Preliminaries

Everything You Always Wanted to Know About Math*

* But were afraid to ask.

Outline

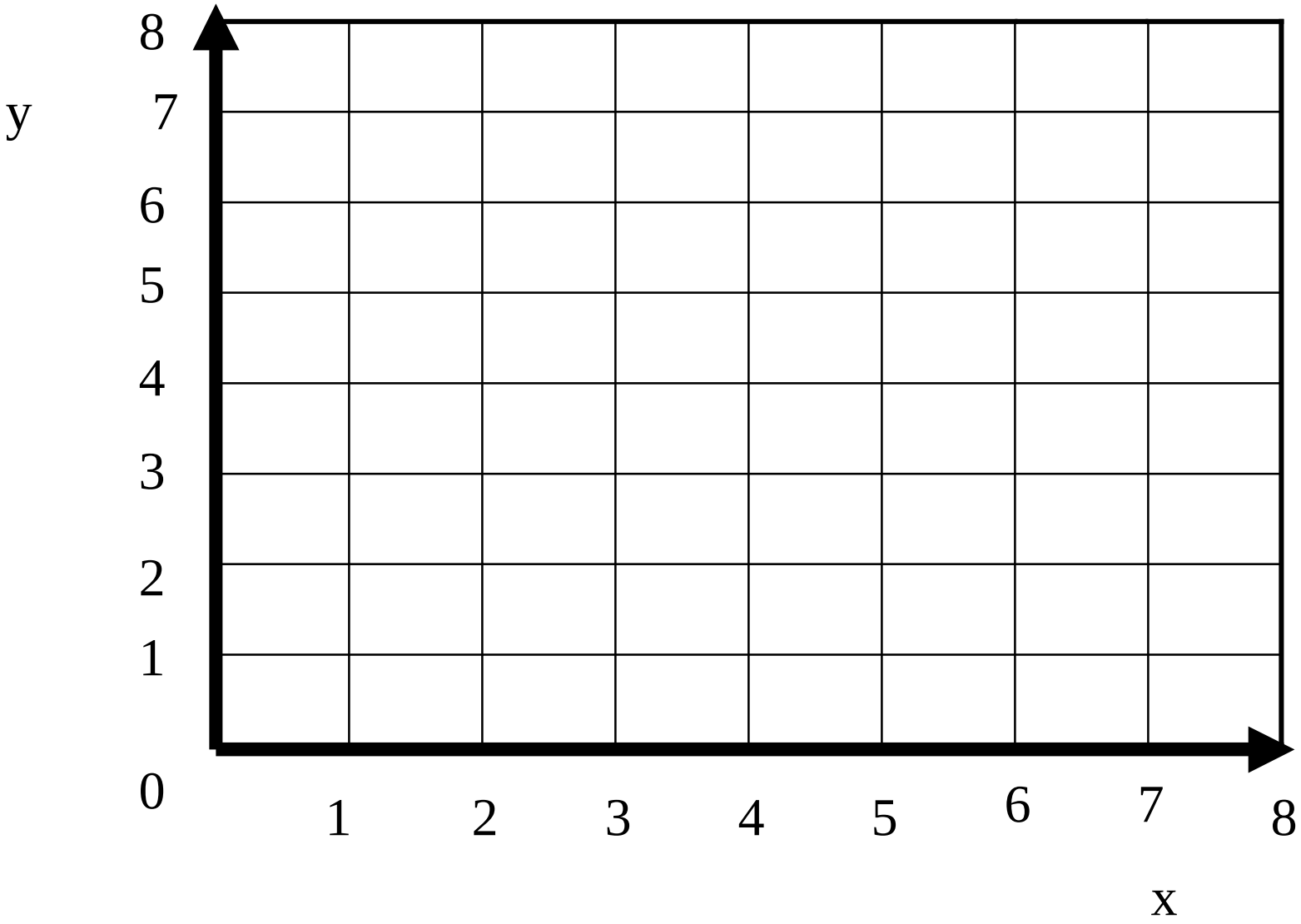
1. Graphs and Equations
2. Depicting and Solving Systems of Equations
3. Levels, Changes and Percentage Change
4. Non-linear relationships and Elasticities

Graphs and Equations

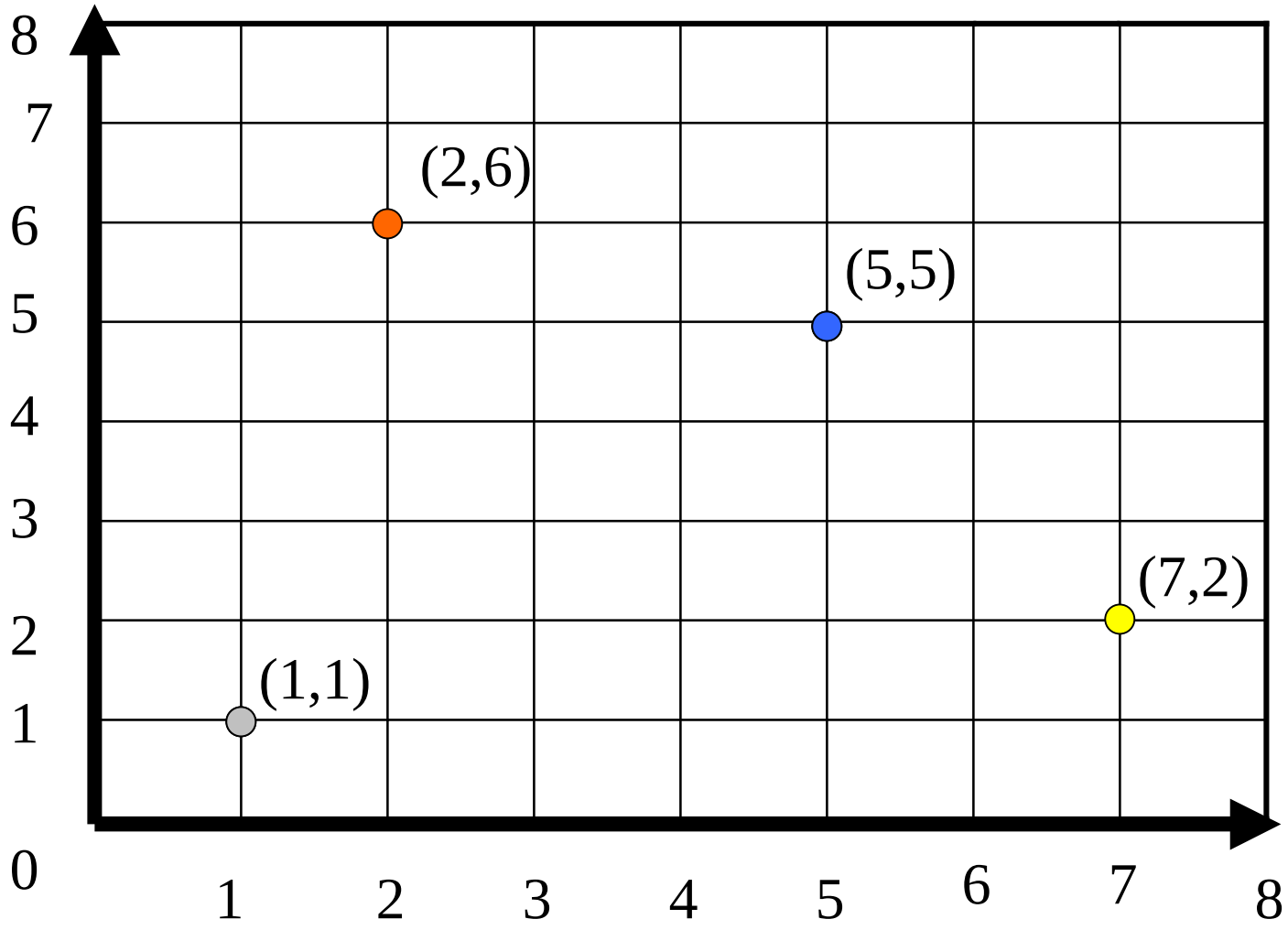
Depicting 2-dimensional relationships

1. Depicting the association between pairs of variables using a Cartesian Plane.
2. Depicting Bivariate (2-variable) functions.

