Chapter 5

Extensive Form Games with Perfect Information

Subgames and their equilibria

- The concept of subgames
- Equilibrium of a subgame
- Credibility problems: threats you have no incentives to carry out when the time comes
- Two important examples
  - Telex vs. IBM
  - Centipede

Game in Extensive Form

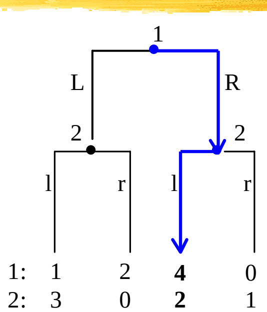
- Who plays when?
- What can they do?
- What do they know?
- What are the payoffs?

Subgame Perfection
(Selten, 1965)

Nash Equilibrium: each player must act optimally given the other players' strategies, i.e., play a best response to the others' strategies.

Problem: Optimality condition at the beginning of the game. Hence, some Nash equilibria of dynamic games involve incredible threats.

Game in Extensive Form: Backward Induction



Unique equilibrium path

Game in Normal Form

		2			
		ll	lr	rl	rr
1	L	1, <u>3</u>	**<u>1</u>, <u>3</u>**	2, 0	<u>2</u>, 0
	R	**<u>4</u>, <u>2</u>**	0, 1	**<u>4</u>, <u>2</u>**	0, 1

- Three Nash equilibria in pure strategies: {R,ll}, {L,lr}, and {R,rl}.
- {L,lr}, and {R,rl} involve incredible threats.

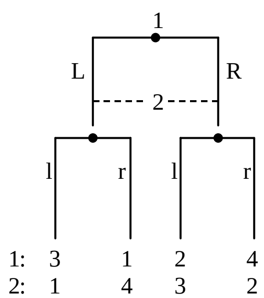
Subgame Perfection with Perfect Information

Consider a game Γ of perfect information consisting of a tree T linking the information sets $i \in I$ (each of which consists of a single node) and payoffs at each terminal node of T. A *subtree* T_i is the tree beginning at information set i, and a *subgame* Γ_i is the subtree T_i and the payoffs at each terminal node of T_i.

Definition

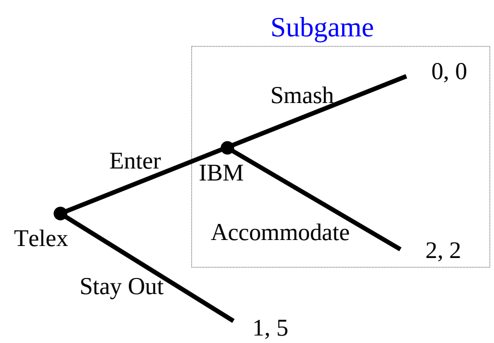
A Nash equilibrium of Γ is *subgame perfect* if it specifies Nash equilibrium strategies in every subgame of Γ. In other words, the players act optimally at every point during the game.

Subgame Perfection with Imperfect Information

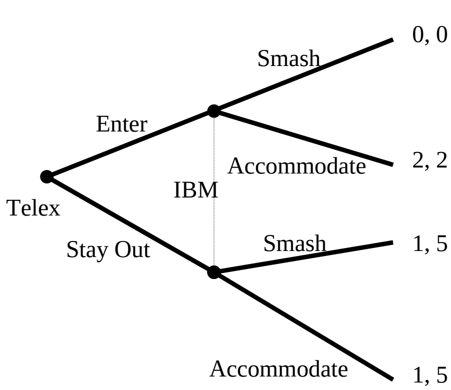


With imperfect information, each information set consisting of a single node determines a subgame. Hence, there are no (proper) subgames in this example.

Telex vs. IBM, extensive form: subgame, perfect information



Telex vs. IBM, extensive form: no subgame



Telex vs. IBM, normal form: The payoff matrix

Telex \ IBM	Smash	Accommodate
Enter	0, 0	2, 2
Stay Out	1, 5	1, 5

Telex vs. IBM, normal form: Strategy for IBM

Telex \ IBM	Smash	Accommodate
Enter	0, 0	2, 2
Stay Out	1, 5	1, 5

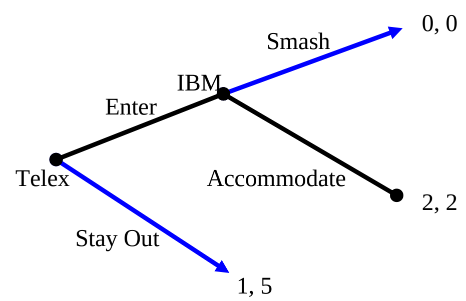
Telex vs. IBM, normal form: Strategy for Telex

