Econ 300, Problem Set 4, Suggested Answers
Professor Cramton

8.4.6.

(a)

$$\frac{dy}{dx} = -\frac{2x + (1/3)x^{-2/3}y^{1/3}}{2y + (1/3)y^{-2/3}x^{1/3}}$$

(b)

$$\frac{dy}{dx} = -\frac{2xy + y^2 + y}{x^2 + 2yx + x}$$

(c)

$$\frac{dy}{dx} = -\frac{(3/x) - 2xy^2}{2x^2y - 4}$$

(d)

$$\frac{\partial y}{\partial x} = -\frac{3x^2 + wy}{3w^3y^2 + wx}$$

(e)

$$\frac{dy}{dx} = -\frac{y}{x(2+y)}$$

9.1.6.

A quadratic function has only one bend and is either concave or convex. The second derivative is constant because $f'(x)$ is linear, therefore, there is no inflection point since the second derivative never changes sign.

9.2.10.

$$\Pi = p\alpha L^{\beta} - (w + b)L$$

The derivative of the profit function

$$\frac{d\Pi}{dL} = \beta p\alpha L^{\beta-1} - (w + b)$$

Optimal demand for labor is derived from setting $\frac{d\Pi}{dL} = 0$.

$$L^* = \left(\frac{w+b}{\beta\alpha p}\right)^{\frac{1}{\beta-1}}$$

In order to see how labor demand changes with b; we take the derivative of the optimal labor demand function with respect to b:

$$\frac{dL}{db} = \frac{1}{\beta - 1}\left(\frac{w+b}{\beta\alpha p}\right)^{\frac{2-\beta}{\beta-1}} < 0$$

The higher benefits the firm has to pay to its workers, the lower demand the firm has for workers.

10.1.2. and 10.2.2.

(a) First order conditions:

$$g_u = 20 - 4u - 2v = 0$$

$$g_v = 16 - 2v - 2u = 0$$

There's a unique stationary point $(u^*, v^*) = (2, 6)$

Second order conditions:

$$g_{uu} = -4$$

$$g_{vv} = -2$$

$$g_{uv} = -2$$

$$g_{uu} * g_{vv} > g_{uv}^2$$

This point is a Maximum.

(b) First order conditions:

$$g_u = -5 + 8u + 8v = 0$$

$$g_v = -9 + 10v + 8u = 0$$

There's a unique stationary point $(u^*, v^*) = (-11/8, 2)$

Second order conditions:

$$g_{uu} = 8$$

$$g_{vv} = 10$$

$$g_{uv} = 8$$

$$g_{uu} * g_{vv} > g_{uv}^2$$

This point is a Minimum.

(c) First order conditions:

$$g_u = u^2 + 3v + 2 = 0$$

$$g_v = 3u - 3v = 0$$

$$g_{uu} = 2u$$

$$g_{vv} = -3$$

$$g_{uv} = 3$$

There are two stationary points (-2,-2) and (-1,-1). Point (-2,-2) is a local maximum and the other point (-1,-1) is a saddle point since

$$g_{uu} * g_{vv} < g_{uv}^2.$$

(d) First order conditions:

$$g_u = 24u^{-2/3}v^{1/3} - 6 = 0$$

$$g_v = 24u^{1/3}v^{-2/3} - 3 = 0$$

$$g_{uu} = -16u^{-5/3}v^{1/3}$$

$$g_{vv} = -16u^{1/3}v^{-5/3}$$

$$g_{uv} = 8u^{-2/3}v^{-2/3}$$

The stationary point is $(u^*, v^*) = (128, 256)$. This point is a maximum. An easier way to solve for the stationary point is to write

$$24u^{-2/3}v^{1/3} = 6$$

$$24u^{1/3}v^{-2/3} = 3$$

Then divide the first equation by the second to get $\frac{v}{u} = \frac{6}{3} = 2$ and use this condition to solve for (u^*, v^*).