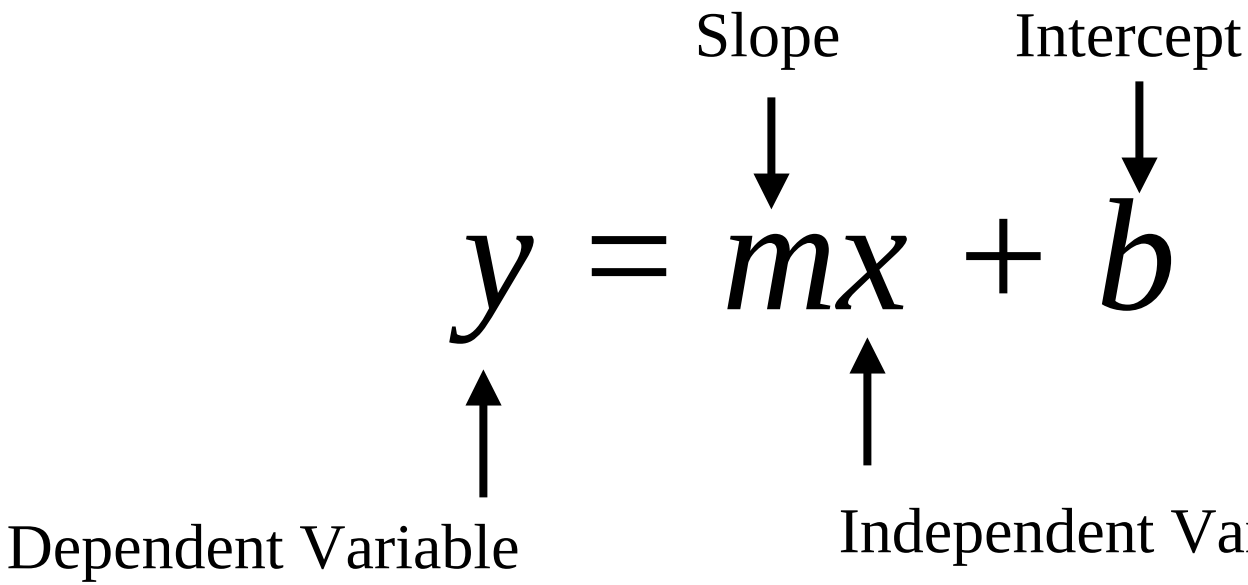
The Cartesian Plane



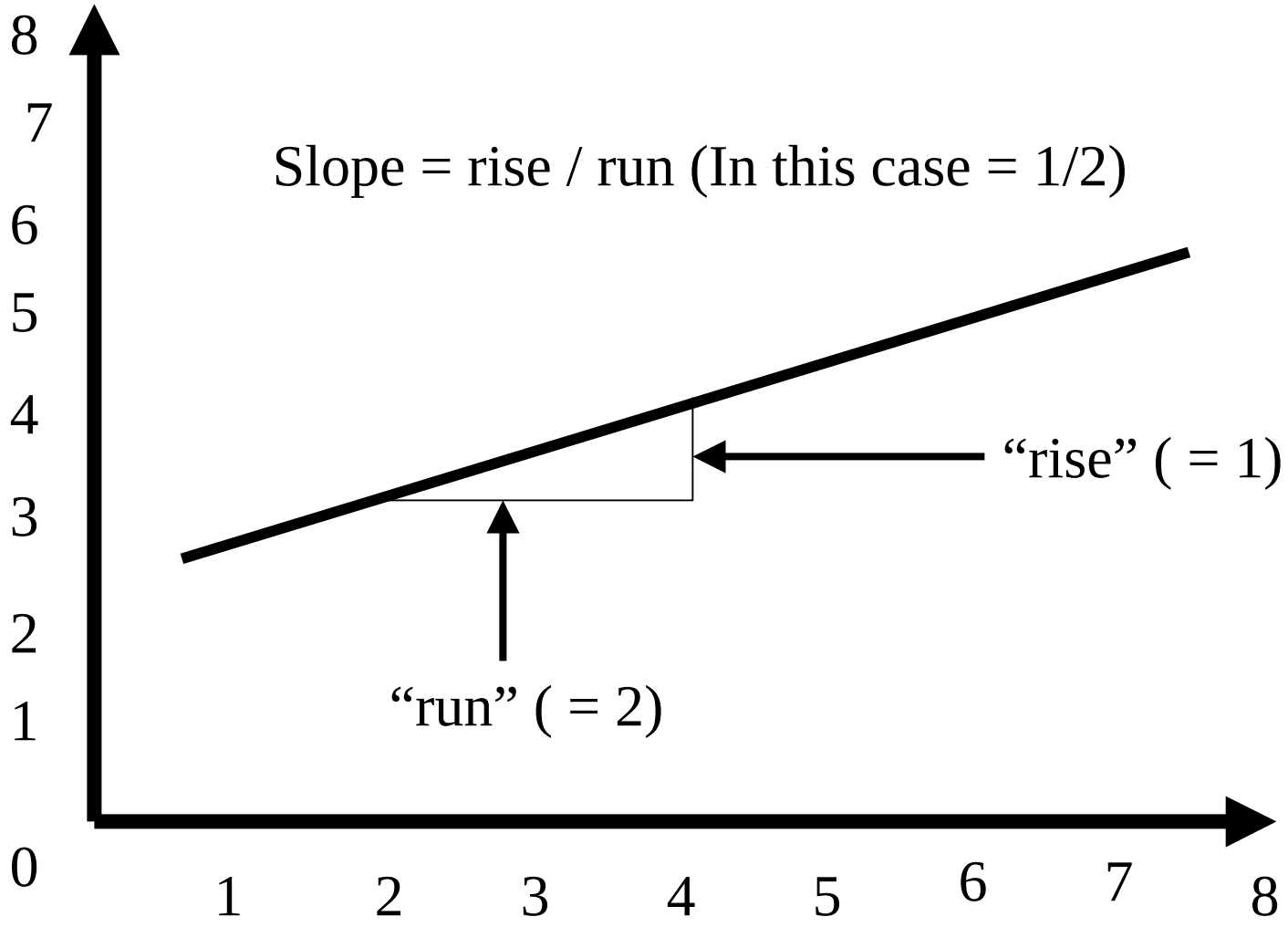
Points in a Cartesian Plane



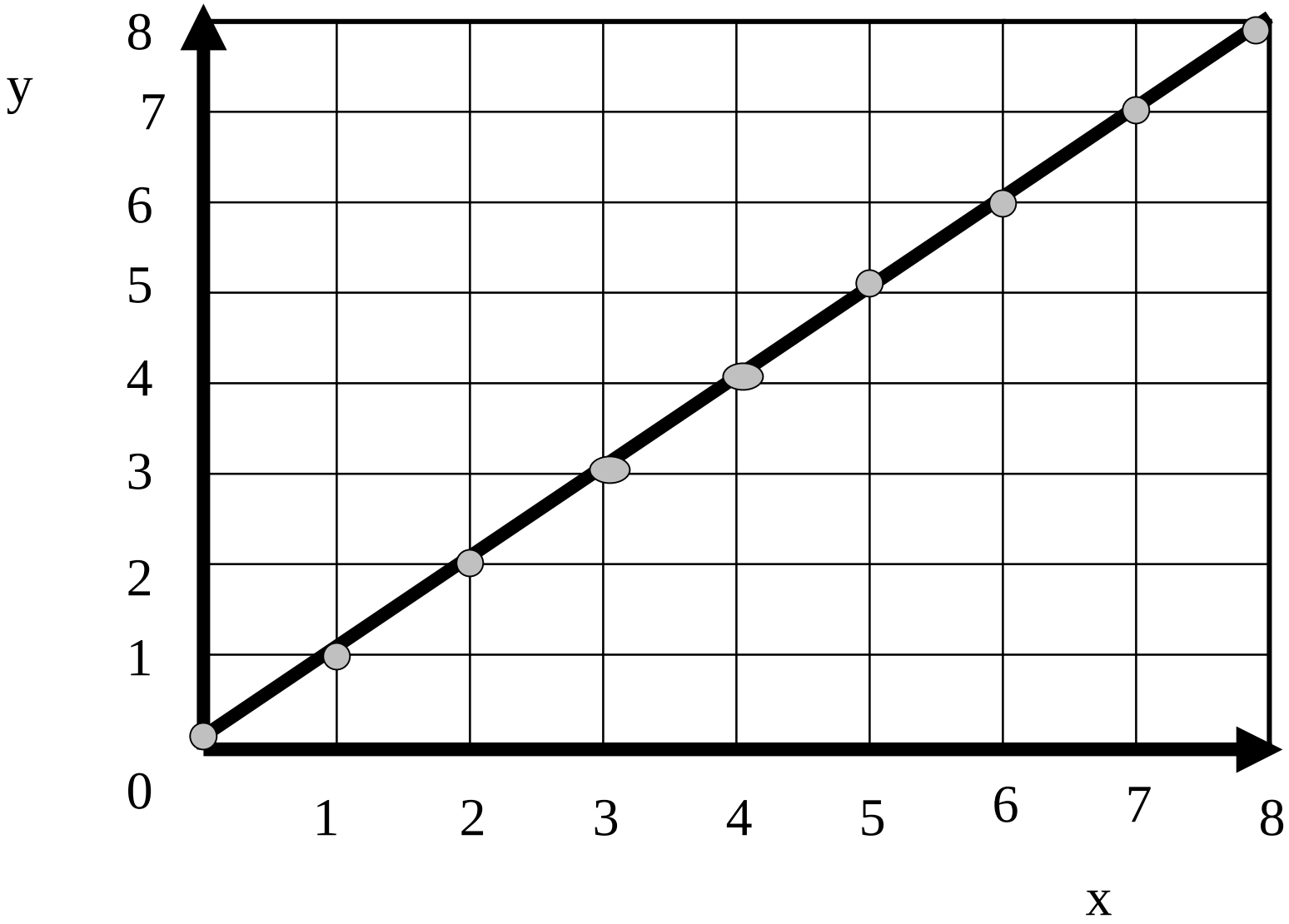
Equation for a straight-line function



