Economics 415
Professor Cramton

Problem Set 1: Due Tuesday, April 18
Midterm, Tuesday, April 25

Question 1.

(a) Consider a private value auction with four bidders and two identical goods. Suppose each bidder demands just a single unit. The values for bidders 1, 2, 3, and 4 are v_1 = 32, v_2 = 26, v_3 = 45, and v_4 = 22. In a Vickrey auction, which bidders get the two goods and how much do they pay?

(b) Now suppose that the bidders have demands for multiple goods. In particular, the values are v_1 = (32, 19), v_2 = (26, 18), v_3 = (45, 35), and v_4 = (22, 14), where the first number is the value for the first good and the second number is the value for the second. In a Vickrey auction, which bidders get the two goods and how much do they pay?

(c) Interpret your answers in (a) and (b) in terms of opportunity cost.

(d) Now suppose the situation in (b), but that the Ausubel auction (i.e., an ascending clock auction with Vickrey pricing) is used to assign the goods. Who gets the items and what do they pay? Describe the process of assignment.

Question 2.

Company *A* (the acquirer) is considering acquiring Company *T* (the target). Company *T*'s value under current management is uniformly distributed between \$0 and \$100, depending on the outcome of a risky project. *T* knows the outcome of the project and hence the value v, but *A* only knows that v is uniformly distributed between 0 and 100. Regardless of the outcome of the project, *T* is worth 50% more to *A* than *T* (*A* values *T* at $1.5v$), so for example if *T* is worth \$50 to *T*, the company is worth \$75 to *A*. Company *A* makes a single take-it-or-leave-it offer for *T*, which *T* accepts or rejects. Both firms are risk neutral.

(a) What price per share should *A* make (i.e., determine the subgame-perfect equilibrium in this game)?

(b) State the winner's curse.

(c) Explain how the winner's curse relates to *A*'s problem.

Question 3. (Hint: See Ausubel and Cramton, "Demand Reduction".)

Consider an independent private value auction with two bidders. The seller has two identical units to sell, and values each at zero. Bidder 1 wants just a single unit, and has a value u, uniformly distributed on [0,1]. Bidder 2 wants up to two units and has a constant marginal value v for each unit, uniformly distributed on [0,1]. Values u and v are independent, and both bidders are risk neutral. Each bidder submits two bids, the first bid for the first unit it wins and the second for the second unit it wins.

(a) Suppose a third-price auction is used: the highest two bids win and both winning units are priced at the highest-rejected bid (i.e., the third-highest bid). What is the best bid for each bidder for the first unit?

(b) What is Bidder 2's optimal bid b for the second unit in the third-price auction?

(c) Now suppose a Vickrey auction is used. What are the bidders' optimal bidding strategies?

(d) Compare both the efficiency and revenues in both the third-price auction and the Vickrey auction.

Question 4 (Extra credit!).

Consider a first-price auction with two bidders for a good with common value $v = x_1 + x_2$. The parameters x_1 and x_2 are independently drawn from the uniform distribution on [0,1]. Bidder i knows x_i but not x_j. Compute the symmetric equilibrium bidding function $b(x)$. [Hint: it is linear.]

Question 5 (Extra credit!).

Consider a first-price auction with n bidders in the independent private-values setting. Each bidder's valuation v is drawn independently from the distribution $F(v) = v^{\alpha}$ on [0, 1] with $\alpha > 0$. Show that, in the symmetric equilibrium, the bid function $b(v)$ is linear in the bidder's valuation.