Econ 300, Problem Set 3, Suggested Answers
Professor Cramton

7.2.2.

(a)

$$\frac{df(x)}{dx} = \frac{2(x^2-1)-(2x+7)2x}{(x^2-1)^2} = \frac{-2x^2-14x-2}{(x^2-1)^2}$$

(b)

$$\frac{df(x)}{dx} = \frac{2bx^3+cx^2+4}{x^2}$$

(c)

$$\frac{df(x)}{dx} = \frac{2e^{2x}(x-1)}{x^3}$$

(d)

$$\frac{df(x)}{dx} = \frac{9x^2-4}{x^2}$$

7.2.8.

First write $Q(t)$ as a function of A_0, K_0, L_0 and t:

$$Q(t) = A_0 K_0^{\alpha} L_0^{1-\alpha} e^{[c+\alpha f+(1-\alpha)g]t}$$

Then write this expression in natural logarithms to get:

$$lnQ(t) = lnA_0 K_0^{\alpha} L_0^{1-\alpha} + [c+\alpha f+(1-\alpha)g]t$$

Finally, differentiate this last expression with respect to t to get the growth rate of output

$$\frac{dlnQ(t)}{dt} = c+\alpha f+(1-\alpha)g$$

7.2.14.

The assumed model for effective labor input L is given by

$$L = 10(1-e^{-0.1t})+2$$

Differentiating this with respect to t gives

$$\frac{dL}{dt} = 10*(-1)(-0.1)e^{-0.1t} = e^{-0.1t} > 0$$

The assumed model implies a positive but decreasing marginal change in the effective labor input with respect to time on the job (as t gets larger, L gets larger too but $\frac{dL}{dt}$ gets smaller).

7.3.4.

The first derivative of the function is $f'(x) = x^2 - 8x + 6$ and the second derivative is $f''(x) = 2x - 8$. Over the interval [0, 4], you can check that $f''(x)$ will always be negative (it will be between -8 and 0). Hence, the function is concave over the given interval.

Confirm the result with the definition of concavity, setting $\lambda = 0.4, x^A = 0$ and $x^B = 4$. First compute $f(0.40 + (1 - 0.4) * 4) = f(2.4) = 3.968, f(0) = 8, f(4) = -10.667$. Then, check that $3.968 \geq 0.4 * 8 + 0.6 * (-10.667) = -3.2$.

7.3.8.

(a) The first derivative of the utility function (marginal utility) is given by

$$U'(c) = a\beta e^{\beta c}$$

and the second derivative is given by

$$U''(c) = a\beta^2 e^{\beta c}$$

If we want to have utility rising at a decreasing rate with the number of cookies, we want $U'(c) = a\beta e^{\beta c} > 0$ and $U''(c) = a\beta^2 e^{\beta c} < 0$. Hence we need $a\beta > 0$ and $a\beta^2 < 0$. This require $a < 0$ and $\beta < 0$ (note that nothing is required for Ψ).

(b) The first derivative of the utility function (marginal utility) is given by

$$U'(c) = \theta\lambda c^{\lambda-1}$$

and the second derivative is given by

$$U''(c) = \theta\lambda(\lambda - 1)c^{\lambda-2}$$

If we want to have utility rising at a decreasing rate with the number of cookies, we want $U'(c) = \theta\lambda c^{\lambda-1} > 0$ and $U''(c) = \theta\lambda(\lambda-1)c^{\lambda-2} < 0$. Two different parameter constellations will satisfy these requirements. First, $\theta < 0$ and $\lambda < 0$ will work. Second, $\theta > 0$ and $0 < \lambda < 1$ will work too.

7.3.10.

The coefficient of absolute risk aversion is given by

$$\theta = -\frac{U''(c)}{U'(c)}$$

For $U(c) = a\sqrt{c}$, we have $U'(c) = a\frac{1}{2}c^{-1/2}$ and $U''(c) = -a\frac{1}{4}c^{-3/2}$. Thus the coefficient of absolute risk aversion is given by

$$\theta = -\frac{-a\frac{1}{4}c^{-3/2}}{a\frac{1}{2}c^{-1/2}} = \frac{1}{2c}$$

For $U(c) = \beta \ln(c)$, we have $U'(c) = \beta c^{-1}$ and $U''(c) = -\beta c^{-2}$. Thus the coefficient of absolute risk aversion is given by

$$\theta = -\frac{-\beta c^{-2}}{\beta c^{-1}} = \frac{1}{c}$$