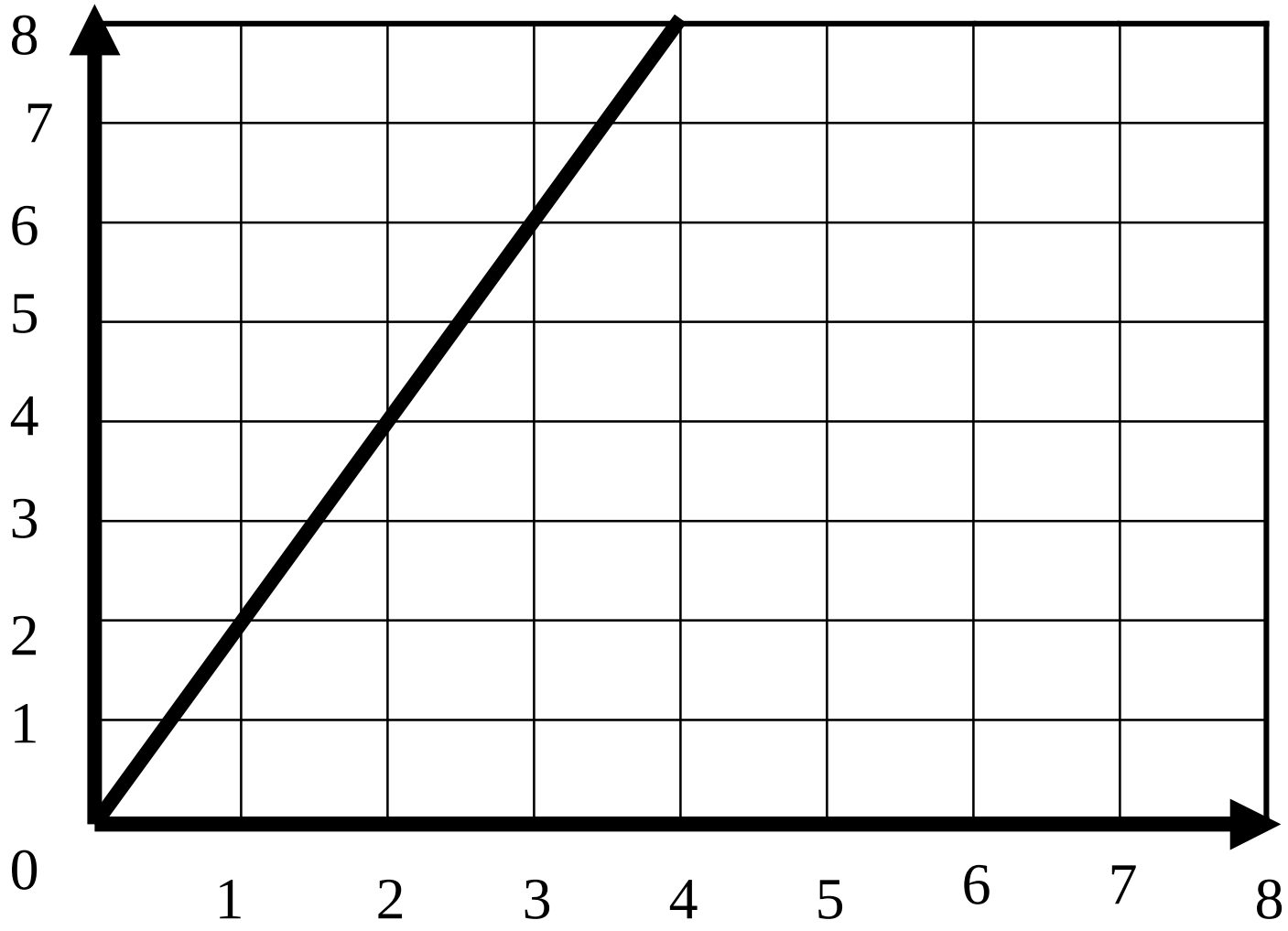
Examples:	**Slope**	**Intercept**
$y = x$	1	0
$y = 3 + 0.25\,x$	0.25	3
$y = 6 - 2\,x$	- 2	6


Slope = rise / run (In this case = 1/2)
“rise” (= 1)
“run” (= 2)
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8