Telex \ IBM	Smash	Accommodate
Enter	0, 0	2, 2
Stay Out	1, 5	1, 5

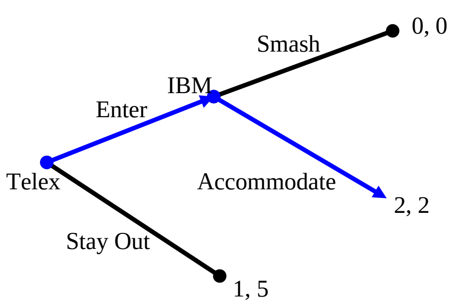
Telex vs. IBM, normal form: Two equilibria

Telex \ IBM	Smash	Accommodate
Enter	0, 0	2, 2
Stay Out	1, 5	1, 5

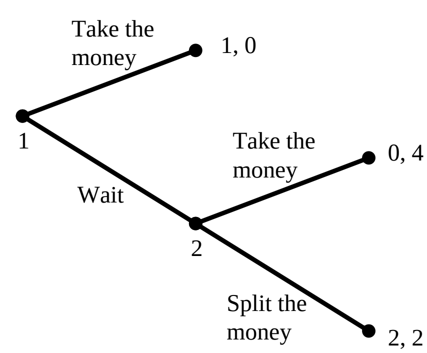
Telex vs. IBM, extensive form: noncredible equilibrium



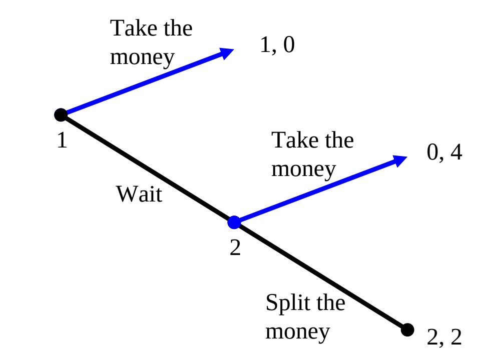
Telex vs. IBM, extensive form: credible equilibrium



Centipede, extensive form



Centipede, extensive form



Centipede, normal form: The payoff matrix

Player 1 \ Player 2	Take the money	Split the money
Take the money	1, 0	1, 0
Wait	0, 4	2, 2

Centipede, normal form: Strategy for player 1

Player 1 \ Player 2	Take the money	Split the money
Take the money	**1**, 0	1, 0
Wait	0, 4	**2**, 2

Centipede, normal form: Strategy for player 2

Player 1 \ Player 2	Take the money	Split the money
Take the money	1, **0**	1, **0**
Wait	0, **4**	2, 2

Centipede, normal form: The equilibrium

Player 1 \ Player 2	Take the money	Split the money
Take the money	**1**, **0**	1, **0**
Wait	0, **4**	**2**, 2

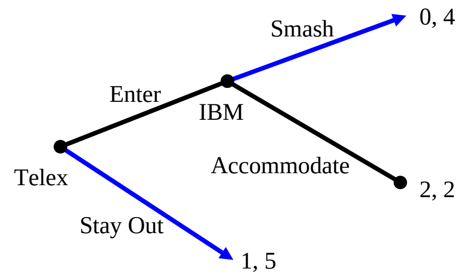
Look Ahead and Reason Back

- This is also called Backward Induction
- Backward induction in a game tree leads to a subgame perfect equilibrium
- In a subgame perfect equilibrium, best responses are played in every subgame

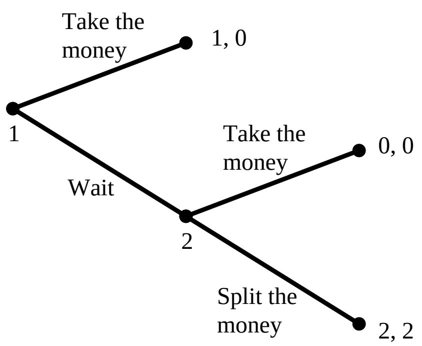
Credible Threats and Promises

- The variation in credibility when money is all that matters to payoff
- Telex vs. Mean IBM
- Centipede with a nice opponent
- The potential value of deceiving an opponent about your type

Telex vs. Mean IBM



Centipede with a nice opponent, extensive form



Centipede with a nice opponent, normal form: The payoff matrix

Player 1 \ Player 2	Take the money	Split the money
Take the money	1, 0	1, 0
Wait	0, 0	2, 2

