Three Theorems About Package Bidding

Based largely on
“Ascending Auctions with Package Bidding”
Larry Ausubel and Paul Milgrom
June 2002

Outline

- Introduction: Complements and the need for package bidding.
- Understanding the laboratory successes of complex auction designs:
  - Theorem 1: proxy auction outcomes are in the (NTU) core with respect to reported preferences.
- Equilibrium in the TU proxy auction.
  - Theorem 2: Equilibrium in semi-sincere strategies (like in matching theory).
- Reasons to reject the Vickrey auction.
  - Theorem 3: “Good performance” of the Vickrey auction (various criteria) is guaranteed if and only if goods are substitutes.
Complements and the Need for Package Bidding

Exposure Problem in the Netherlands

- Variant of SAA completed February 18, 1998 after 137 rounds.
- Raised NLG 1.84 billion.
- Prices per band in millions of NLG
  - Lot A: 8.0
  - Lot B: 7.3
  - Lots 1-16: 2.9-3.6
Prices: Substitutes & Complements

- **Theorem**: If all items are **mutual substitutes** then (despite indivisibilities), a competitive equilibrium exists.

- **Theorem (Milgrom, Gul-Stacchetti)**. If the set of possible valuations strictly includes the ones for which items are substitutes, then it includes a profile for which no CE exists.

<table>
<thead>
<tr>
<th></th>
<th>Item A</th>
<th>Item B</th>
<th>Package AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidder 1</td>
<td>a</td>
<td>b</td>
<td>a+b+c</td>
</tr>
<tr>
<td>Bidder 2</td>
<td>a+αc</td>
<td>b+αc</td>
<td>a+b</td>
</tr>
</tbody>
</table>

- Market clearing prices do not exist if .5<α<1.

**Understanding the lab successes of complex auction designs**
FCC-Cybernomics Experiment

<table>
<thead>
<tr>
<th>Complementarity Condition:</th>
<th>None</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAA (No packages)</td>
<td>97%</td>
<td>90%</td>
<td>82%</td>
<td>79%</td>
</tr>
<tr>
<td>SAAPB (“OR” bids)</td>
<td>99%</td>
<td>96%</td>
<td>98%</td>
<td>96%</td>
</tr>
<tr>
<td><strong>Revenues</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAA (No packages)</td>
<td>4631</td>
<td>8538</td>
<td>5333</td>
<td>5687</td>
</tr>
<tr>
<td>SAAPB (“OR” bids)</td>
<td>4205</td>
<td>8059</td>
<td>4603</td>
<td>4874</td>
</tr>
<tr>
<td><strong>Rounds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAA (No packages)</td>
<td>8.3</td>
<td>10</td>
<td>10.5</td>
<td>9.5</td>
</tr>
<tr>
<td>SAAPB (“OR” bids)</td>
<td>25.9</td>
<td>28</td>
<td>32.5</td>
<td>31.8</td>
</tr>
</tbody>
</table>

Scheduling Trains in Sweden

- Paul Brewer and Charles Plott
- Lab environment
  - Additive values for trains
  - Single N-S track
  - Complex “no crashing” constraint
- Ascending offer process
- Efficient outcomes
The General Proxy Model

- Each bidder $i$ has
  - a finite set of feasible offers $X_i$ and
  - a strict ordering over them represented by $u_i$.
- Auctioneer has
  - a feasible set $X = X_1 \times \ldots \times X_L$.
  - a strict ordering over $X$ represented by $u_0$.
- Proxy auction rules
  - Auction proceeds in a sequence of rounds
  - Provisional winning bidders make no new bid
  - Others add “most preferred” remaining bid, unless “no trade” is preferred to that bid.
  - Auctioneer takes at most one bid per bidder to maximize $u_0$.

Proxy Auction Analysis

- Generalized Proxy Auction
  - By round $t$, proxy has proposed null bid and all packages for bidder satisfying a minimum profit constraint: $u_i(x_i) \geq \pi_i^t$
- At round $t$, the auctioneer tentatively accepts the feasible bid profile that maximizes $u_0(x^t)$.
  - Therefore, utility vector $\pi^t$ is unblocked by any coalition $S$.
- Bidders not selected reduce their target utilities to include one new offer, but do not reduce below “zero” (the value of no trade).
  - Therefore, when the auction ends, the utility allocation is feasible.
Theorem 1. The generalized proxy auction terminates at a (non-transferable-utility) core allocation relative to reported preferences.

