Economics 603 | Professor Peter Cramton

PROBLEM SET 9

1.Prove, using a fixed-point theorem, that the following system of two equations has at least one solution in the set $\{(x,y) : -1 \le x \le 1 \text{ and } -1 \le y \le 1\}$:

$$x - \frac{(y+1)^3}{10} = 0$$
$$y - \frac{(x-1)^3}{10} = 0$$

2.Three oligopolists operate in a market where the inverse demand function is given by $P(Q) = 120 - 2Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has a constant marginal and average cost of 0. The firms choose their (nonnegative) quantities as follows: (i) firm 1 chooses q_1; (ii) firm 2 observes q_1, and then chooses q_2; and (iii) firm 3 observes q_1 and q_2, and then chooses q_3.
Solve for the subgame-perfect equilibrium.

3.Gibbons, page 133, problem 2.6.

4.Consider an auction with three bidders, each of whom can consume the good only in quantities of zero or one. Assume utility functions of:

$U_1(0) = 0$; $U_1(1) = 30$; $U_1(2) = 30$.
$U_2(0) = 0$; $U_2(1) = 20$; $U_2(2) = 20$.
$U_3(0) = 0$; $U_3(1) = 10$; $U_3(2) = 10$.

In other words, bidder 1's valuation for the good is \$30, bidder 2's valuation for the good is \$20, and bidder 3's valuation for the good is \$10. Each of these valuations are known by all the bidders.

a. Suppose that there is one such good for sale, and that it is sold via a second-price auction. Determine which bidder wins the good, and the price which she pays. (Make sensible assumptions as needed.)

b. Suppose that there are ***two identical*** such goods for sale, and that they are sold sequentially via ***two*** second-price auctions (one held immediately after the other). Each of the three bidders participates in both of the auctions; but remember that each bidder can consume at most one of the goods. Determine which bidder wins each of the two goods, and the price which each pays, if the goods are sold by second-price auction (one after the other). Also briefly explain the result.

5.Consider the four-period, alternating-offer bargaining game. The seller's valuation equals zero and the buyer's valuation equals one. Each player uses a discount factor $\delta \in (0,1)$ between periods. In period one, the seller makes an offer. If rejected, the buyer makes a counteroffer in period two. If rejected, the seller makes an offer in period three. If rejected, the buyer makes a counteroffer in period four. If the game reaches the fourth period and if the period-four counteroffer is rejected, the game ends and each player earns zero utility. Calculate the unique subgame-perfect equilibrium by backward induction.

6.Consider the infinite-period bargaining game in which the seller makes all the offers. The seller's valuation equals zero and the buyer's valuation equals one. Each player uses a discount factor $\delta \in (0,1)$ between periods. In period one, the seller makes an offer. If rejected, the seller makes another offer in period two. If rejected, the seller makes another offer in period three, etc. The game never ends until an offer is accepted, and the buyer never gets to make any counteroffers.

Determine the subgame-perfect equilibrium of this game.

7.Consider the game in which the seller's valuation equals zero and the buyer's valuation equals one, and each player uses a discount factor $\delta \in (0,1)$ between periods. In period one, a general coin is tossed: with probability $\pi \in (0,1)$, the seller is chosen to make an offer; with probability $1 - \pi$, the buyer is chosen to make an offer. As usual, the opposite party can immediately accept or reject. If he accepts, the game ends and utilities accrue; if he rejects, the game continues to period two. In period two, the coin is again tossed, with the same probabilities and independently of the previous period, determining who is chosen to make an offer, etc.

Determine the subgame-perfect equilibrium of this game.