Economics 603 | Professor Lawrence Ausubel
Fall 2004

Final Exam

Answer all four questions, each in a *different* blue book.
Write your student number on each blue book and include your name on the book for Question 1.
Please return this question sheet at the same time that you turn in the blue books.
Write legibly, show your work, and justify your responses.

1. [20 points] Consider the two-player static game of complete information in the matrix below:

Player II

Player I		L	R
	T	0 , 2	2 , 2
	M	1 , 3	1 , 1
	B	2 , 1	0 , 3

(a) Find all strictly dominated and weakly dominated strategies.

(b) Determine all pure-strategy Nash equilibria.

(c) Show that there are a continuum of Nash equilibria in which Player I plays a (nondegenerately-) mixed strategy. Describe them exactly.

(d) Show that there are a continuum of Nash equilibria in which Player I plays a pure strategy. Describe them exactly.

2. [30 points] Consider the following game of complete information. There are two players (i = 1,2). Each player i simultaneously and independently selects a strategy $x_i \in [0,5]$. For any parameter value of α, where α is restricted to values $\alpha \in (0,1]$, the payoffs to the players are given by:

$$U_1(x_1, x_2) = \alpha x_1 - \alpha x_2 \; - \; \alpha^2 x_2^{\,2}.$$
$$U_2(x_1, x_2) = \alpha x_2 - \alpha x_1 \; - \; \alpha^2 x_1^{\,2}.$$

(a) State why this might be considered to be a game in which a higher level of x_i represents a lower level of "cooperation" (or higher level of "defection") by the players.

(b) Argue rigorously that there is a unique Nash equilibrium of this game for each $\alpha \in (0,1]$.

(c) Which strategies $x_i \in [0,5]$ would maximize the sum of the players' payoffs?

Parts (d) – (e) concern the ***infinitely repeated*** version of the above game.

(d) Calculate the minimum discount factor such that there exists a subgame perfect equilibrium using trigger strategies in which the sum of the players' payoffs is maximized (i.e., the players "cooperate" in equilibrium)?

(e) Evaluate how the minimum discount factor of part (d) depends on α. Based on this, state whether you agree or disagree with the following general claim about repeated games: "An increase in the private returns from defection necessarily makes it more difficult to sustain cooperation in a subgame perfect equilibrium of a repeated game." Provide a convincing intuition for your conclusion.

3. [30 points] Contest with incomplete information.

There are two risk-neutral players, whose types (denoted t_1 and t_2, respectively) are private information and which are drawn from i.i.d. random variables that are uniformly distributed on the interval [0,1]. After observing her own type, each of the two players simultaneously and independently decides whether to 'Enter' or 'Not Enter' a contest:

- If both players enter the contest, then the player *i* with the higher type $(t_i > t_j)$ wins the contest and obtains a payoff equaling her own type (t_i). The player *j* with the lower type loses the contest and obtains a payoff of zero. In the event of a tie, the winner of the contest is determined by the toss of a fair coin.
- If only one player enters the contest, then that player *i* automatically wins the contest and obtains a payoff equaling her own type (t_i). The player who does not enter the contest obtains a payoff of $\frac{1}{2}$ for second place, independent of her type.
- If neither player enters the contest, then both players obtain payoffs of $\frac{1}{2}$, independent of their types.

(a) Write this game in matrix form.

(b) Verbally describe what is meant by a *strategy* of this game, and verbally describe the conditions that a *Bayesian-Nash equilibrium* of this game must satisfy.

(c) Write algebraic expressions for the conditions that a Bayesian-Nash equilibrium of this game must satisfy.

(d) Solve for the Bayesian-Nash equilibrium of this game.

4. [20 points] Consider an economy in which there are two goods, *x* and *y*. The market for good *x* is a monopoly. Good *y* can be considered to be money, and the price of good *y* is normalized to 1. The monopolist has a constant marginal and average cost of producing good *x* equal to zero. There are two consumers in the market. The first consumer's utility function is given by:

$$u_1(x, y) = 3x - \tfrac{1}{2}x^2 + y .$$

The second consumer's utility function is given by:

$$u_2(x, y) = 4x - \tfrac{1}{2}x^2 + y .$$

(a) Calculate the monopolist's profit-maximizing price schedule under second-degree price discrimination (i.e., using a nonlinear price menu, charged uniformly across all consumers), *assuming that the monopolist sells to both consumers*.

(b) Calculate the monopolist's profit-maximizing price schedule under second-degree price discrimination (i.e., using a nonlinear price menu, charged uniformly across all consumers), *assuming instead that the monopolist sells only to the second consumer*.

(c) State whether the monopolist does better under part (a) or (b), providing a brief intuition.