Proof. The payoff vector is unblocked at every round, and the allocation is feasible when the auction ends. QED

The Quasi-linear (TU) Case

Seller's revenue at round $t$ is given by:

\[
\pi_t^f = \max_{x \in X} \sum_{i=0} B_i^f(x_i) \\
= \max_{x \in X} \sum_{i=0} \max(0, v_i(x_i) - \pi_i^f) \\
= \max_{x \in X} \left[ \max_{S \subseteq L} \sum_{i \in S \cup \emptyset} v_i(x_i) - \pi_i^f \right] \\
= \max_{S \subseteq L} \left[ \max_{x \in X} \sum_{i \in S \cup \emptyset} v_i(x_i) - \pi_i^f \right] \\
= \max_{S \subseteq L} \left[ w(S) - \sum_{i \in S \cup \emptyset} \pi_i^f \right] \\
\therefore (\forall S) w(S) \leq \sum_{i \in S} \pi_i^f
\]

Payoffs are unblocked at every round

“Coalitional second price auction”
Applications (w/o Proxies!?!)

- **Train Schedules (Brewer-Plott)**
  - Bidders report additive values for each train
  - Auctioneer maximizes total bid at a round, respecting scheduling constraints (to avoid crashes).

- **FCC package auctions**
  - Bidders report valuations of packages
  - Final outcome is a “core allocation” (for the reported preferences).

- **Package Auctions with Budget Constraints**
  - Bidders report valuations and a budget limit.
  - Final outcome is a “core allocation.”

A Novel “Matching” Procedure

- Uniquely among deferred acceptance algorithms:
  - Offers are multidimensional and/or package offers
  - Feasible sets may be arbitrarily complex
  - The algorithm is *not* monotonic over “held offers”: it may backtrack to take previously rejected offers
  - The analysis does *not* employ a “substitutes” condition.
  - The outcome may *not* be a bidder-Pareto-optimal point in the core.

- Unique in matching theory analysis
  - Equilibrium will be characterized with complex offers.
Equilibrium in a TU Proxy Auction

Formulation

- Assume that all payoffs are quasi-linear
  - For bidders: value received less money paid.
  - For seller: value of allocation plus money received.
- Consider limiting process as the size of the bid increments goes to zero.
  - Focus shifts to transferable utility core.
  - Call this the “TU-proxy auction.”
The Substitutes Case

- Theorem. In the TU-proxy auction, suppose that the set of possible bidder values \( V \) includes all the purely additive values. Then these three statements are equivalent:
  - The set \( V \) includes only values for which goods are substitutes.
  - For every profile of bidder valuations drawn from \( V \), sincere bidding is an \textit{ex post} Nash equilibrium of the proxy auction.
  - For every profile of bidder valuations drawn from \( V \), sincere bidding results in the Vickrey allocation and payments for all bidders.

“Semi-Sincere” Bidding

- Definitions. A strategy in a direct revelation trading mechanism is “semi-sincere” if it can be obtained from sincere reporting by changing the utility of the “no trade” outcome.

- Theorem. In the TU-proxy auction, fix any pure strategy profile of other bidders and let \( \pi_l \) be bidder \( l \)’s maximum profit. Then, bidder \( l \) has a semi-sincere best reply, which is report to its proxy that its values are given by \( v_l(x) - \pi_l \).
  - An anti-collusion property.
Selected Equilibria

- **Selection criterion**
  - All bidders play semi-sincere strategies
  - Losers play sincere strategies

- **Theorem 2**. Let $\pi$ be a bidder-Pareto-optimal point in $\text{Core}(L,w)$ with respect to actual preferences. Then in the TU-proxy auction, semi-sincere strategies with values reduced by $\pi$ constitute a (full-information) Nash equilibrium. Moreover, for any equilibrium satisfying the selection criterion, the payoff vector has bidder profits in $\text{Core}(L,w)$.

