Auctioning Many Similar Items

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Examples of auctioning similar items

• Treasury bills
• Stock repurchases and IPOs
• Telecommunications spectrum
• Electric power
• Emission allowances
Ways to auction many similar items

- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)
Pay-as-bid Auction:
All bids above $P_0$ win and pay bid
Uniform-Price Auction:
All bids above $P_0$ win and pay $P_0$
Vickrey Auction:
All bids above $P_0$ win and pay opportunity cost

\[
Q_i(p) = Q_i(p_0)
\]

\[
Q_S - \sum_{j \neq i} Q_j(p)
\]

Demand

Residual Supply

Price

Quantity
Vickrey Auction: m Discrete Items

- Allocate m items efficiently: m highest marginal values
- Winning bidder pays k\textsuperscript{th} highest losing bid of others on k\textsuperscript{th} item won
- Payment = social opportunity cost of items won

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Payment rule affects behavior

\[ Q_i(p_0) \]

\[ Q_S - \sum_{j \neq i} Q_j(p) \]

Price

Quantity

Pay-as-bid

Uniform-Price

Vickrey

\( p_0 \)
More ways to auction many similar items

• Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  – Standard ascending-bid
  – Ausubel ascending-bid (Ausubel 1997)
Standard Ascending-Bid Auction: All bids at $P_0$ win and pay $P_0$
Ausubel Ascending-Bid:
All bids at $P_0$ win and pay price at which clinched

\[
\text{Residual Supply: } Q_S - \sum_{j \neq i} Q_j(p)
\]

\[
Q_i(p_0)
\]

Excess Demand

Demand $Q_i(p)$

Price

Clock

Quantity
More ways to auction many similar items

• Ascending-bid
  – Simultaneous ascending auction (FCC spectrum)

• Sequential
  – Sequence of English auctions (auction house)
  – Sequence of Dutch auctions (fish, flowers)

• Optimal auction
  – Maskin & Riley 1989
Research Program

How do standard auctions compare?

• Efficiency
  – FCC: those with highest values win

• Revenue maximization
  – Treasury: sell debt at least cost
Efficiency
(not pure common value; capacities differ)

- Uniform-price and standard ascending-bid
  - Inefficient due to demand reduction
- Pay-as-bid
  - Inefficient due to different shading
- Vickrey
  - Efficient in private value setting
  - Strategically simple: dominant strategy to bid true demand
  - Inefficient with affiliated information
- Ausubel ascending-bid
  - Same as Vickrey with private values
  - Efficient with affiliated information
Inefficiency Theorem

In any equilibrium of uniform-price auction, with positive probability objects are won by bidders other than those with highest values.

• Winning bidder influences price with positive probability
• Creates incentive to shade bid
• Incentive to shade increases with additional units
• Differential shading implies inefficiency
Inefficiency from differential shading

Large bidder makes room for smaller rival
Vickrey inefficient with affiliation

- Winner’s Curse in single-item auctions
  - Winning is bad news about value

- Winner’s Curse in multi-unit auctions
  - Winning more is worse news about value
  - Must bid less for larger quantity
  - Differential shading creates inefficiency in Vickrey
What about seller revenues?

Price

Pay-as-bid

Uniform-Price

Vickrey

Residual Supply

$Q_s - \sum_{j \neq i} Q_j(p)$

Demand

$Q_i(p)$

$p_0$

$Q_i(p_0)$

Quantity
Exercise

• 2 bidders (L and R), 2 identical items
• L has a value of $100 for 1 and $200 for both
• R has a value of $90 for 1 and $180 for both
• Uniform-price auction
  – Submit bid for each item
  – Highest 2 bids get items
  – 3rd highest bid determines price paid
• Ascending clock auction
  – Price starts at 0 and increases in small increments
  – Bidders express how many they want at current price
  – Bidders can only lower quantity as price rises
  – Auction ends when no excess demand (i.e. just two demanded); winners pay clock price
Uniform price may perform poorly

• Independent private values uniform on [0,1]
• 2 bidders, 2 units; L wants 2; S wants 1
• Uniform-price: unique equilibrium
  – S bids value
  – L bids value for first and 0 for second
  – Zero revenue; poor efficiency
• Vickrey
  – price = \(v_{(2)}\) on one unit, zero on other
Standard ascending-bid may be worse

- 2 bidders, 2 units; L wants 2; S wants 2
- Uniform-price: two equilibria
  - Poor equilibrium: both L and S bid value for 1
    - Zero revenue; poor efficiency
  - Good equilibrium: both L and S bid value for 2
    - Get $v_{(2)}$ for each (max revenue) and efficient
- Standard ascending-bid: unique equilibrium
  - Both L and S bid value for 1
    - S’s demand reduction forces L to reduce demand
    - Zero revenue; poor efficiency
Efficient auctions tend to yield high revenues

**Theorem.** *With flat demands drawn independently from the same regular distribution, seller’s revenue is maximized by awarding good to those with highest values.*

Generalizes to non-private-value model with independent signals:

\[ v_i = u(s_i, s_{-i}) \]

Award good to those with highest signals if downward sloping MR and symmetry.
Downward-sloping demand: \[ p_i(q_i) = v_i - g_i(q_i) \]

**Theorem.** *If intercept drawn independently from the same distribution, seller’s revenue is maximized by*

- *awarding good to those with highest values if constant hazard rate*
- *shifting quantity toward high demanders if increasing hazard rate*

*Note: uniform-price shifts quantity toward low demanders*
But uniform price has advantages

