Risk Theory

Professor Peter Cramton

Economics 300

Is expected value a good criterion to decide between lotteries?

- One criterion to choose between two lotteries is to choose the one with a higher expected value

- Does this criterion provide reasonable predictions? Let’s examine a case…
  - Lottery A: Get $3125 for sure (i.e. expected value= $3125)
  - Lottery B: win $4000 with probability 0.75,
  - and win $500 with probability 0.25
  - (i.e. expected value also $3125)
- Which do you prefer?

Is expected value a good criterion to decide between lotteries?

- Probably most people will choose Lottery A because they dislike risk (risk averse)
- However, according to the expected value criterion, both lotteries are equivalent. Expected value is not a good criterion for people who dislike risk
- If someone is indifferent between A and B it is because risk is not important for him (risk neutral)

Expected utility: The standard criterion to choose among lotteries

- Individuals do not care directly about the monetary values of the prizes
  - they care about the utility that the money provides
- *U(x)* denotes the utility function for money
- We will always assume that individuals prefer more money than less money, so:

$$U'(x_i) > 0$$

Expected utility: The standard criterion to choose among lotteries

- The expected utility is computed in a similar way to the expected value
- However, one does not average prizes (money) but the utility derived from the prizes
- The formula of expected utility is:

$$EU = \sum_{i=1}^{n} p_i U(x_i) = p_1 U(x_1) + p_2 U(x_2) + \ldots + p_n U(x_n)$$

- The individual will choose the lottery with the highest expected utility
- Utility is invariant to linear transformation
  V(x) = a + bU(x) for b > 0 is an equivalent utility function