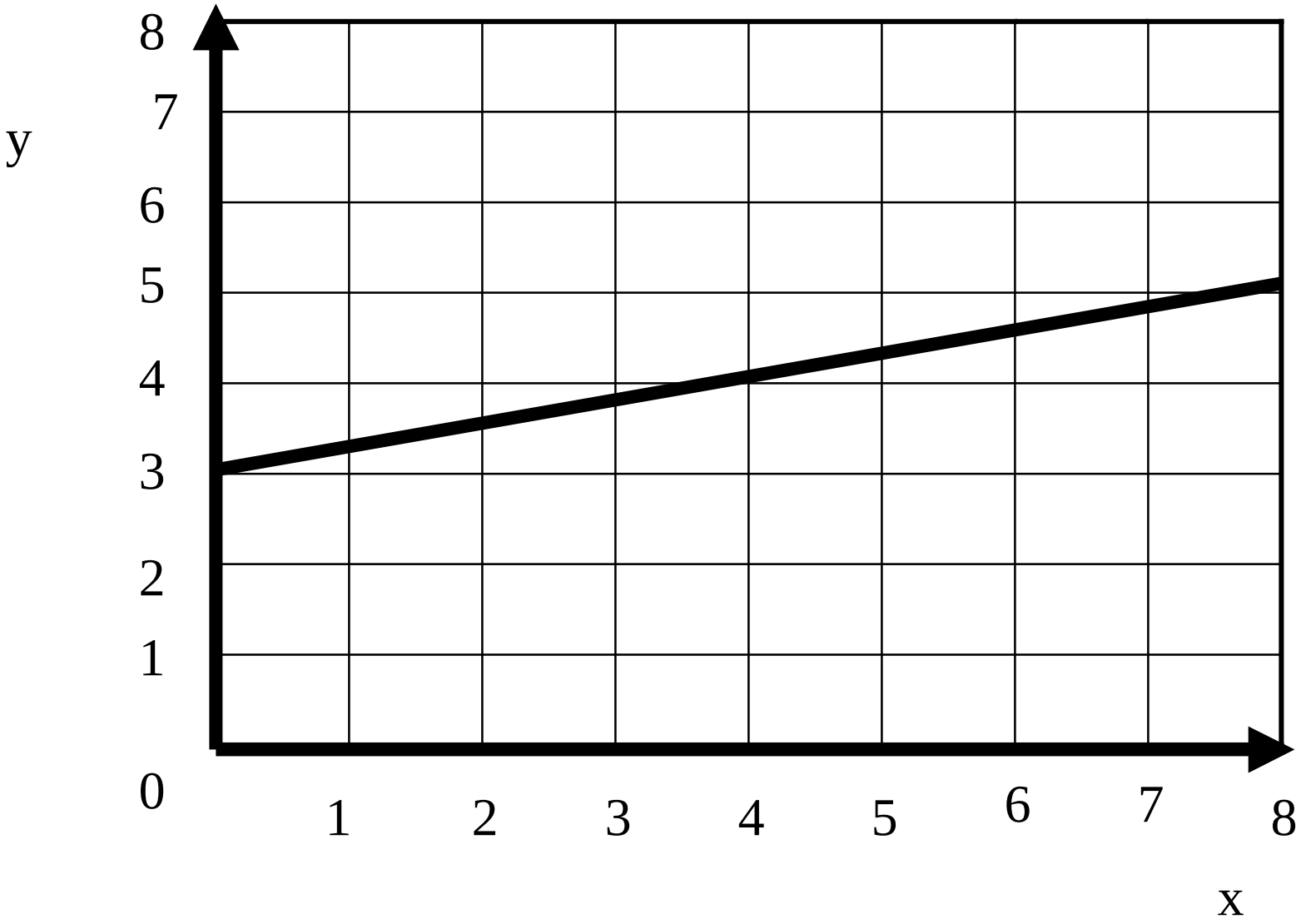
Plotting the Function $y = x$



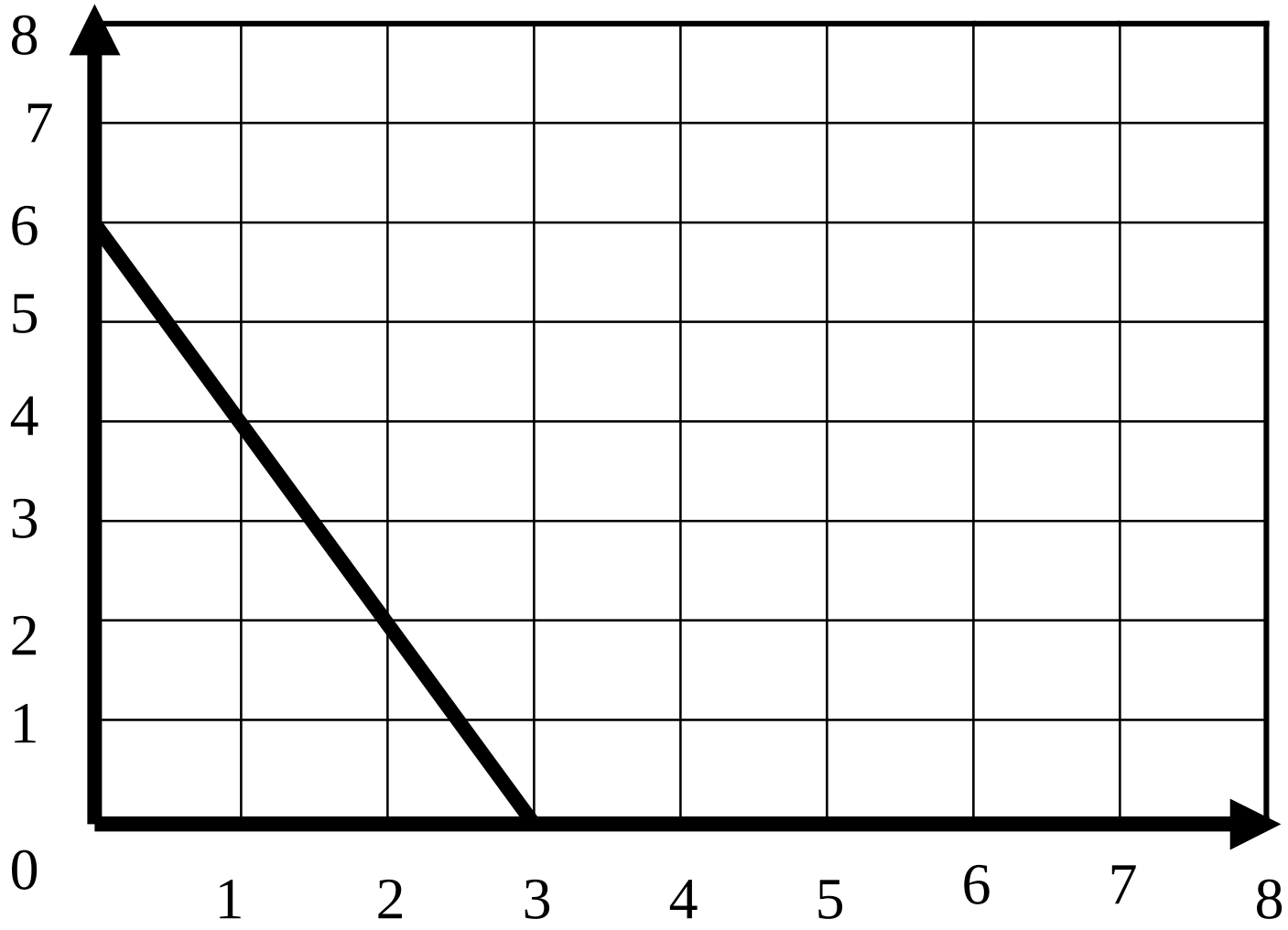
Plotting the Function $y = 2x$



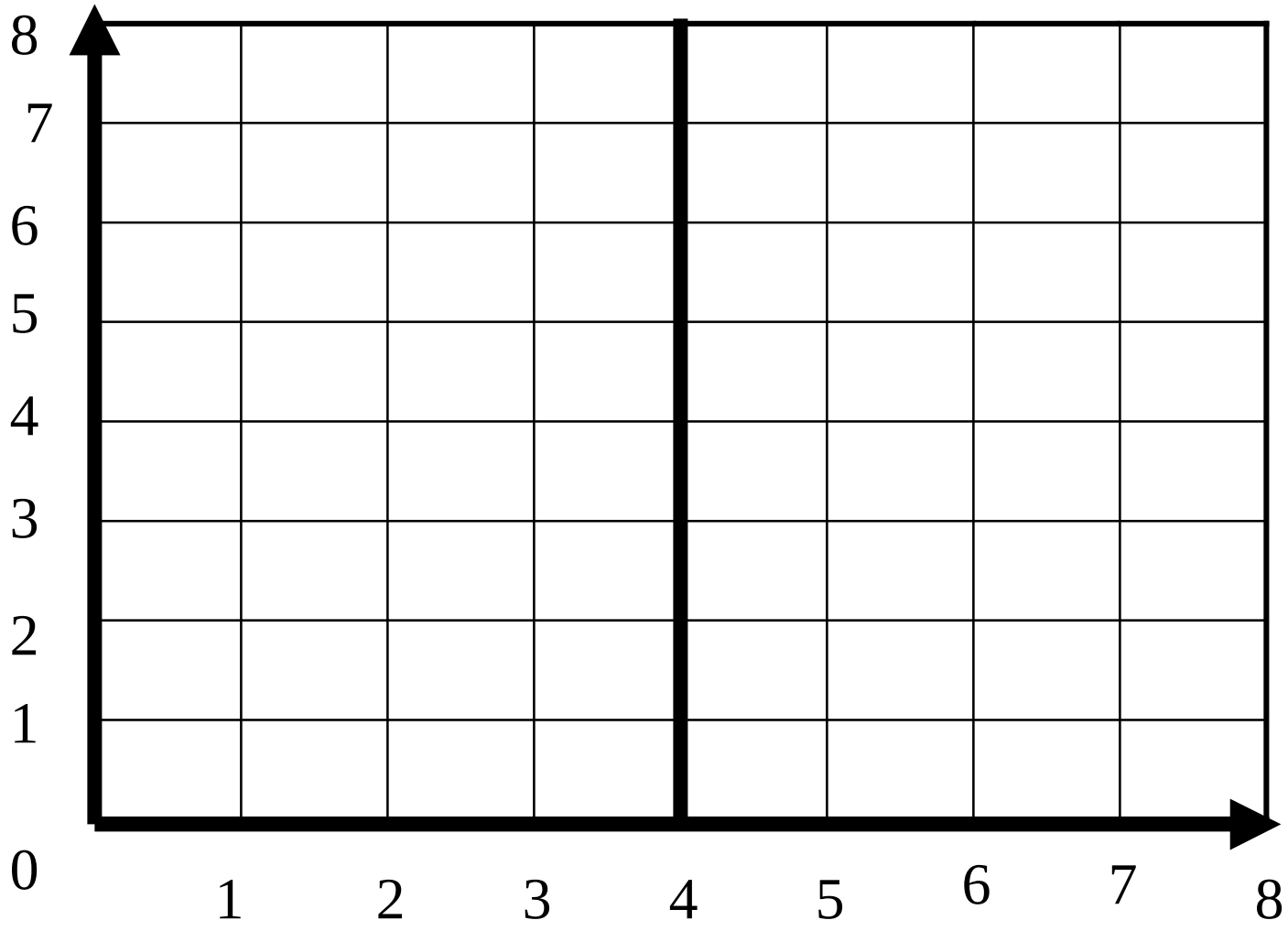
Plotting the Function $y = 3+0.25x$



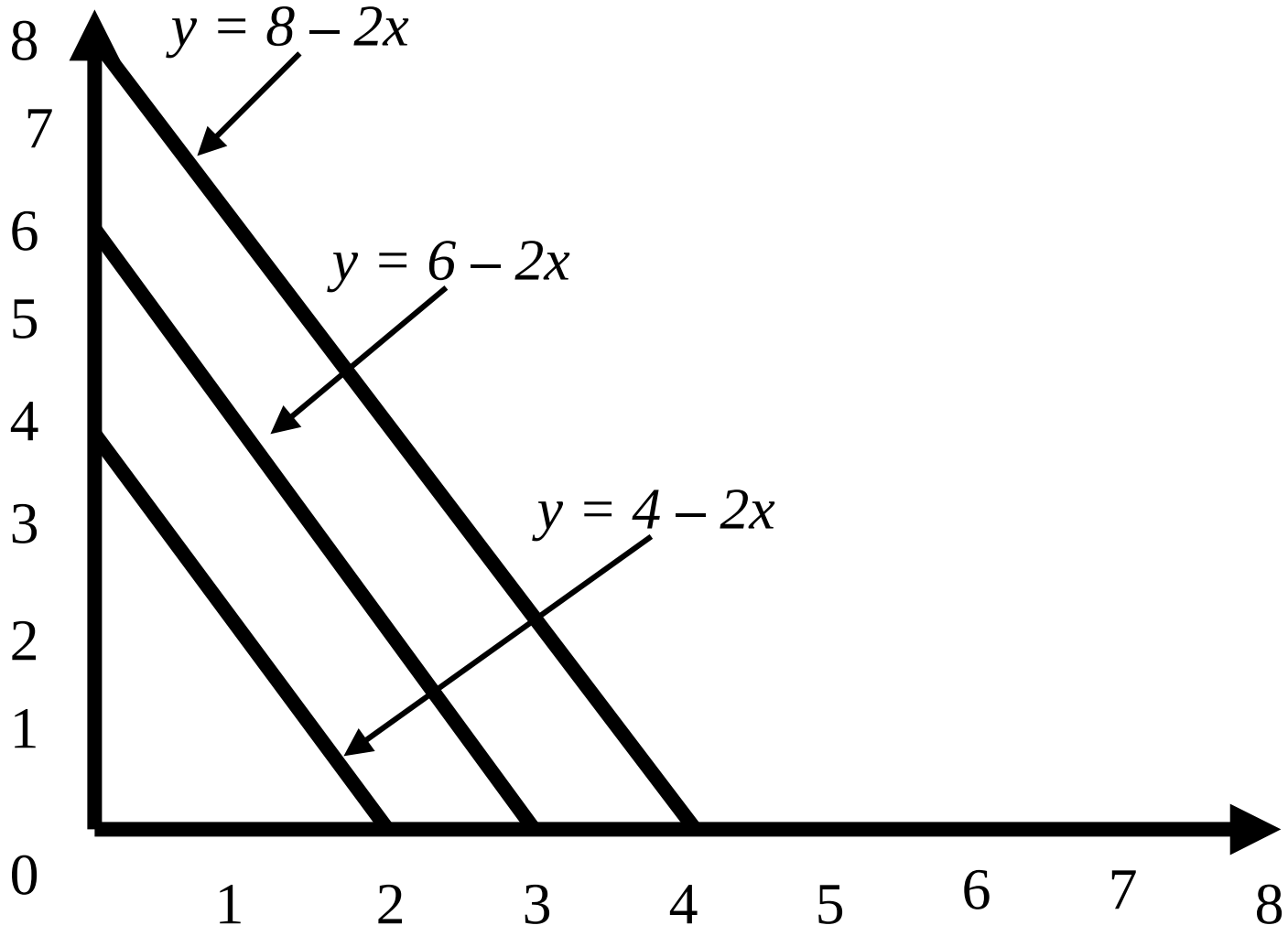
