Economics 603 Professor Peter Cramton

PROBLEM SET 10

1.Consider the following version of the Prisoner's Dilemma game:

Player II (columns S, C); Player I (rows S, C)

	S	C
S	6 , 6	0 , 8
C	8 , 0	1 , 1

a. Explain fully why this game "has the structure of Prisoner's Dilemma."

b. Write (algebraically) and sketch (graphically) the "Folk Theorem" region for the infinitely-repeated version of this game.

c. Find the critical discount factor which supports (S,S) in every period with a trigger-strategy equilibrium (i.e., calculate the lowest discount factor such that, for all larger discount factors, silence in every period can be supported as a subgame-perfect Nash equilibrium).

d. Consider the "tit-for-tat" strategy for each player: Player i plays S provided that the other player j played S in the previous period (and player i starts the game by playing S in the first period); player i plays C provided that the other player j played C in the previous period. Show that the tit-for-tat strategies do *not* form a subgame-perfect Nash equilibrium of the infinitely-repeated version of this game, for any discount factor.

2.Consider the infinitely-repeated version of the Bertrand game:

a. Write (algebraically) and sketch (graphically) the "Folk Theorem" region for the repeated Bertrand game.

b. Find the critical discount factor to support a division of $(\frac{1}{2}\pi_m, \frac{1}{2}\pi_m)$ with a trigger-strategy equilibrium (i.e., calculate the lowest discount factor such that, for all larger discount factors, an equal division of joint monopoly profits can be supported as a subgame-perfect Nash equilibrium).

c. Write down a pair of trigger strategies which would provide a division of approximately $(\frac{3}{4}\pi_m, \frac{1}{4}\pi_m)$ for discount factors near one.

d. Find the critical discount factor to support the trigger strategies of part (c) as a subgame-perfect Nash equilibrium.

3.Consider an infinitely-repeated Cournot game with three firms, with common discount factor $\delta < 1$ between periods, common unit costs of $c > 0$, and inverse market demand function of $P(Q) = a - bQ$, where $a > c$ and $b > 0$. Furthermore, suppose that after every period there is a small probability of “Doomsday”: a random variable is (independently) drawn after every period and, with a probability $\gamma > 0$, the market ceases to continue after that moment.

Determine the exact conditions on δ and γ such that the symmetric joint monopoly outputs of $(q_1, q_2, q_3) = (Q^m/3, Q^m/3, Q^m/3)$ can be sustained with trigger strategies which call for $(Q^m/3, Q^m/3, Q^m/3)$ to be played if no one has previously deviated and for the Nash equilibrium of the one-period Cournot game to be played otherwise.

4.Consider an infinitely-repeated Stackelberg game with two firms, with common unit costs of $c > 0$, and inverse market demand function of $P(Q) = a - bQ$, where $a > c$ and $b > 0$. Within each period, firm 1 is the leader, which picks its quantity first. Firm 2 is the follower, which observes firm 1's choice and then chooses its quantity. Firm 1 uses discount factor $\delta_1 > 0$, to discount between periods, and firm 2 uses discount factor $\delta_2 > 0$. There is no discounting between firm 1's choice and firm 2's choice.

Determine the exact conditions on δ_1 and δ_2 such that the symmetric joint monopoly outputs of $(q_1, q_2) = (Q^m/2, Q^m/2)$ can be sustained with trigger strategies which call for $(Q^m/2, Q^m/2)$ to be played if no one has previously deviated and for the Nash equilibrium of the one-period Stackelberg game to be played otherwise.

5.Gibbons, page 169, problem 3.2.

6.Gibbons, page 169, problem 3.3.