Classification

$$U''(X) < 0, \text{ strictly concave U(X)} \Rightarrow \text{Risk averse}$$

$$U''(X) = 0, \text{ linear U(X)} \Rightarrow \text{Risk neutral}$$

$$U''(X) > 0, \text{ strictly convex U(X)} \Rightarrow \text{Risk lover}$$

Examples of commonly used Utility functions for risk averse individuals

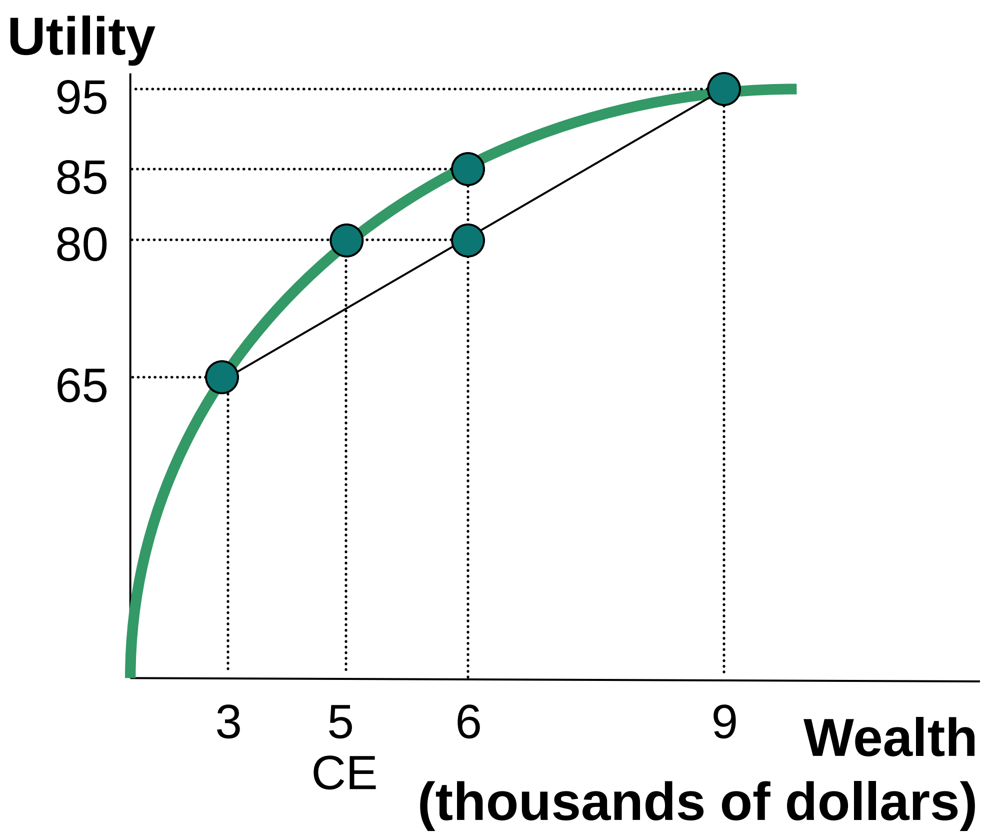
$$U(x) = \ln(x)$$

$$U(x) = \sqrt{x}$$

$$U(x) = x^a \quad \text{where} \quad 0 < a < 1$$

$$U(x) = 1 - \mathrm{e}^{-ax} \quad \text{where} \quad a > 0$$

Example: Alex is considering a job, which is based on commission & pays $3000 with 50% probability & $9000 with 50% probability.



$3000 is worth 65 units of utility to Alex, and $9000 is worth 95 units of utility.

The utility of the job's earnings is the average of 65 & 95, or 80 units of utility.

We can see from the TU curve that a job paying $6000 with certainty would be worth more to Alex (85 units of utility).

A job that paid $5000 with certainty would be worth the same level of utility to Alex as the risky job.

Measuring Risk Aversion

- The most commonly used risk aversion measure was developed by Pratt

$$r(X) = -\frac{U''(X)}{U'(X)}$$

- For risk averse individuals, $U''(X) < 0$
  - $r(X)$ will be positive for risk averse individuals
  - $r(X)$ = coefficient of absolute risk aversion
- r(X) is same for any equivalent U (i.e., a+bU)