Plotting the Function $y = 6 - 2x$



Plotting the Function $x = 4$



Shifting Lines: Changing the Intercept



Systems of Equations

Solving Systems of Equations

$$y = 6 - 2x$$

$$y = 3 + x$$

1. Solve out for y

$$6 - 2x = 3 + x$$

2. Isolate x

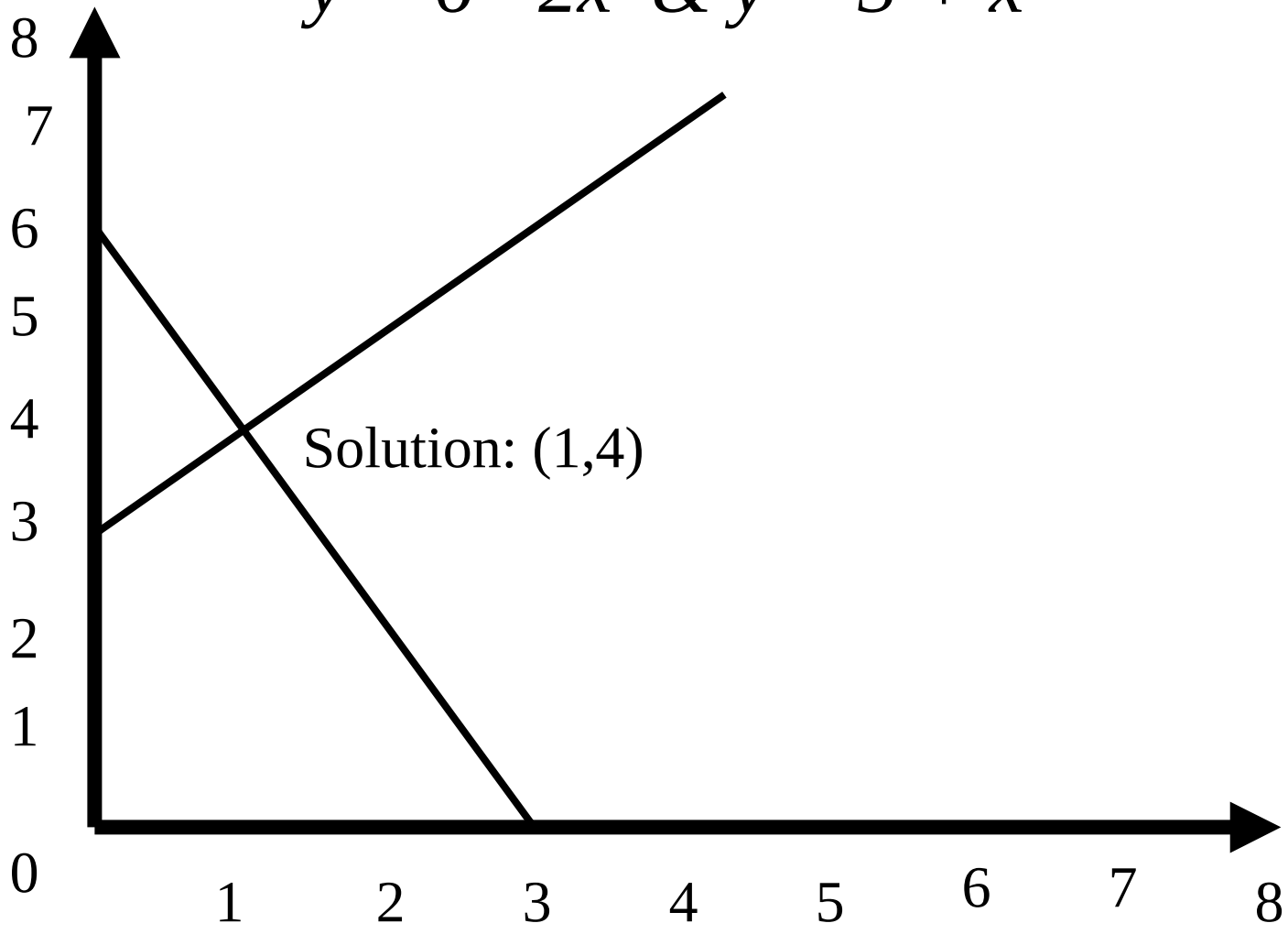
