A sealed-bid procedure for package bidding
- Bidders specify values
- Finds bidder-Pareto optimal point in Core
  - Efficient assignment
  - Competitive revenues
Clock-proxy auction

• A clock auction followed by a final proxy round
  – Bidders directly submit bids in clock auction phase
  – When clock phase concludes, bidders have a single opportunity to input proxy values
  – Proxy phase concludes the auction
Clock-proxy auction

- All bids are kept “live” throughout auction (no bid withdrawals)
- Bids from clock phase are also treated as package bids in the proxy phase
- All bids are treated as mutually exclusive (XOR)
- Activity rules are maintained within clock phase and between clock and proxy phases
Advantages of clock-proxy auction

• Clock phase
  – Simple for bidders
  – Provides price discovery
  – Interdependent values
  – Economize on package evaluation costs

• Proxy phase
  – Efficient allocations
  – Competitive revenues
  – Reduces opportunities for collusion
Key issues and variations

- Making discrete rounds continuous
- Reducing exposure
- Promoting price discovery
- Reducing demand reduction
Making discrete rounds continuous
Making discrete rounds continuous
Making discrete rounds continuous

![Diagram showing the process of making discrete rounds continuous](image-url)
Reducing exposure

• Treatment of bids that cause demand < supply
  – Ration or reject reduction
    • Exposure problem
    • Supply = demand
  – Allow reduction
    • No exposure problem
    • Undersell
Promoting price discovery

• Revealed-preference activity rule
  – Compare times s and t (s < t),
    Prices: $p_s$, $p_t$    Demands: $x_s$, $x_t$
  – At time s, $x_s$ is better than $x_t$: $v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t$
  – At time t, $x_t$ is better than $x_s$: $v(x^t) - p^t \cdot x^t \geq v(x^s) - p^t \cdot x^s$

• Adding inequalities yields the RP activity rule:

\[
(RP) \quad (p^t - p^s) \cdot (x^t - x^s) \leq 0.
\]
Reducing demand reduction

Theorem (Ausubel-Milgrom): Payoff vector resulting from proxy auction is in the core relative to the reported preferences.

• Interpretations
  – Core outcome assures competitive revenues for seller
  – Core outcome assures allocative efficiency (ascending proxy auction is not subject to inefficient demand reduction)
Advantages of clock-proxy

- **Clock**
  - Take linear prices as far as they will go
  - Simplicity and flexibility for bidders and auctioneer
  - Expand substitution possibilities
  - Minimize scope for collusion
  - No exposure problem; no threshold problem

- **Proxy**
  - Core outcome
  - Efficiency
  - Substantial seller revenues