Economics 603 Professor Peter Cramton

Problem Set 11

1.Consider an auction with two bidders, in which the two bidders' valuations are private information and the valuations are i.i.d. random variables uniformly distributed on the interval [1,2].

(a)Calculate the linear equilibrium of the first-price auction.

(b)Calculate the linear equilibrium of the second-price auction.

2.Consider a sealed-bid ***second-price*** auction for a single object, where there are only two allowable bids. The two risk-neutral bidders have valuations which are private information and which are drawn from i.i.d. random variables that are uniformly distributed on the interval [0,1]. After observing her own valuation, each of the two bidders simultaneously and independently submits a bid selected from the two-element set $\{0,2/3\}$ (i.e., the only allowable bids are 0 and 2/3). The high bidder wins the object and pays the amount of the losing bid; in the event of a tie, the winner is determined by the toss of a fair coin.

(a)Recall that in the standard second-price auction game where *all bids* between 0 and 1 are allowed, a bidder never finds it in her interest to bid above her value. Does this property continue to hold in the current game where there are only two allowable bids? Explain precisely.

(b)Solve for the Bayesian-Nash equilibrium of the second-price auction game where there are only *two allowable bids*. [***Hint***: A strategy for this game must fully specify which of the two allowable bids is selected by each bidder, for each possible type $t_i \in [0,1]$.]

(c)Compare the efficiency of the Bayesian-Nash equilibrium of this auction game with the Bayesian-Nash equilibrium of the standard auction game where *all bids* between 0 and 1 are allowed.

3.In the sealed, simultaneous-bid bargaining model with two-sided incomplete information, in which the two bidders' valuations are i.i.d. random variables uniformly distributed on the interval [0,1], verify that the following bidding functions constitute a Bayesian-Nash equilibrium:

$$p_s(v_s) = 2/3\ v_s + 1/4$$

$$p_b(v_b) = 2/3\ v_b + 1/12\ .$$

4.The Hotelling Model with One-Sided Incomplete Information.

Consider a town consisting of a single Main Street of length 1. Locations on the street are denoted by the variable *x*, where $x \in [0,1]$. Consumers are uniformly distributed on the interval [0,1]. There are two firms, located at $x = 0$ and $x = 1$, which each produce the identical physical good at zero marginal (and average) cost. Consumers incur transportation costs of *t* per unit of distance. The firms name prices simultaneously and independently. Assume that all parameters in this problem are chosen such that all consumers consume exactly one unit of the good in equilibrium, and such that both firms always choose to produce positive quantities of the good.

(a)Given that the firms charge prices p_1 and p_2, show that the consumer who is indifferent between buying the good at Firm 1 and buying the good at Firm 2 is located at:

$$\tilde{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t} \ .$$

(b)Write expressions for the demand functions, $q_1(p_1,p_2)$ and $q_2(p_1,p_2)$, respectively, for the goods sold by the two firms.

In parts (c), (d) and (e), now assume that we are in a game of incomplete information. The transportation cost, *t*, is a random variable which takes the following values:

$$t = \begin{cases} t_H \text{ , with probability } \theta \\ t_L \text{ , with probability } 1\text{-}\theta \ . \end{cases}$$

Firm 1 observes *t* immediately after it is realized, and before firms name their prices. Firm 2 does not observe *t* until after firms must name their prices, but Firm 2 knows the distribution of *t* as given above. The consumers observe *t* before they must choose between purchasing at Firm 1 or Firm 2.

(c)Define what is meant by a Bayesian-Nash equilibrium of this game. Be sure to be precise about the structure of each of the firms' strategies.

(d)Determine all three of the first-order conditions which a Bayesian-Nash equilibrium of this game must satisfy.

(e)[Algebraically fairly difficult]. Solve the three simultaneous equations of part (d), and thus solve for the Bayesian-Nash equilibrium of this game.