$$3 = 3x \quad so \quad x = 1$$

3. Solve for y using either equation:

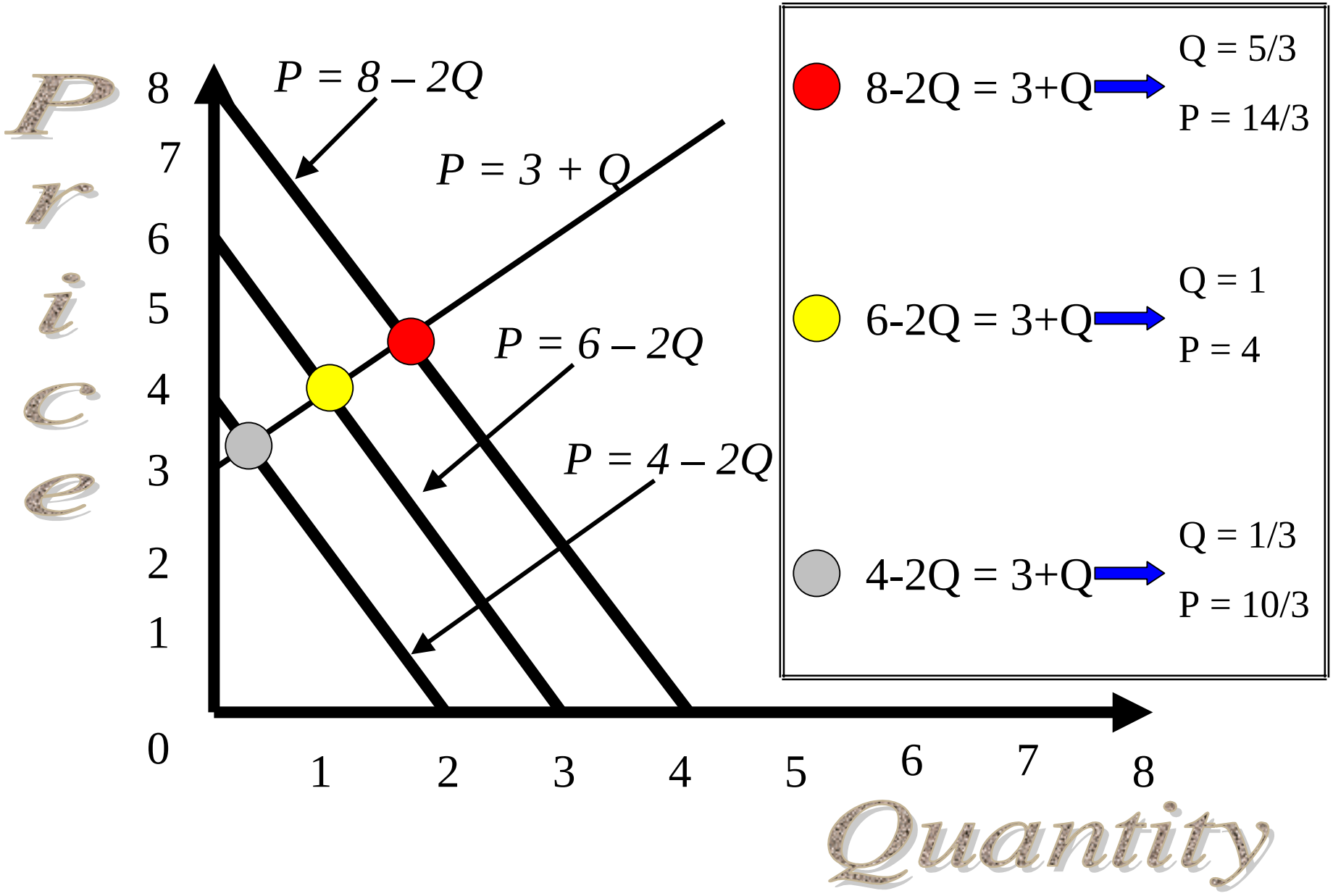
$$y = 6-2 = 3+1 = 4$$

Solving Systems of Equation

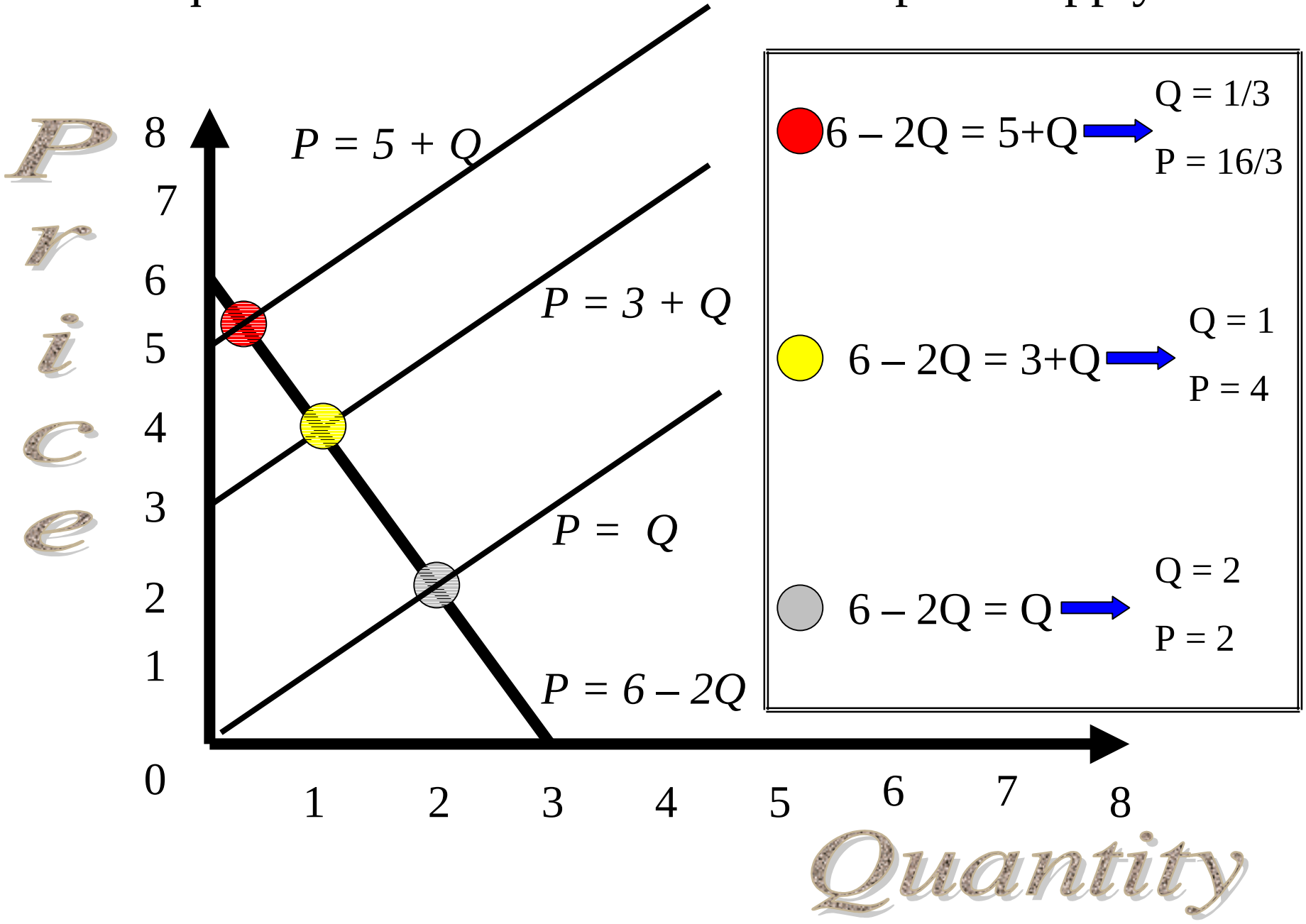
$$y = 6 - 2x \;\; \& \; y = 3 + x$$



Equilibriums with Different Intercepts: “Demand Shift”



