

Amateur Participation in a Matching Game

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1. Introduction

A game theoretic analysis of matching games with indivisible goods was first attempted by Gale and Shapley (1962). Their focus was on marriage and college admissions and showed a market was stable if it ensured that all parties were matched. They also proved that a stable market that weakly dominates all other stable markets must exist. They refer to this market as “the core.”

The analysis by Gale and Shapley and subsequent papers that followed on their research dealt primarily with two sided matching games. In these games an indivisible good, such as houses, must be assigned to an equal number of buyers. This model is problematic in several respects. It assumes that every individual desires exactly one match. It also assumes that participants in this game separate their roles as buyers and sellers.

These assumptions made in the benchmark matching game make it inapplicable in certain situations. One such example presented itself at the House of Lords Pool and Billiards Hall located in Hicksville, NY. A weekly tournament held in this venue, used an auction to allocate the rights to the prize winnings of the participants. This situation is unique because the rules of the tournament, as will be described in length, force participants to act simultaneously as buyers and sellers of the same good. This paper will analyze the strategies used by participants in this game. It will then analyze changes the rules of the game designed to eliminate certain strategies.

2. Data

A. Unchanged Tournament Rules

A weekly tournament is held for amateur 9-ball pool players. As it is a local tournament in a relatively unknown location (no professional tournaments have ever been held in this locale) the participants are primarily repeat locals players with limited experience in both 9-ball and auction strategy.

Nine Ball Rules

The rules of tournament 9-ball are well established and identical in practically all locations. The game has two players who use nine balls numbered in order plus a cue ball. The game begins with the balls racked in a diamond shape on the opposite side of the table. The player who is not breaking this rack may arrange the balls in the order of his choosing provided the number 9 ball is in the middle. The initial breaker, who is determined by coin flip, hits the cue ball into the diamond shaped rack from the opposite end of the table. The shooter continues to shoot until he does not sink a ball into a pocket or until he commits a foul. Players must hit the lowest numbered ball first, but after hitting the lowest numbered ball any ball that subsequently goes into a pocket is a good shot. The game is won by whoever sinks the number 9 ball. All other balls are irrelevant. In the event of a foul such as not hitting the lowest numbered ball first, the player loses his turn and the opposing player can place the cue ball anywhere on the table. A more thorough explanation of the rules of 9-ball can be found at <http://www.texasexpress.com>.

General Tournament Rules

Rankings

All participants in the tournament are ranked based on their ability and performance in the tournament. The rankings are 5, 6, 7, 8, 9, +1, +2, +3, and +4. A lower ranking makes it easier to win the tournament. Initial rankings are assigned solely based on the assessment of a player's ability as made by the commissioner of the tournament.

In a match between two players with the same ranking the normal 9-ball rules as described above will apply. In the event two players have different rankings, the match will be weighted to assist the lower ranked player in the following manner. For each level of separation, a "spot ball" will be given to the lower ranked player. The spot ball gives a second method of winning the game to the lower ranked player. For example, if a player was getting the number 7 ball as a "spot" he could win the game by making either the number 7 ball or the number 9 ball, while the higher ranked player could only win by making the number 9 ball. The following rules govern the amount of weight given to the lower player:

Difference in Rank	Weight given to lower ranked player
1	Number 8 ball spot
2	Number 7 ball spot
3	Number 6 ball spot
4	Number 5 ball spot
5	Number 5 ball spot and one free game win
6	Number 5 ball spot and two free game wins
7	Number 5 ball spot and three free game wins
8 (theoretical max)	Number 5 ball spot and four free game wins

A player must win six games to win the match.

3-5-10 Drop Rule

The weighted match system obviously causes participants to desire a lower ranking. The “3-5-10 rule” determines how long a player must wait before he is allowed to fall to the next lower ranking.

The first time a player is at a specific rank he must lose for three consecutive participatory weeks before dropping to the next lowest ranking. The second time a player is at that ranking he must lose for five consecutive participatory weeks before dropping to the next lowest ranking. Finally the third time a player is at that ranking he must lose for ten consecutive participatory weeks before dropping to the next lowest ranking. The cycle then repeats. Drops between each level are independent from one another. This means that the wait for a player to drop from the “9” to the “8” has no bearing on the wait for a drop from the “8” to the “7”, etc.

For example, if a player starts the tournament ranked as a “7”. He loses for three straight weeks. His new rank is now a “6”. The player then wins pushing his rank back up to “7” (rules for increasing rank will be described below). Now the player must lose for five consecutive weeks before dropping to the “6” rank again.

Tournament Bracket

After the final list of participants and their relevant rank is determined, the participants are split in half by rank. The highest and lowest ranked players are grouped together. In the event of a tie, the older players, by age not tournament experience, are grouped with the higher ranked players. After the two groups are established, they are

each assigned at random to a single elimination tournament bracket. Players from the two groups will only meet in the final match.

