Econ 300, Problem Set 1, Suggested Answers
Professor Cramton

2.1.2.

(a) Yes, this is a function. It maps each x in the domain into one and only one y in the range.

(b) No, this is not a function. It maps each x in the domain into more than one y. For instance, for x = 2, we can have y = 1 but also y = 0 or any other number smaller or equal to 2.

(c) Yes, this is a function. It maps each x in the domain (which is restricted to the interval $(0, \infty)$ into one and only one y in the range.

(d) Yes, this is a function. It maps each x in the domain into one and only one y in the range. Note however that it is not a invertible function (it is not one-to-one).

(e) No, this is not a function. For any strictly positive x, we have two associated y's, while for any strictly negative x, there is no y fulfilling the condition.

(f) Yes, this is a function, but we need to exclude x = 3 from its domain. This is because $\frac{1}{x-3}$ is not defined for x = 3.

(g) No, this is not a function. For any x in the domain, there will be two associated y's (for example for x = 2, we have y = 4 and y = -4).

2.1.8.

(a) This function is NOT continuous over the interval $(0, \infty)$ because it has a point of discontinuity at x = 7.

(b) This function is continuous over the interval $(-\infty, \infty)$. It is simply a linear function (it can also be written as $y = \frac{4}{7} - \frac{a}{7}x$).

(c) This function is continuous over the interval $(0, \infty)$. The point of discontinuity at x = -7 is not a problem since it is not in the given interval.

(d) This function is continuous over the interval [-3, 3]. Although the function would not be differentiable at the point x = 2, there is no problem of discontinuity.

2.2.8.

Take any two a and b in the domain of the function $(a, b \in X)$ such that $a < b$.

When the function is strictly increasing, we know that $f(a) < f(b)$. Hence, the differences $\Delta y = f(b) - f(a)$ and $\Delta x = b - a$ are both positive. It follows that $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} > 0$.

When, on the other hand, the function is strictly decreasing, we know that $f(a) > f(b)$. Hence,$\Delta y = f(b) - f(a)$ is negative while $\Delta x = b - a$ is positive. It follows that $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} < 0$.

2.2.10.

The graph of the function $y = 8 + 10x - x^2$ over the domain $[0, 7]$ is represented in figure 1 below.

(a) The value x' is given by $x' = \lambda x_A + (1 - \lambda)x_B = 0.4 * 1 + 0.6 * 4 = 2.8$. The value of $f(x')$ is

$f(x') = f(2.8) = 8 + 10 * 2.8 - 2.8^2 = 28.16$.

(b) First observe that $f(1) = 8 + 10 * 1 - 1^2 = 17$

$f(4) = 8 + 10 * 4 - 4^2 = 32$. Then compute the value of y' as

$y' = \lambda f(x_A) + (1 - \lambda) f(x_B) = 0.4f(1) + 0.6f(4) = 0.4 * 17 + 0.6 * 32 = 26$.

(c) We found above $f(x') = 28.16$ and $y' = 26$. Thus, we have $f(x') > y'$ since $28.16 > 26$, which says that the function is strictly concave.

2.3.2.

(a)

$$x^a \cdot x^b \cdot x^c \div x^d = x^{(a+b+c-d)}$$

(b)

$$x^{1/2}x^{3/2} \div x^{1/3} = x^{(\frac{1}{2}+\frac{3}{2}-\frac{1}{3})} = x^{(2-\frac{1}{3})} = x^{\frac{5}{3}}$$

(c)

$$((x^{1/3})^8)^{1/2} \cdot x^2 \div x^{3/4} = x^{((\frac{1}{3})(8)(\frac{1}{2})+2-\frac{3}{4})} = x^{(\frac{4}{3}+2-\frac{3}{4})} = x^{(\frac{16}{12}+\frac{24}{12}-\frac{9}{12})} = x^{\frac{31}{12}}$$

(d) This question was not graded because too confusing.

$$x^2y^3 \cdot x^3 \div xy^2 \cdot x^{-2}y = x^{2+3-1-2}y^{3+2+1} = x^2y^6$$

2.3.12.

(a) To find the tax rate(s) consistent with raising tax revenues equal to 60 billion, we need to solve the following quadratic equation

$$60 = 350t - 500t^2 \quad or \quad 0 = -500t^2 + 350t - 60.$$

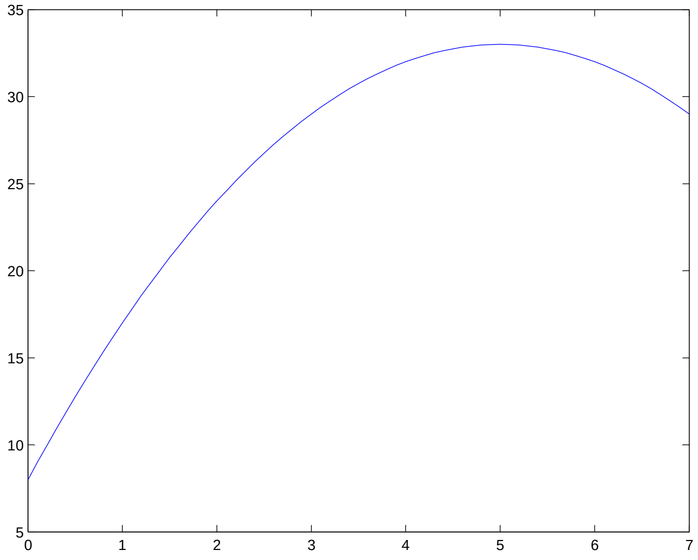

Figure 1: Graph of the function $y = 8 + 10x - x^2$ over the domain $[0, 7]$.

We use the quadratic formula to find the roots of this quadratic equation

$$t_1, t_2 = \frac{-350 \pm \sqrt{350^2 - 4(-500)(-60)}}{2(-500)}$$

The roots are $t_1 = 0.3$ and $t_2 = 0.4$.

(b) In the same way, in order to find the tax rate(s) consistent with raising tax revenues equal to 61.25 billion, we need to solve the following quadratic equation We use the quadratic formula to find the roots of this quadratic equation

$$t_1, t_2 = \frac{-350 \pm \sqrt{350^2 - 4(-500)(-61.25)}}{2(-500)}$$

There is one multiple root given by t = 0.35 (or two equal roots). This is because the term $b^2 - 4ac$ is equal to 0 in this case.