Dynamic Games with Incomplete Information (continued)

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Takeover Game

- Target's value v is uniform on [0,100]
- Target knows value; Acquirer does not
- Acquirer's value is 1.5 v
- What offer p should Acquirer make to Target? This is take-it-or-leave-it bargaining game.

Auctions

- Auctions are important institution.
- Understanding auctions should help us understand the formation of markets by modeling the competition on one side of the market.
- Auctions represent an excellent application of game theory, since in an auction the rules of the game are made explicit.

Simple Auctions

Auctions typically take one of four simple forms:

Oral		Sealed Bid
English (↑ price)		2nd Price
Dutch (↓ price)	≡	1st Price

Simple Auctions

- *English*: price increases until only one bidder is left; the remaining bidder gets the good and pays the highest bid.
- *Dutch*: prices decreases until a bidder accepts the price; this bidder gets the good and pays the price at acceptance.
- *Second Price*: each bidder submits a bid in a sealed envelope; the highest bidder gets the good and pays the second highest bid.
- *First Price*: each bidder submits a bid in a sealed envelope; the highest bidder gets the good and pays the amount of his bid.

Auction Exercise

- Bid for single object
- Common value = $1 per bean
- On slip of paper write:
  - Name
  - Estimate (# of beans $\times$ $1)
  - Bid in first-price sealed-bid auction
  - Bid in second-price sealed-bid auction

Models of Private Information

(1) Independent Private Value:

$v_i \sim F_i$ independently of v_j for $j \neq i$.

(2) Common Value:

$e_i = v + \varepsilon_i$, $\varepsilon_i \sim F_i$ w/ mean 0.

(3) Affiliated Value:

$v_i(x,s)$, my value depends on private information $x = (x_1,...,x_n)$ and state of world s.

Winner's Curse

I won. Therefore, I overestimated the most. My bid only matters when I win, so I should condition my bid on winning (i.e., that I overestimated the most).

- Winning is bad news about my estimate of value. This is a form of adverse selection that arises in any exchange setting: if you want to trade with me, it must be that no one else offered more, because they did not think that the item is worth what I am willing to pay.

Models of Private Information

- Independent private value model: It makes sense if differences in value arise from heterogeneous preferences over the attributes of the item
- Common Value: It makes sense if the bidders have homogeneous preferences, so they value the item the same ex post, but have different estimates of this true value.
- Affiliated value model: In this model, each bidder has private information that is positively correlated with the bidder's value of the good.

Auction Multiple Items

- 2 bidders (L and S), 2 identical items
- L has a value of $100 for 1 and $200 for both
- S has a value of $90 for 1 and $180 for both
- Uniform-price auction
  - Submit bid for each item
  - Highest 2 bids get items
  - 3rd highest bid determines price paid
- Ascending clock auction
  - Price starts at 0 and increases in small increments
  - Bidders express how many they want at current price
  - Bidders can only lower quantity as price rises
  - Auction ends when no excess demand (i.e. just two demanded); winners pay clock price

What if private information?

- 2 bidders (L and S), 2 identical items
- L has constant marginal value u drawn U[0,1]
- S has constant marginal value v drawn U[0,1]
- Uniform-price auction
  - Submit bid for each item
  - Highest 2 bids get items
  - 3rd highest bid determines price paid
- Ascending clock auction
  - Price starts at 0 and increases in small increments
  - Bidders express how many they want at current price
  - Bidders can only lower quantity as price rises
  - Auction ends when no excess demand (i.e. just two demanded); winners pay clock price