Prize Money

Each player pays a \$10 entry fee for the right to participate in the tournament. In addition, the “rights” to each player are sold off auction style to all participants. Whoever purchases the “rights” to a player is entitled to one half of the prize money that the player wins. The tournament itself is revenue neutral. One hundred percent of collected money is allocated to prizes.

First place winner and the runner up are entitled to prize money. The first place prize is 75% of the total money. The remaining 25% of the prize money is awarded to the runner-up.

The cost of the winning bid is split equally by the winning bidder and the player. Players are permitted to bid on and therefore win the “rights” to themselves.

The auction is held after the players are split into the two groups, but prior to the random assignment to the tournament bracket.

B. Original Rules

Rank Increase

After the tournament, the rank of the first place winner increases by two levels. The rank of the runner-up increases by one level. This increase in rank is made independent of the number of tournament participants or the amount of prize money.

Auction Format

The auction is a sequential ascending English auction. The auctions are held in the same order in which the players signed up to participate in the tournament. Players are required to make a minimum \$10 bid for the “rights” to themselves.

3. Analysis

The following assumptions will be made to facilitate analysis.

1. The bidders are risk neutral when bidding on other players rights.
2. The bidders are risk averse when bidding on their own rights. (bidders desire to win the rights to themselves)
3. The bidders are symmetric.
4. Players ability fluctuates from week to week.
5. Players are better able to estimate their own ability based this probability distribution than they are able to estimate other players’ ability.

A. Expected Value of a Winning Bid

Players ability changes based on external conditions. The ability of player i is derived from distribution Z . So, $a(i) = z \in Z$. An individual players probability of winning is a function of their ability, the ability of all other players, and the random tournament draw.

Two components determine the expected value of a player “rights”. The first aspect is the probability of the player winning or coming in second in the tournament. The sum of the probability of all players $i \in X$ is 1.

The second component is the total prize money to be won. The prize money is identical no matter who wins the tournament, making this a common value situation.

However, the common value y , the aggregation of all bids, is unknown. Further, the common value element is a function of the individual auction closing prices.

For any individual bidder, the expected value of a player is equal to the probability of winning times one half of the prize money.

$$E(i) = p(i) * .5y$$

B. Information Rents and Signaling

Bidders face a problem. Each player is bidding on a signal. Each individual player will have the least noise in estimating their own signal, but will face great difficulty in learning other players' signals. This creates a situation where the winners curse applies. The generalized winners curse says that a bid only matters at the margin. When a bid wins an auction, it is because it is greater than all other bids and therefore too high. A rational bidder must condition their bid on the negative information learned from winning.

The use of an ascending auction with public information is beneficial in learning these signals. Cramton (1998) found that the winners curse is of much less concern in ascending auctions in non independent private value situations, because information is revealed in the auction process. Bidders can condition their bids on their own information and the information learned in the bidding process. The most relevant piece of information this process will reveal is the bidding strategy each player uses on their own "rights". Aggressive bidding on oneself reveals the player has a strong signal about their ability.

Bidders face a second signal problem. Bidders in the early auctions have the least information about the common value prize money. The competitiveness of later auctions will determine the value of the prizes. Bidders are once again facing a winners curse. Bidders in the early auctions must estimate the revenue to be generated from later auctions. However, they must adjust their bid to condition on the negative information learned from winning. This however is offset to a certain extent because the common value is dependent on the auction prices. Higher prices in early auctions will increase the prices in later auctions.

C. Bidder Strategy

Intentional Loss

One problem with the tournament design is strict increase in rankings of the winning players. In a tournament with few participants, the penalty of winning is great. Winning a small tournament directly reduces the probability of a player winning a future tournament. The better players in the tournament who win on a regular basis know this. They lose the desire to win small tournaments. This penalty associated with winning is a major element in determining a player's signal. A player may have the ability to shoot well, but choose not to because it is more desirable to remain a lower rank for a larger tournament in the future.

The Winners Curse (or benefit)?

Typically, the winners curse is thought of as a mutually harmful element to bidders and sellers. Bidders are damaged by the likely overestimation of value. Sellers

are damaged by the bid shading and reduced revenue that the winners curse will generate from rational bidders.

In this tournament, the winner curse actually benefits individual players, because it forces down the cost of participation. Players desire to be auctioned off in a situation with the most winners curse, because it will have lower prices. The participant is responsible to pay one half of the cost of the closing price for their “rights”. They must pay this cost even if they did not participate in the auction. The winners curse is hugely beneficial to individual players as it lowers cost, but detrimental to the revenue generation ability of the overall tournament. Essentially, the winners curse is an externality of the tournament.

The question players’ then face is when the winners curse will be the greatest. In this auction process the winners curse decreases with each subsequent auction because bidders have the most information about common value and the signal of other players as learned through their bidding strategies. As expected, participants in the tournament rush to be the first on the sign up list as the auctions are conducted in that order. The advantage, from participating in an auction with winners curse is so great, that some players choose not to participate at all if they are not near the top of the auction list.

