Economics 603 Professor Lawrence Ausubel
Fall 2003–2004

Final Exam

Answer all four questions, each in a *different* blue book. Each question is worth 25 points.
Write your student number on each blue book and include your name on the book for Question 1.
Please return this question sheet at the same time that you turn in the blue books.
Write legibly, show your work, and justify your responses.

1. Consider the following static game:

		Player II		
		L	C	R
Player I	T	4 , 4	6 , 2	0 , 0
	M	2 , 6	8 , 8	0 , 0
	B	0 , 0	0 , 0	x , $-x$

(a) For *each* possible value of $x \in \mathbb{R}$, determine the outcome of iterated elimination of strictly dominated strategies.

(b) For *each* possible value of $x \in \mathbb{R}$, determine *all* of the Nash equilibria of this game.

Parts (c) – (e) concern the ***infinitely repeated*** version of the above static game.

(c) Assume that $x = 2$. Draw an accurate sketch of the "Folk Theorem" region for the infinitely-repeated version of the above static game.

(d) Write down a pair of trigger strategies that would provide average payoffs to the two players of approximately (5, 7) for discount factors near one. State the trigger strategies precisely.

(e) Argue whether the trigger strategies of part (d) form a subgame perfect equilibrium for discount factors, δ, sufficiently close to one. If so, solve for the critical δ that supports the subgame perfect equilibrium. If not, explain why no such δ exists.

2. Consider a first-price, sealed-bid auction **with *n* bidders** ($n \geq 2$), in which the bidders' valuations v_i are i.i.d. random variables distributed on the interval [0,1] **according to the distribution function $F(x) = x^2$**. As usual, each bidder knows her own valuation but only the probability distribution of her opponents' valuations, and each bidder is risk-neutral. The bidders simultaneously and independently submit sealed bids $b_i \in [0,1]$, and the high bidder wins the item and pays the amount of her bid. If bidder i wins the auction with bid b_i, her payoff (as usual) is given by $v_i - b_i$. If bidder i loses the auction, her payoff equals zero.

(a) [7 points] In a symmetric equilibrium with an increasing bid function, determine the probability that bidder i with valuation v_i wins the auction.

(b) [18 points] Derive and calculate the symmetric Bayesian-Nash equilibrium of the first-price auction described above.

3. Consider a **dynamic** version of the Hotelling model, with two firms. In period 1, Firm 1 announces a price of p_1. In period 2 (and after observing p_1), Firm 2 announces a price of p_2.

Otherwise, the model is identical in payoff structure to the standard (static) Hotelling model. The customers are uniformly distributed on the unit interval [0,1]. Each customer has a unit demand for the good (i.e., purchases either zero or one unit of the good). Each customer has a sufficiently high valuation, v, for the good that he purchases the good in equilibrium. Firm 1 is located at 0, and Firm 2 is located at 1. A customer located at point $x \in [0,1]$ incurs a transportation cost of tx purchasing from Firm 1 and a transportation cost of $t(1-x)$ purchasing from Firm 2, where $t > 0$. Except for location, the goods produced by Firm 1 and Firm 2 are identical, so that each customer purchases from the firm such that the price paid + transportation cost incurred is minimized.

(a) [15 points] Solve for the subgame perfect equilibrium of the dynamic Hotelling game, showing your work.

(b) [10 points] How do the profits of each firm change in the dynamic Hotelling game, as compared to the respective firm's profits in the standard (static) Hotelling game? Does the change go in the same way or in a different way, as in comparing the static Cournot game and the dynamic version of the same game (i.e., the Stackelberg game)? Provide both a comparison of the exact numbers and an intuitive explanation for your result.

4. Consider an economy in which there are two goods, x and y. The market for good x is a monopoly. Good y can be considered to be money, and the price of good y is normalized to 1. The monopolist has a constant marginal and average cost of producing good x equal to $c > 0$.

(a) Suppose that there are N identical consumers in the market, each with utility function $u_j(x,y)$ given by:

$$u_j(x,y) = \tfrac{3}{2}x^{2/3} + y\ .$$

Calculate the optimal two-part tariff for the monopolist.

(b) Instead, suppose that there are $4N$ consumers in the market.
N of the consumers have utility functions $u_j(x,y)$ as described above, and $3N$ of the consumers have utility functions $u_i(x,y)$ given by:

$$u_i(x,y) = x^{2/3} + y\ .$$

Calculate the monopolist's profit-maximizing price schedule under second-degree price discrimination (i.e., using a nonlinear price schedule, charged uniformly across all consumers). Fully explain and justify your answer, and indicate the monopolist's profits therein.