Risk Aversion

- If utility is logarithmic in consumption

$$U(X) = ln\ (X)$$

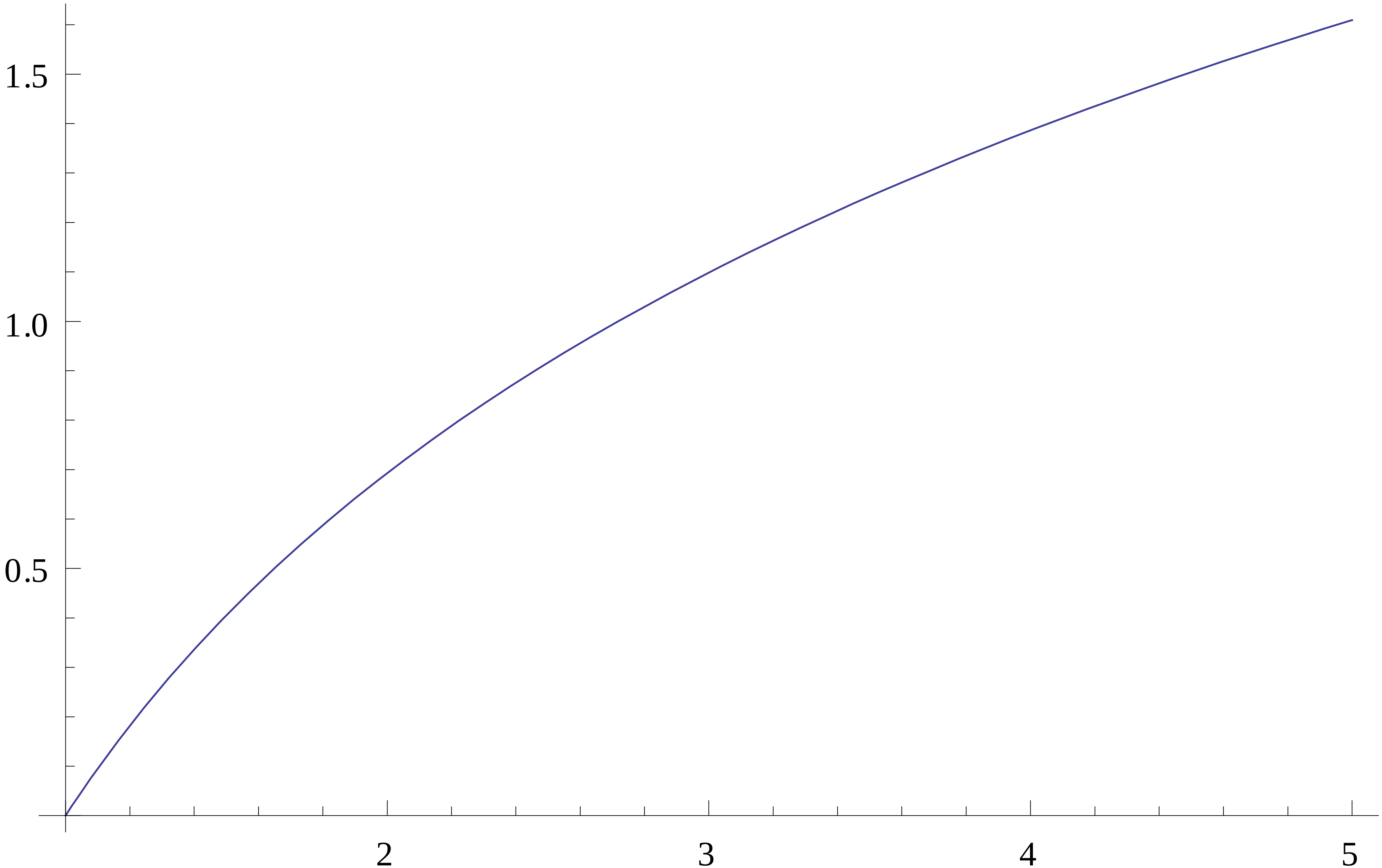
where $X > 0$

- Pratt's risk aversion measure is

$$r(X) = -\frac{U''(X)}{U'(X)} = -\frac{-X^{-2}}{X^{-1}} = \frac{1}{X}$$

- Risk aversion decreases as wealth increases

ln(x) becomes “more linear”