Retaliatory Bidding

Every participant in this tournament has an identical vulnerability. That is, they must pay one half the cost of their own closing price. This creates an easy opportunity to develop a reputation for using a retaliatory bidding strategy. In other auctions, retaliatory bidders can drive up the cost to their competitors, but when faced with this situation, a bidder can always decide to drop out of the auction. In this situation that luxury is not

provided. A typical strategy in the tournament is to earn a reputation as both a retaliatory bidder, and a player who will intentionally lose the tournament if anyone else purchases their “rights”.

Bidders using this strategy will bid up the price of anyone who bid on their “rights”. The earlier in the order an auction is held the easier it becomes to use this strategy. An early auction prevents mutual retaliation, because the bidding is already finished on the person implementing the strategy. It also ensures that the players against whom the retaliation is directed have not yet to been auctioned off.

Conclusions

Participants in this auction have a clear incentive to be among the first few auctions. Early auctions will close unambiguously further below their true values when compared to later auctions.

Ultimately this was the downfall of this tournament format. The market equilibrium caused bidders to realize the benefits of early auction listing. Reforms were implemented to address the problems created by this format.

D. New Tournament Rules

Ranking Changes

Under the new format, players’ ranking adjusts based on the number of participants in the tournament. In a small tournament, of zero to fifteen players, a player’s rankings will not increase after a win. A win in a small tournament will restart the wait time before a player is allowed to drop to the next lowest ranking. In a medium

sized tournament of sixteen to twenty-four players, the winner of the tournament increases by one ranking, while the runner-up stays at the same ranking. Finally, in a large tournament with more than twenty four players, the old ranking rules apply.

Auction Format

The most significant of changes in the tournament rules is the change in auction format. A sequential ascending English auction is still used. However, bidders no longer are bidding on the “rights” to a specific player. Instead, they are bidding for the option to take the “rights” to any unselected player in the tournament. For example, the first winning bidder can select the “rights” to any player in the tournament. The cost of his bid will be split between the winning bidder and the player who is selected. The second auction allows the winning bidder to select the “rights” to any player except the player chosen in the first auction.

Analysis

The new format has many desirable features when compared with the prior format. First, the new format significantly reduces the winner curse that arises from estimates of the common value of the auction. Participants can reasonably estimate with low degrees of error the size of the total tournament based on the size of the first bid. This is because the first bid is the most valuable one. Each subsequent bid is of less value. The English auction format now reveals information in two ways. The ascending price provides information about the value of the specific item up for auction. The closing price for each auction also provides information that the next auction should be slightly less valuable. The common value element of the auction is strictly dependent on

the auction closing values. Therefore, the high bidders in the first auction can set the size of all subsequent auctions. A large closing price in the first auction increases the value of every auction that follows. This makes the winners curse largely irrelevant. Even if a bidder over estimates the value in an auction, it will not matter, because all future bidder will raise their prices because of the increased common value.

The new format also takes advantage of individual bidder risk aversion in bidding on themselves. Given two players with identical probabilities of winning, one being themselves, and the other some arbitrary player, all players would rather own the “rights” to themselves. This is simply because the tournament is intended more as a recreational activity and not a money making venture. In general, players would rather enjoy the benefits of their personal success. Milgrom and Weber (1982) showed that in an auction with risk averse participants, the revenue from a first price auction will be greater than or equal to an that of an English auction. While this new format is not exactly a first price auction, the intuition is identical to the one used by Milgrom and Weber. The fear of losing the “rights” to oneself causes aggressive bidding in the auction and therefore higher revenue.

Finally, the new format reduces the ability to implement a retaliatory bidding strategy. In the new format the order of signup irrelevant. While a retaliatory strategy can still be used, the target of an attempted retaliation may have already been purchased. The only method of retaliation is intentional loss or to wait until the next week to bid up a competitor, but by that time the target may have forgotten who he purchased in the prior weeks tournament.

The new format has one significant drawback. Bidders no longer reveal information about a specific participant. This increases the value of information rents that bidders achieve due to the private information they hold on their personal ability.

4. Conclusions

The new auction has eliminated the benefits of certain strategies used in the old auction format. It remains unclear in theoretical terms if the new auction format will yield higher revenue. The new format will increase revenue if the gains from a lower winners curse in the common value estimation, reduced retaliatory bidding, increased participation, and less value from intentional loss outweigh the losses due to higher information rents.

5. Works Cited

- Cramton, Peter. 1998. Applications of Auction Theory: Ascending Auctions. *European Economic Review* 42. 745-756
- Gale, D., and Shapley, L. 1962. College Admission and the Stability of Marriage, *Amer. Math. Monthly* 69, 9–14.
- Milgrom, P. and Weber, R. 1982. A Theory of Auctions and Competitive Bidding. *Econometrica* 50. 1089-1122