Market Games

Professor Peter Cramton

Economics 300

Quantity Competition between two firms

- Cournot competition with 3 strategies
- Cournot equilibrium lies between monopoly and perfect competition

Cournot competition for two firms

Market Price, $P = 130 - Q$ when $Q \leq 130$
$= 0$ otherwise

Market Quantity, $Q = x_1 + x_2 + \ldots + x_n = \sum x_i$

Quantity vector, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$
here x_i represents firm i's quantity delivered to the market

$\therefore$ For a market with 2 firms,
$Q = x_1 + x_2$ *and* $\mathbf{x} = (x_1, x_2)$

Constant average variable cost = c

Cournot competition for two firms: A firm's profits

Firm i's profits:

$u_i(\mathbf{x})$ = revenue - cost

$= Px_i - cx_i$

$= (P - c)x_i$

$\therefore \quad u_1(\mathbf{x}) = (P - c)x_1$ and

$u_2(\mathbf{x}) = (P - c)x_2$

Cournot competition for two firms: Profits for any given level of output

Let's take $x_1 = x_2 = 30$ and $c = \$10$

$\therefore$ Q = 30 + 30 = 60 and

P = \$130 - \$60 = \$70

$u_1(\mathbf{x}) = (70 - \mathit{10}) \times 30 = \$1800$

$u_2(\mathbf{x}) = (70 - \mathit{10}) \times 30 = \$1800$

Combination of profits will be different for different **x**s.

Cournot competition, two firms: Payoffs from different production plans

Firm 1 \ Firm 2	$X_2 = 30$	$X_2 = 40$	$X_2 = 60$
$X_1 = 30$	1800, 1800	1500, 2000	900, 1800
$X_1 = 40$	2000, 1500	1600, 1600	800, 1200
$X_1 = 60$	1800, 900	1200, 800	0, 0

Cournot competition, two firms: Strategy for firm 1

Firm 1 \ Firm 2	$X_2 = 30$	$X_2 = 40$	$X_2 = 60$
$X_1 = 30$	1800, 1800	1500, 2000	**900**, 1800
$X_1 = 40$	**2000**, 1500	**1600**, 1600	800, 1200
$X_1 = 60$	1800, 900	1200, 800	0, 0

Cournot competition, two firms: Strategy for firm 2

Firm 1 \ Firm 2	$X_2 = 30$	$X_2 = 40$	$X_2 = 60$
$X_1 = 30$	1800, 1800	1500, **<u>2000</u>**	900, 1800
$X_1 = 40$	2000, 1500	1600, **<u>1600</u>**	800, 1200
$X_1 = 60$	1800, **<u>900</u>**	1200, 800	0, 0

Cournot competition, two firms: The equilibrium

Firm 1 \ Firm 2	$X_2 = 30$	$X_2 = 40$	$X_2 = 60$
$X_1 = 30$	1800, 1800	1500, **2000**	**900**, 1800
$X_1 = 40$	**2000**, 1500	**1600**, **1600** ☆	800, 1200
$X_1 = 60$	1800, **900**	1200, 800	0, 0

Cournot competition for two firms: The Cournot equilibrium

Cournot Equilibrium:

$\mathbf{x^*} = (40, 40)$

$Q^* = 40 + 40 = 80$

$P^* = 130 - 80 = 50$

$u_1(\mathbf{x^*}) = (50 - 10) \times 40 = 1600$

$u_2(\mathbf{x^*}) = 1600$

Cournot competition for two firms

Cournot competition between two firms leads to an outcome between monopoly and perfect competition

Cournot Competition, two firms, Deriving a Firm's Best Response

Utility function of firm i: $u_i(\mathbf{x}) = (P - c)x_i$

<u>Consider firm 1</u>

Firm 1 maximizes its profit by producing up to the point where marginal profit equals zero:

$$0 = \partial u_1/\partial x_1 = (P - c) + x_1 \partial P/\partial x_1$$

$\Rightarrow \quad 0 = (120 - x_1 - x_2) + x_1(-1)$

$\Rightarrow \quad 0 = 120 - 2x_1 - x_2$

Finding Cournot best responses

- Firm 1's first-order condition is:

  $2x_1 + x_2 = 120$

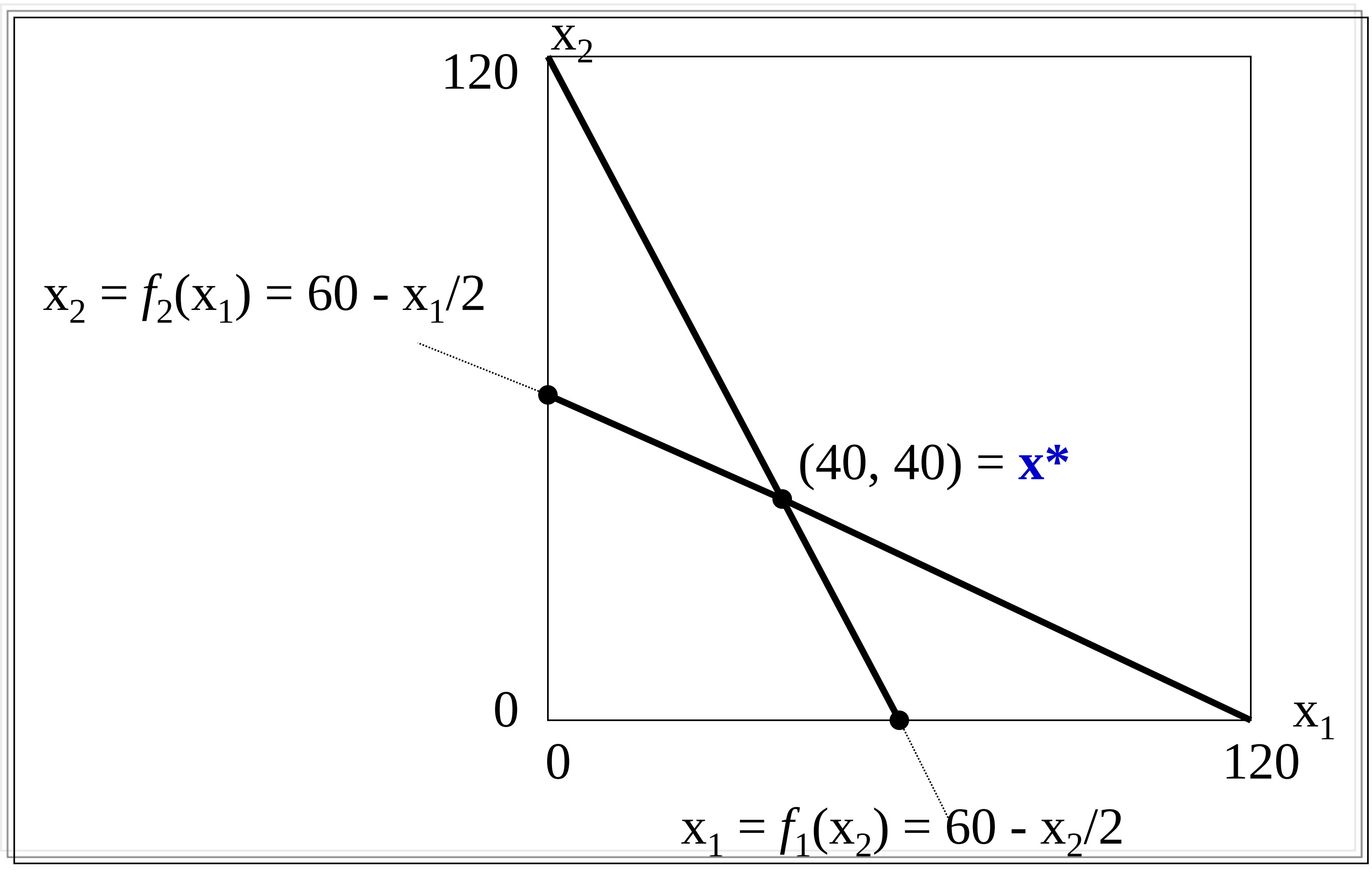
- Solving for x_1 as a function of x_2 yields firm 1's best-response function:

  $x_1 = f_1(x_2) = 60 - x_2/2$

- Similarly, firm 2's best-response function:

  $x_2 = f_2(x_1) = 60 - x_1/2$

Cournot best responses, **x*** = Cournot equilibrium



Perfect Competitive Equilibrium for two firms

- Market price equals marginal cost
- In this market, marginal cost = c = \$10
- Q = 130 - P = 130 - 10 = 120
- **x*** = (60, 60)
- Profit for any firm = (10 - 10) $\times$ 60

  = 0

Monopoly Equilibrium for two firms

- Market profits are as large as possible
- A monopoly will maximize total market profit, $u = u_1 + u_2 = (P - c) Q$

  $\Rightarrow \quad u = (120 - Q) Q$
- For maximizing profit, marginal utility of producing one more unit needs to be zero

  $\Rightarrow \quad 0 = \partial u / \partial Q = 120 - 2Q$

  $\therefore Q^* = 60$ and

  total profits = $(120-60) \times 60 = \$3600$

Cournot equilibrium in the market

- Monopoly is associated with the highest price, lowest quantity, and highest profit
- Perfect Competition is associated with the lowest price, highest quantity, and zero profit
- Cournot equilibrium lies in between on all three dimensions

The Cournot Limit Theorem

- The Cournot limit theorem: the higher the number of firms, the closer Cournot equilibrium gets to perfect competition
- The Cournot limit is good for the economy

Cournot market game:
Market outcomes compared