Centipede with a nice opponent, normal form: Strategy for player 1

Player 1 \ Player 2	Take the money	Split the money
Take the money	**1**, 0	1, 0
Wait	0, 0	**2**, 2

Centipede with a nice opponent, normal form: Strategy for player 2

Player 1 \ Player 2	Take the money	Split the money
Take the money	1, **0**	1, **0**
Wait	0, 0	2, **2**

Centipede with a nice opponent, normal form: The equilibrium

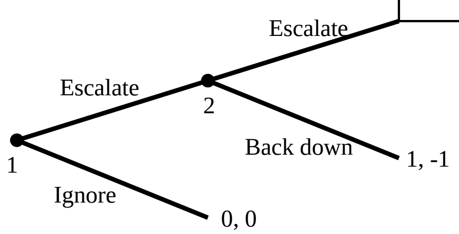
Player 1 \ Player 2	Take the money	Split the money
Take the money	1, 0 ★	1, 0
Wait	0, 0	2, 2 ★

Mutually Assured Destruction

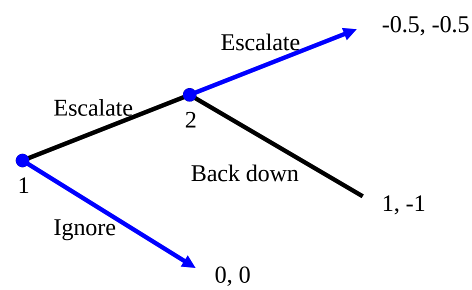
- The credibility issue surrounding weapons of mass destruction
- A game with two very different subgame perfect equilibria
- Subgame perfection and the problem of mistakes

MAD, extensive form: entire game

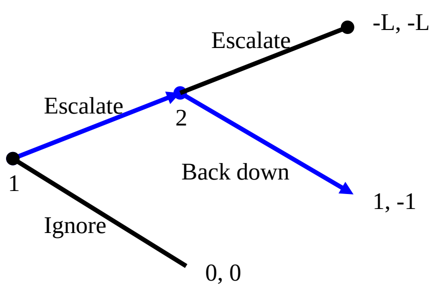
Player 1 \ Player 2	Doomsday	Back down
Doomsday	-L, -L	-L, -L
Back down	-L, -L	-0.5, -0.5



MAD, extensive form: path to final backing down



MAD, extensive form: path to Doomsday



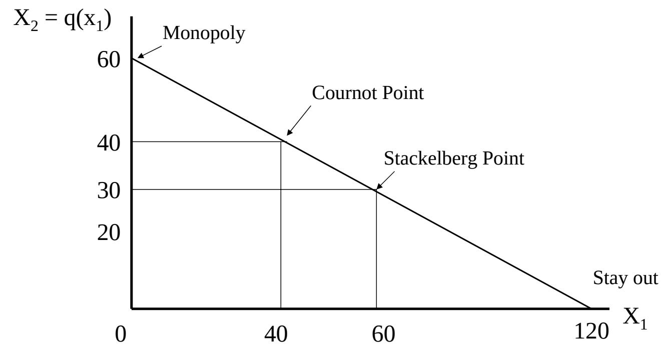
MAD, normal form: b = Back down; e = Escalate; D = Doomsday; i = Ignore; ★ = equilibrium; ⬣ = subgame perfect equilibrium

Country 1 \ Country 2	e, D	e, b	b, D	b, b
e, D	-L, -L	-L, -L	⬣ 1, -1	1, -1 ★
e, b	-L, -L	-0.5, -0.5	1, -1	1, -1
i, D	0, 0 ★	0, 0	0, 0	0, 0
i, b	0, 0 ★	⬣ 0, 0	0, 0	0, 0

Credible Quantity Competition: Cournot-Stackelberg Equilibrium

- The first mover advantage in Cournot-Stackelberg competition
- One firm sends its quantity to the market first. The second firm makes its moves subsequently.
- The strategy for the firm moving second is a function
- Incredible threats and imperfect equilibria

Cournot-Stackelberg Equilibrium: firm 2's best response



Cournot-Stackelberg Equilibrium for two firms

Market Price, P = 130 - Q

Market Quantity, Q = $x_1 + x_2$

Constant average variable cost, c = \$10

Firm 1 ships its quantity, x_1, to market first

Firm 2 sees how much firm 1 has shipped and then ships its quantity, x_2, to the market