Equilibriums with Different Intercepts: “Supply Shift”



Percent Changes

Levels, Changes, and Percentage Changes

Level	Change	Percentage Change
x_t	$\Delta x_t = x_t - x_{t-1}$	$\%\Delta x_t = 100[(x_t - x_{t-1}) / x_{t-1}]$
100		
120	20	20%
140	20	16.67%

Formula for Percentage Change

$$\%\Delta x_t = 100[(x_t - x_{t-1}) / x_{t-1}]$$

$$= 100[(x_t / x_{t-1}) - 1]$$

Some General Rules

For $z = xy$, with small percentage changes

$$\%\Delta z \cong \%\Delta x + \%\Delta y$$

For $z = y/x$, with small percentage changes

$$\%\Delta z \cong \%\Delta y - \%\Delta x$$

Examples

$x_t = 10$	$x_{t+1} = 11$	$\%\Delta\, x_t = 10\%$
$y_t = 20$	$y_{t+1} = 24$	$\%\Delta\, y_t = 20\%$

z = xy $z_t = 200$ $z_{t+1} = 264$

$\%\Delta\, z_t = 100([264/200]\text{-}1) = 32\ \%$

z = y/x $z_t = 2$ $z_{t+1} = 2.18182$

$\%\Delta\, z_t = 100([2.18182\ /2]\text{-}1) = 9.091\ \%$

Elasticities

Non-Linear Relationships

- May want to associate percentage change with percentage change, rather than change with change.

$y = 2x + 3 \Rightarrow \Delta y = 2\,\Delta x$, <u>but not</u> $\%\Delta y = 2\%\,\Delta x$

- One function that relates $\%\Delta y$ to a constant $\%\,\Delta x$ takes the form