Risk Aversion

- If utility is exponential

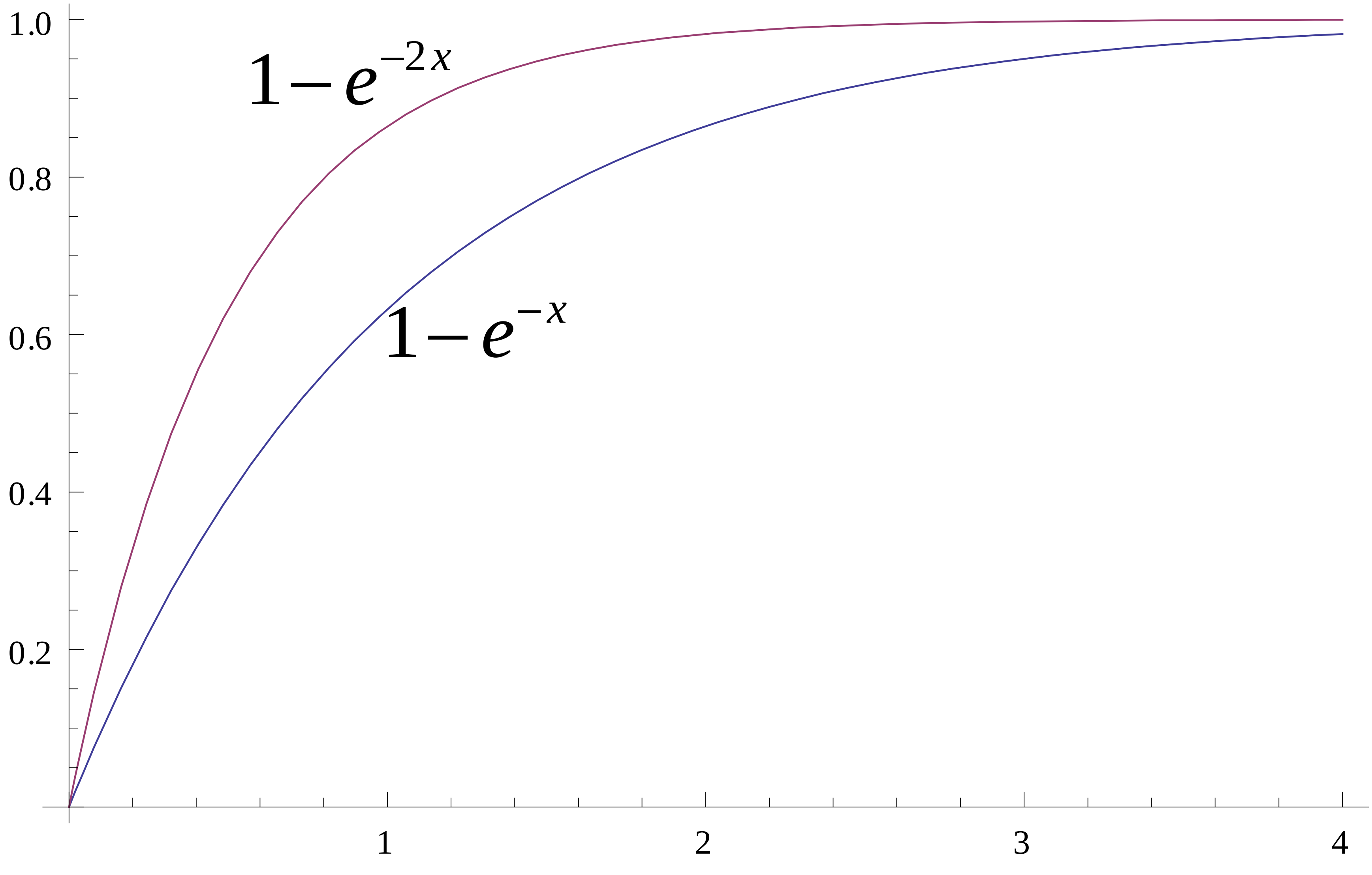
$$U(X) = -e^{-aX} = -exp\,(-aX)$$

where *a* is a positive constant

- Pratt's risk aversion measure is

$$r(X) = -\frac{U''(X)}{U'(X)} = -\frac{-a^2 e^{-aX}}{ae^{-aX}} = a$$

- Risk aversion is constant as wealth increases
  CARA = constant absolute risk aversion



1.0
0.8
0.6
0.4
0.2
1
2
3
4
$1-e^{-2x}$
$1-e^{-x}$

Willingness to Pay for Insurance

- Consider a person with a current wealth of \$100,000 who faces a 25% chance of losing his automobile worth \$20,000
- Suppose also that the utility function is

$$U(X) = ln\ (x)$$

Willingness to Pay for Insurance

- The person's expected utility will be

$$E(U) = 0.75U(100{,}000) + 0.25U(80{,}000)$$

$$E(U) = 0.75\ ln(100{,}000) + 0.25\ ln(80{,}000)$$

$$E(U) = 11.45714$$

Willingness to Pay for Insurance

- The individual will be willing to pay more than $5,000 to avoid the gamble. How much will he pay?

$$E(U) = U(100{,}000 - y) = ln(100{,}000 - y) = 11.45714$$

$$100{,}000 - y = e^{11.45714}$$

$$y = 5{,}426$$

- The maximum premium he is willing to pay is $5,426

  $426 more than "actuarially fair" insurance of $5,000

Summary

- Expected value is an adequate criterion to choose among lotteries if the individual is risk neutral
- However, it is not adequate if the individual dislikes risk (risk averse)
- If someone prefers to receive $B rather than playing a lottery in which expected value is $B then we say that the individual is <u>risk averse</u>
- If $U(x)$ is the utility function then we always assume that $U'(x)>0$
- If an individual is risk averse then $U''(x)<0$, that is, the marginal utility is decreasing with money ($U'(x)$ is decreasing).
- If an individual is risk averse then his utility function, $U(x)$, is concave
- We have studied a standard measure of risk aversion and insurance