Cournot-Stackelberg Equilibrium for two firms: Firm 2 maximizes its profits

Firm 2 faces the demand curve,

$$P = (130 - x_1) - x_2$$

Firm 2 maximizes its profits,

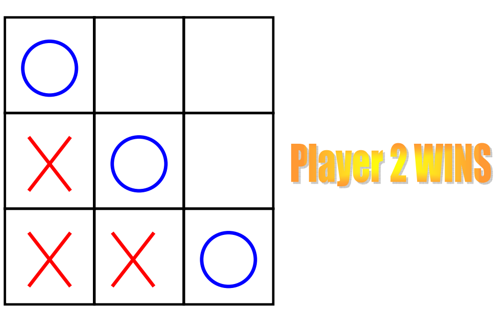
$$\max u_2(\mathbf{x}) = x_2(130 - x_1 - x_2 - 10)$$

Differentiating $u_2(\mathbf{x})$ with respect to x_2:

$$0 = \partial u_2/\partial x_2 = 120 - x_1 - 2x_2$$

$$\Rightarrow \quad x_2 = g(x_1) = 60 - x_1/2$$

Games like Chess: Tic-Tac-Toe



Player draft

- Each team takes turn choosing players
- Is it best to always chose most preferred player?

Cournot-Stackelberg Equilibrium: Firm 1 also wants to maximize its profits

Firm 1's profit function is given by:
$u_1(\mathbf{x}) = [130 - x_1 - g(x_1) - 10]\, x_1$

Substituting $g(x_1)$ into that function:
$u_1(\mathbf{x}) = (120 - x_1 - 60 + x_1/2)\, x_1$

$\therefore$ Firm 1's profits depend only on its shipment

Taking the first order condition for $u_1(\mathbf{x})$:
$0 = 60 - x_1$

The Cournot-Stackelberg Equilibrium for two firms

The Cournot-Stackelberg equilibrium value of firm 1's shipments, $x_1^* = 60$

Firm 2's shipments, $x_2^* = 60 - 60/2 = 30$

Market Quantity, Q = 60 + 30 = 90

Market Price, P = 130 - 90 = $40

This equilibrium is different from Cournot competition's equilibrium, where $x_1^* = x_2^* = 40$, Q = 80 and P = $50

Credible Price Competition: Bertrand-Stackelberg Equilibrium

- Price is the strategic behavior in Bertrand-Stackelberg competition
- The strategy for the firm moving second is a function
- Firm 2 has to beat only firm 1's price which is already posted
- The second mover advantage in Bertrand-Stackelberg competition

Bertrand -Stackelberg Equilibrium for two firms

Market Price, P = 130 - Q and
Constant average variable cost, c = $10

Firm 1 first announces its price, p_1

Firm 2's profit maximizing response to p_1:

p_2 = $70	if p_1 is greater than $70
$p_2 = p_1$ - $0.01	if p_1 is between $70 and $10.02
$p_2 = p_1$	if $p_1 = 10.01$
p_2 = $10	otherwise

Get competitive outcome; no extra profits!

Market Games with Differentiated Products

- Price and quantity competition when products are differentiated
- Cournot and Bertrand equilibrium still different, but the difference is muted
- Monopolistic competition as the limit of market game equilibrium

Differentiated Products

All **differentiated products** have one thing in common: if the price is slightly above the average price in the market, a firm doesn't lose all its sales

Two firms in a Bertrand competition

- The demand function faced by firm 1:
  $x_1(\mathbf{p}) = 180 - p_1 - (p_1 - \text{average price})$
- The demand function faced by firm 2:
  $x_2(\mathbf{p}) = 180 - p_2 - (p_2 - \text{average price})$

Two firms in a Bertrand competition

- Firm 1 has profits
  $u_1(p_1,p_2) = (p_1 - 20)\, x_1$
  $= (p_1 - 20)\,(180 - 2p_1 + \text{average price})$
  $= (p_1 - 20)\,(180 - 1.5p_1 + 0.5p_2)$
- Firm 2's profit function
  $u_2(p_1,p_2)$
  $= (p_2 - 20)\,(180 - 1.5p_2 + 0.5p_1)$

Maximizing profits

Firm 1 maximizes its profits when its marginal profit is zero:

$0 = \partial u_1/\partial p_1$

$= (p_1 - 20)\,(-1.5) + (180 - 1.5p_1 + 0.5p_2)$

$\Rightarrow 0 = 210 - 3p_1 + 0.5p_2$

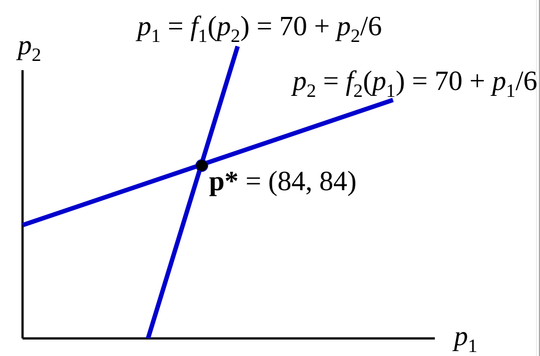
Firm 1's best response function:

$p_1 = f_1(p_2) = 70 + p_2/6$

Similarly, Firm 2's best response function:

$p_2 = f_2(p_1) = 70 + p_1/6$

Bertrand best responses, two firms, differentiated products



Bertrand Equilibrium

- The Bertrand equilibrium of the market game is located at (84, 84)
- The market price is $84, significantly higher than the marginal price which is given at $20
- Each firm sales (180 - 84) units = 96 units
- Each firms profit = (84 - 20) × 96
  = $6144
- Therefore, each firm could spend over $6000 in differentiating its products and can still come out ahead

Bertrand competition with n firms

- Firm 1's market demand
  $x_1 = 180 - p_1 - (n/2)\,(p_1 - \text{average price})$
- Firm 1's profit function
  $u_1(\mathbf{p}) = (p_1 - 20)\, x_1$
- When the first order condition is satisfied
  $0 = \partial u_1/\partial p_1 = (p_1 - 20)(-1-n/2+1/2) + x_1$
  $\Rightarrow K\,(p_1 - 20) = 180 - p_1$
  where $K = (n + 1)/2$
  $\therefore p_1^* = 180/(K+1) + 20K/(K+1)$

Bertrand competition with infinite number of firms

- $n \to \infty \Rightarrow K \to \infty$ and $1/K \to 0$
- Taking limit of p_1^* as n goes to infinity
  $\lim p_1^* = \lim 20/(1 + 1/K) = 20$
- In this limit, price is equal to marginal cost and profits vanish.
- This limit is monopolistic competition

Differentiated Products

- Product differentiation mutes both types of mover advantage
- A mover disadvantage can be offset by a large enough cost advantage

Two firms in a Bertrand-Stackelberg competition

The demand function faced by firm 1:

$x_1(\mathbf{p}) = 180 - p_1 - (p_1 - \text{average } p)$

$\Rightarrow \quad x_1 = 180 - 1.5p_1 + 0.5p_2$

Similarly, the demand function faced by firm 2:

$x_2 = 180 + 0.5p_1 - 1.5p_2$

Constant average variable cost, $c = \$20$

Two firms in a Bertrand-Stackelberg competition: Determining optimum p_2

Firm 2 wants to maximize its profits, given p_1:

$\max\ (p_2 - 20)(180 + 0.5p_1 - 1.5p_2)$

Profit maximizes when the first order condition is satisfied: $0 = 180 + 0.5p_1 - 3p_2 + 30$

Solving for optimal price p_2, we get

$p_2^* = g(p_1) = 70 + p_1/6$

Two firms in a Bertrand-Stackelberg competition: Equilibrium prices

Knowing that firm 2 will determine p_2 by using $g(p_1)$, firm1 tries to maximize its profit:

$\max\ (p_1 - 20)[180 - 1.5p_1 + 0.5(70 + p_1/6)]$

Profit maximizes when the first order condition is satisfied: $0 = 215 - (17/12)p_1 + (p_1 - 20)(-17/12)$

$\therefore \quad p_1^* = 2920/34 = \$85.88$

Firm 2, which moves last, charges slightly lower price than p_1^*:

$p_2^* = 70 + p_1^*/6 = 70 + \$14.31 = \$84.31$

Two firms in a Bertrand-Stackelberg competition: Profits for the two firms

Firm 1 sells less than firm 2 does:

$x_1^* = 93.34$ and $x_2^* = 96.48$

Firm 1's profit, $u_1^* = (93.34)(85.88 - 20) = \$\ 6149.24$

Firm 2's profit, $u_2^* = (96.48)(84.31 - 20) = \$\ 6204.63$

Firm 2, the second mover, makes more money

This Offer is Good for a Limited Time Only

- The credibility problems behind the marketing slogan
- The principle of costly commitment
- Industries where the slogan is credible

An example of "This offer is good for a limited time only"

- Exploding job offers
  - An early job offer with a very short time to decide on whether to take the job.
  - Risk-averse people often end up accepting inferior job offers

Ultimatum Games in the Laboratory

- Games with take-it-or-leave-it structure
- In experiments, subjects playing such games rarely play subgame perfect equilibria
- The nice opponent explanation vs. the expected payoff explanation