- Participation
  - Encourages participation by small bidders (since quantity is shifted toward them)
  - May stimulate competition

- Post-bid competition
  - More diverse set of winners may stimulate competition in post-auction market
Auctioning Securities

A pure common-value model with affiliation

- $n$ risk-neutral symmetric bidders
- Each bidder has pure common value $V$ for security and can purchase any quantity (flat demand curve w/o capacity)
Models

• Common uncertainty
  – Bidders have no private information

• Affiliated private signals
  – Bidder i gets signal $S_i$
  – Random variables $V$, $S_1$, …, $S_n$ are affiliated
Results: Common Uncertainty

Proposition. (Wilson ‘79; Maxwell ‘83; Back & Zender ‘93)

- **Wide range of prices can be supported as equilibrium in uniform-price auction, even if supply is stochastic; highest yields EV**

Proposition. (Wang & Zender ‘02)

- **Many equilibria in pay-as-bid auction, even if supply is stochastic; highest yields EV**
- **Indeterminacy avoided if set reserve price (even 0)**
Results: Common Uncertainty

Theorem.

- Vickrey auction has a unique equilibrium that survives elimination of weakly-dominated strategies
- Vickrey auction has a unique symmetric equilibrium consistent with stochastic supply
- This equilibrium revenue-dominates all equilibria of all auction formats consistent with voluntary bidder participation
Results: Affiliated Private Signals

• With affiliated signals, each auction format has a “simple equilibrium” where bidders submit flat demand curves

• Conjecture: These simple equilibria provide upper bounds on revenues from each format

• Alt. ascending-bid > Vickrey > Pay-as-bid

• Std. ascending-bid > Uniform > Pay-as-bid
Results: Affiliated Private Signals

Vickrey and Ausubel ascending-bid eliminate bottom end of revenue indeterminacy:

Revenues

Pay-as-Bid  Uniform Price  Vickrey  Standard Ascending Bid  Ausubel Ascending Bid
Conclusion

• Efficient auctions should be favored
• Treasury should try Ausubel ascending-bid
• IPOs should be auctioned
Competitive Bidding Behavior in Uniform-Price Auction Markets

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6 January 2004
Summary

- Marginal cost bidding is a useful benchmark, but not a norm of behavior.
- Profit maximization is an appropriate norm of behavior in markets.
- Profit maximization should be expected and encouraged.
- Market rules should be based on this norm.
Uniform-price auction:
All bids below $p_0$ win and get paid $p_0$
Residual demand removes supply of other bidders

\[ \text{Residual demand} = q - q_i \]

\[ p_0 \]

\[ q_0 \]

\[ q \]

\[ q_{-i} \]

\[ q_i \]

\[ p \]

\[ p \]

\[ p \]
Residual demand curve

\[ D_i(p) = D(p) - \sum_{j \neq i} S_j(p) \]

As-bid supply

\[ S_i(p) \]

Residual demand curve

Price

Quantity

\[ p_0 \]

\[ q_i \]
Bidding strategy with perfect competition

\[ p_0 = \text{Loss} \]

As-bid supply: \[ S_i = MC_i \]

Residual demand: \[ D_i \]
Incentive to bid above marginal cost: tradeoff higher price with reduced quantity

\[ p \]

\[ q \]

\[ \text{Residual demand} \]

\[ D_i \]

\[ \text{As-bid supply} \]

\[ S_i \]

\[ MC_i \]

\[ p_0 \]

\[ \text{Gain} \]

\[ \text{Loss} \]

\[ q_i \]
Optimal bid balances marginal gain and loss

\[ p \]

\[ \text{As-bid supply} \]

\[ S_i \]

\[ p_0 \]

\[ \text{Gain} \]

\[ \text{Loss} \]

\[ q_i \]

\[ MC_i \]

\[ D_i \]

\[ 0 \]
Still bid above marginal cost when others bid marginal cost

\[ S_i = MC_i \]

Other bidders

Firm \( i \)

\[ D_i \]

\[ MC_i \]

\[ S_i \]

\[ D \]

\[ q_i \]

\[ q_{-i} \]

\[ p_i \]

\[ p_0 \]
Residual demand response reduces incentive to inflate bids
Residual demand is steeper for large bidders

For the large bidder:
- The demand curve is steeper, indicating a more elastic demand for energy. This means that a small change in price leads to a larger change in quantity demanded.
- The supply curve is flatter, indicating a less elastic supply of energy. This means that a small change in price leads to a smaller change in quantity supplied.

For the small bidder:
- The demand curve is flatter, indicating a more inelastic demand for energy. This means that a small change in price leads to a smaller change in quantity demanded.
- The supply curve is steeper, indicating a more elastic supply of energy. This means that a small change in price leads to a larger change in quantity supplied.
Large bidder makes room for its smaller rivals

Large bidder

Small bidder

\( p \)

\( q_l \)

\( q_s \)

\( p_0 \)

\( D_l \)

\( S_l \)

\( D_s \)

\( S_s \)

\( MC_l \)

\( MC_s \)
Economic vs. Physical Withholding
Forward contracts mitigate incentive to bid above marginal cost

\[ p \]

\[ q \]

\[ S_i \text{ no forward} \]

\[ S_i \text{ with forward} \]

\[ p' \]

\[ p_0 \]

Forward sale

\[ q_i' \]

\[ q_F \]

\[ q_i \]

\[ q_S \]

Residual demand

\[ MC_i \]

\[ D_i \]
California not more concentrated

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Sources