Market Games

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Economics 300
Quantity Competition between two firms

- Cournot competition with 3 strategies
- Cournot equilibrium lies between monopoly and perfect competition
Cournot competition for two firms

Market Price, \( P = 130 - Q \) when \( Q \leq 130 = 0 \) otherwise

Market Quantity, \( Q = x_1 + x_2 + \ldots + x_n = \sum x_i \)

Quantity vector, \( x = (x_1, x_2, \ldots, x_n) \)
here \( x_i \) represents firm i’s quantity delivered to the market

\( \therefore \) For a market with 2 firms,
\( Q = x_1 + x_2 \) and \( x = (x_1, x_2) \)

Constant average variable cost = \( c \)
Cournot competition for two firms: A firm’s profits

Firm i’s profits:

\[ u_i(x) = \text{revenue} - \text{cost} \]
\[ = Px_i - cx_i \]
\[ = (P - c)x_i \]

\[ \therefore u_1(x) = (P - c)x_1 \]
\[ \text{and} \]
\[ u_2(x) = (P - c)x_2 \]
Cournot competition for two firms: Profits for any given level of output

Let’s take $x_1 = x_2 = 30$ and $c = $10

$Q = 30 + 30 = 60$ and

$P = $130 - $60 = $70

$u_1(x) = (70 - 10) \times 30 = $1800$

$u_2(x) = (70 - 10) \times 30 = $1800$

Combination of profits will be different for different xs.
Cournot competition, two firms: Payoffs from different production plans

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>X₂ = 30</th>
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<tbody>
<tr>
<td>X₁ = 30</td>
<td>1800, 1800</td>
<td>1500, 2000</td>
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Cournot competition, two firms:
Strategy for firm 1

Firm 2

Firm 1

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Cournot competition, two firms: Strategy for firm 2

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Cournot competition, two firms: The equilibrium

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Cournot competition for two firms: The Cournot equilibrium

Cournot Equilibrium:

\[ x^* = (40, 40) \]

\[ Q^* = 40 + 40 = 80 \]

\[ P^* = 130 - 80 = 50 \]

\[ u_1(x^*) = (50 - 10) \times 40 = 1600 \]

\[ u_2(x^*) = 1600 \]
Cournot competition for two firms

Cournot competition between two firms leads to an outcome between monopoly and perfect competition
Cournot Competition, two firms, Deriving a Firm’s Best Response

Utility function of firm i: \( u_i(x) = (P - c)x_i \)

Consider firm 1

Firm 1 maximizes its profit by producing up to the point where marginal profit equals zero:

\[
0 = \frac{\partial u_1}{\partial x_1} = (P - c) + x_1 \frac{\partial P}{\partial x_1}
\]

\[
\Rightarrow 0 = (120 - x_1 - x_2) + x_1(-1)
\]

\[
\Rightarrow 0 = 120 - 2x_1 - x_2
\]
Finding Cournot best responses

- Firm 1’s first-order condition is:
  \[2x_1 + x_2 = 120\]
- Solving for \(x_1\) as a function of \(x_2\) yields firm 1’s best-response function:
  \[x_1 = f_1(x_2) = 60 - \frac{x_2}{2}\]
- Similarly, firm 2’s best-response function:
  \[x_2 = f_2(x_1) = 60 - \frac{x_1}{2}\]
Cournot best responses, \( x^* = \) Cournot equilibrium

\[
x_2 = f_2(x_1) = 60 - \frac{x_1}{2}
\]

\[
x_1 = f_1(x_2) = 60 - \frac{x_2}{2}
\]

\((40, 40) = x^*\)
Perfect Competitive Equilibrium for two firms

- Market price equals marginal cost
- In this market, marginal cost = \( c = $10 \)
- \( Q = 130 - P = 130 - 10 = 120 \)
- \( x^* = (60, 60) \)
- Profit for any firm = \((10 - 10) \times 60 = 0\)
Monopoly Equilibrium for two firms

- Market profits are as large as possible
- A monopoly will maximize total market profit, \( u = u_1 + u_2 = (P - c) Q \)
  \[ \Rightarrow u = (120 - Q) Q \]
- For maximizing profit, marginal utility of producing one more unit needs to be zero
  \[ \Rightarrow 0 = \frac{\partial u}{\partial Q} = 120 - 2Q \]
  \[ \therefore Q^* = 60 \text{ and} \]
  total profits = \((120-60) \times 60 = 3600\)
Cournot equilibrium in the market

- Monopoly is associated with the highest price, lowest quantity, and highest profit
- Perfect Competition is associated with the lowest price, highest quantity, and zero profit
- Cournot equilibrium lies in between on all three dimensions
The Cournot Limit Theorem

- The Cournot limit theorem: the higher the number of firms, the closer Cournot equilibrium gets to perfect competition
- The Cournot limit is good for the economy
Cournot market game:
Market outcomes compared

Monopoly equilibrium
Cournot equilibrium
Perfectly competitive equilibrium