**Vickrey auctions for complements?**
Vickrey Auction Rules

- Bids and allocations
  - One or more goods of one or more kinds
  - Each bidder $i$ makes bids $b_i(x)$ on all bundles
  - Auctioneer chooses the feasible allocation $x^* \in X$ that maximizes the total bid accepted

- Vickrey (“pivot”) payments for each bidder $i$ are:
  $$p_i = \max_{x \in X} \sum_{j \neq i} b_j(x_j) - \sum_{j \neq i} b_j(x_j^*)$$

- Vickrey auction advantages are well known, but there are also important disadvantages.

Direct vs. Indirect Mechanisms

- The Vickrey auction is a direct mechanism, requiring the bidder to evaluate $2^N$ packages to make its bids.

- Indirect mechanisms may be favored (CRA Report to FCC: Milgrom, et al) to economize on valuation efforts.
Theorem 3. Suppose that the set of possible bidder values $V$ includes all the purely additive values. Then these six statements are all equivalent:

- The set $V$ includes only values for which goods are substitutes.
- For every profile of bidder valuations drawn from $V$, Vickrey auction revenue is isotone in the set of bidders.
- For every profile... $V$, Vickrey payoffs are in the core.
- For every profile... $V$, there is no profitable shill (“false name”) bidding strategy in the Vickrey auction.
- For every profile... $V$, there is no profitable joint deviation by losing bidders in the Vickrey auction.

Monotonicity and Revenue Problems

- Vickrey Auction and the Core
  - Two identical spectrum bands for sale
  - Bidders 1 wants the pair only and will pay up to $2$ billion.
  - Bidders 2 and 3 want single license and will pay up to $2B$.
  - Outcome:
    - Bidders 2 and 3 acquire the licenses.
    - Price is zero.

- Problems in this example:
  - Adding bidder 3 reduces revenue from $2B$ to zero.
  - The Vickrey outcome lies outside the core.

- Conclusions change if 1 will pay up to $1B$ each.
  - Substitutes condition is the key.
The Shills Problem


- Example: two identical spectrum bands for sale
  - Bidder 1 wants only the pair, will pay up to $2B.
  - Bidder 2 is willing to pay $0.5B each, $1B for the pair
  - By bidding $2B for each license using two names, bidder 2 can win both licenses at a price of zero.

- The Vickrey auction is vulnerable to shill bidders.

- Conclusion changes if 1 will pay up to $1B each.
  - Substitutes condition is the key.

Loser Collusion

- Example: two identical spectrum bands for sale
  - Bidder 1 wants only the pair, will pay up to $2B.
  - Bidder 2 is willing to pay $0.5B for one
  - Bidder 3 is willing to pay $0.5B for one
  - Losing bidders 2 and 3 have a profitable joint deviation, bidding $2B each, winning both licenses at a price of zero.

- The Vickrey auction is unique in its vulnerability to collusion even among losing bidders

- Conclusion changes if 1 will pay up to $1B each.
  - Substitutes condition is the key.
Vickrey’s “Efficiency Problem”

- Example: 2 licenses, East and West
  - Bidder 1 has value $1.2B for the pair
  - Bidder 2 has value of $1B for East
  - Bidder 3 has value $1B for West
  - Merged bidders 2 & 3 have value $2.5 for E-W package

- Vickrey price and profit effects of a merger
  - Unmerged firms total price is $400 million, profit of $1.6B.
  - Merged firm’s price is $1.2 billion, profit $1.3B

- Incentive is not to merge; value is not maximized.
  - Result reverses if 1’s value is $0.6B per license.

Comparing Auctions

- + means “has the property generally”
- * means “has the property when goods are substitutes”

<table>
<thead>
<tr>
<th>Property</th>
<th>Vickrey Auction</th>
<th>Proxy Auction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sincere bidding is a Nash equilibrium.</td>
<td>+</td>
<td>*</td>
</tr>
<tr>
<td>Equilibrium outcomes are in the core.</td>
<td>*</td>
<td>+</td>
</tr>
<tr>
<td>No profitable shill bids</td>
<td>*</td>
<td>+</td>
</tr>
<tr>
<td>No profitable joint deviations for losers</td>
<td>*</td>
<td>+</td>
</tr>
<tr>
<td>Competing technologies property</td>
<td>No</td>
<td>+</td>
</tr>
<tr>
<td>Fully adaptable to limited budgets</td>
<td>No</td>
<td>+</td>
</tr>
</tbody>
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The End