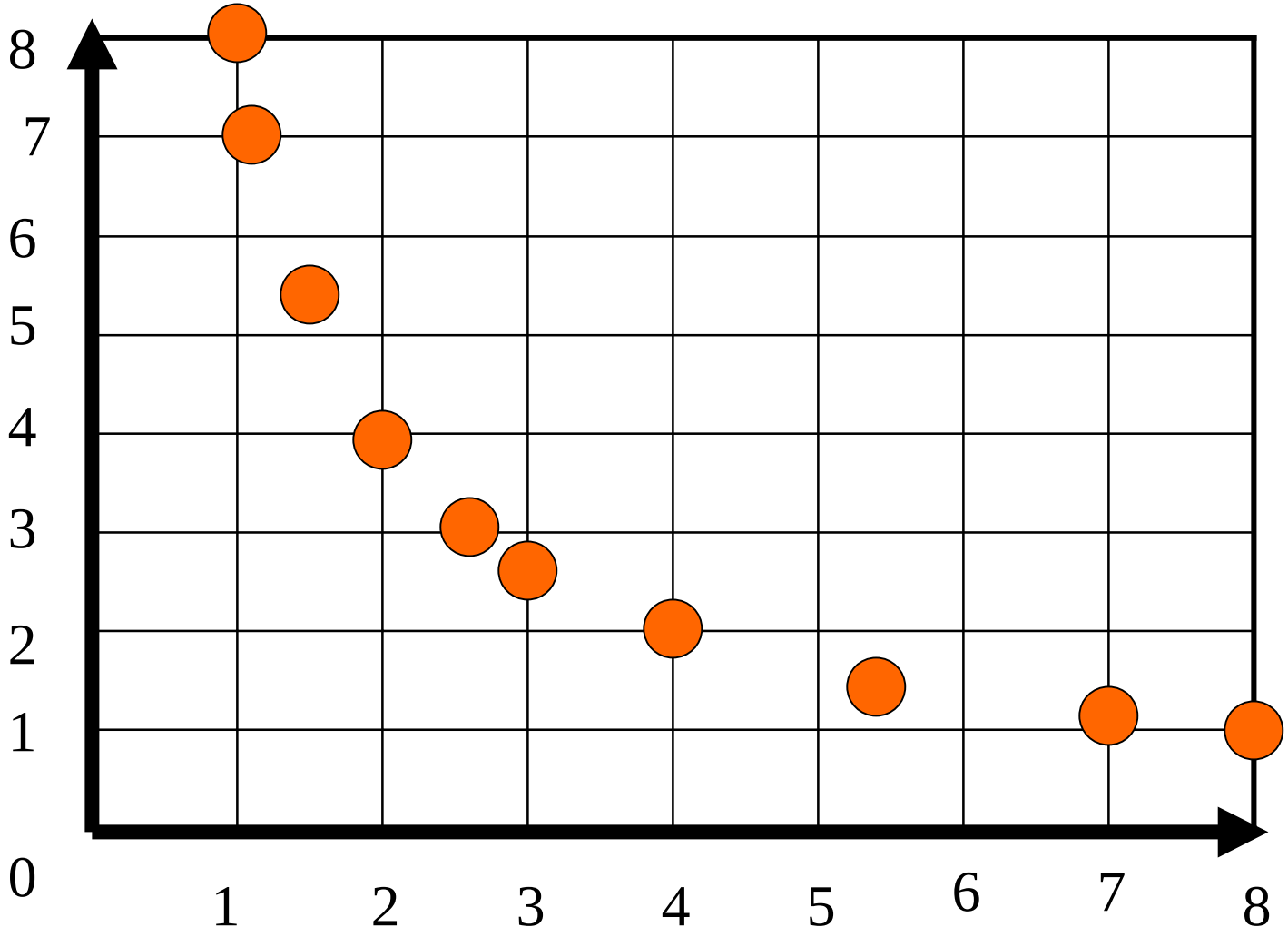
$$y = bx^a$$

Where a & b are constant parameters

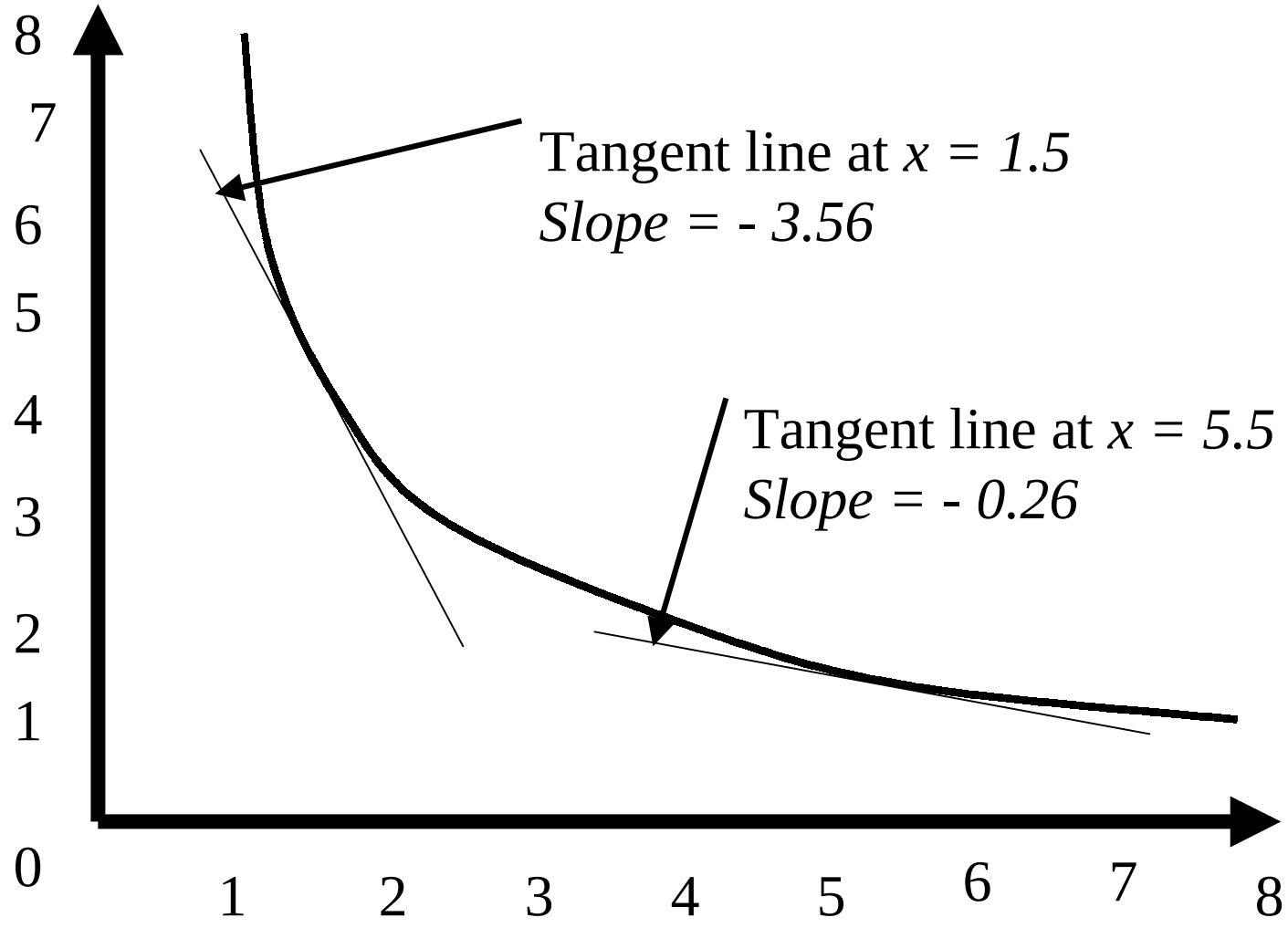
Rules of Exponents

Rule	Example
$x^0 = 1$	$2^0 = 1$
$x^1 = x$	$2^1 = 2$
$x^{-1} = 1 / x$	$2^{-1} = 1 / 2$
$(x^a)^b = (x^b)^a = x^{ab}$	$(2^1)^3 = (2^3)^1 = 8$
$x^a x^b = x^{a+b}$	$2^2 2^3 = 2^5 = 32$
$x^a / x^b = x^{a-b}$	$2^3 / 2^2 = 2^1 = 2$
$x^a y^a = (xy)^a$	$2^2 3^2 = 6^2 = 36$
$x^a / y^a = (x/y)^a$	$4^2 / 2^2 = (4/2)^2 = 4$
$x^{1/a} = \sqrt[a]{x}$	$9^{1/2} = \sqrt{9} = 3$

Plotting $y = 8 / x$



A Tangent Line to a Hyperbola Shows the Slope at a Point



Elasticities

An elasticity relates the percent change in one variable to the percent change in another variable;

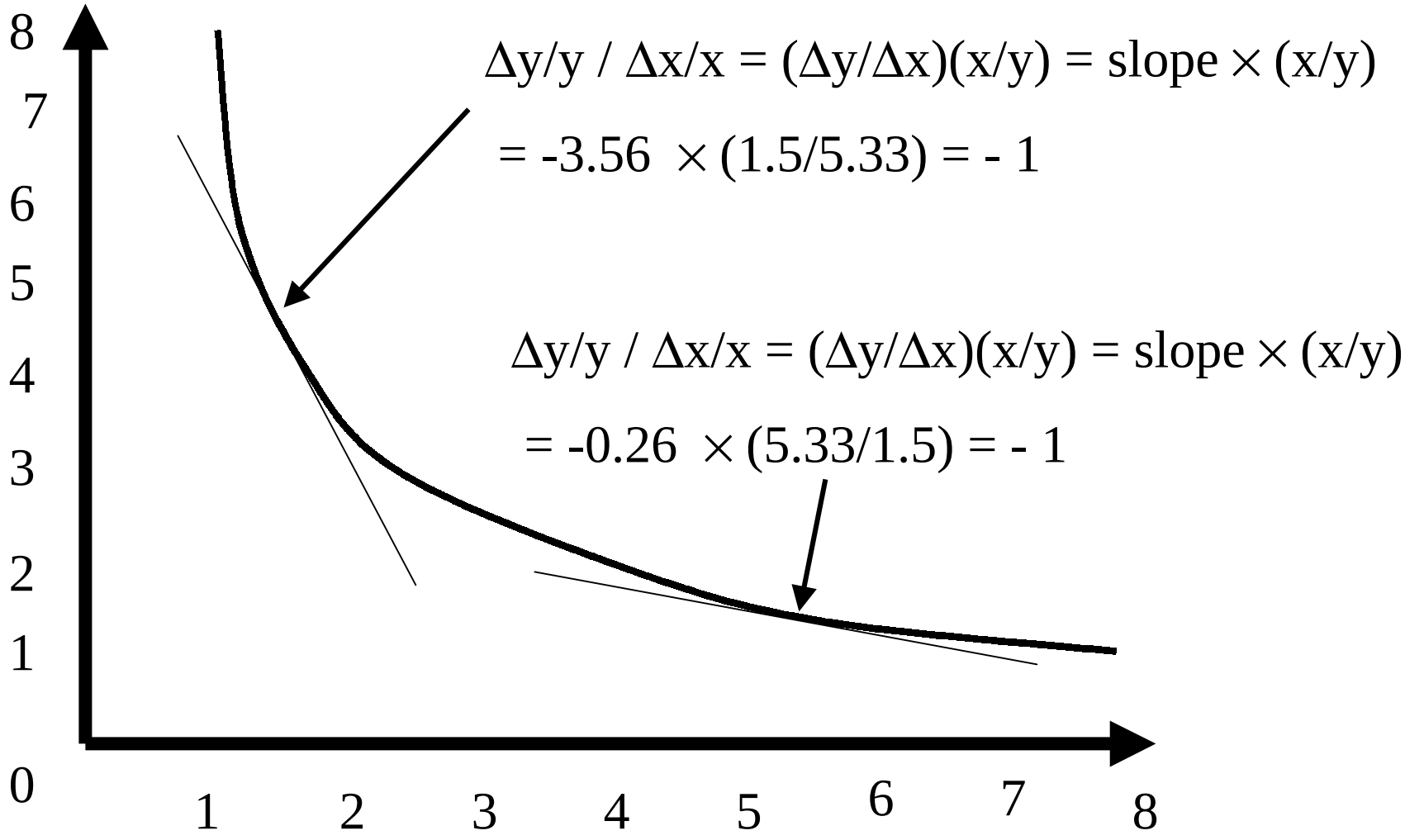
Elasticity between x & y

$$= \%\Delta y / \% \Delta x$$

$$= (\Delta y / y) / (\Delta x / x)$$

$$= (\Delta y / \Delta x) \times (x / y)$$

Constant Elasticities with a Hyperbola



Exercises

1. In a Cartesian plane, plot the following points: (0,5), (4,2), (6,1), (3,3)
2. Graph the following linear equations
   1. $y = 2x + 3$
   2. $y = 21 - 4x$
3. Solve the system of two equations given by the equations in question (2) above. Also solve the system for the case where equation 2.*1* changes to $y = 15 - 4x$ and show how this change in equation 2.*1* is represented in a graph.
4. Graph the equation $y = x^{0.5}$. Calculate the percentage change in the dependent variable between the points where $x=4$ and $x=4.41$. Determine the elasticity between these two points.