Risk Theory

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Economics 300
Is expected value a good criterion to decide between lotteries?

• One criterion to choose between two lotteries is to choose the one with a higher expected value

• Does this criterion provide reasonable predictions? Let’s examine a case…
  – **Lottery A**: Get $3125 for sure (i.e. expected value= $3125)
  – **Lottery B**: win $4000 with probability 0.75,
    – and win $500 with probability 0.25
    – (i.e. expected value also $3125)

• Which do you prefer?
Is **expected value** a good criterion to decide between lotteries?

- Probably most people will choose Lottery A because they dislike risk (risk averse)
- However, according to the expected value criterion, both lotteries are equivalent. Expected value is not a good criterion for people who dislike risk
- If someone is indifferent between A and B it is because risk is not important for him (risk neutral)
Expected utility: The standard criterion to choose among lotteries

- Individuals do not care directly about the monetary values of the prizes
  - they care about the utility that the money provides
- $U(x)$ denotes the utility function for money
- We will always assume that individuals prefer more money than less money, so:

$$U'(x_i) > 0$$
**Expected utility**: The standard criterion to choose among lotteries

- The expected utility is computed in a similar way to the expected value
- However, one does not average prizes (money) but the utility derived from the prizes
- The formula of expected utility is:

\[
EU = \sum_{i=1}^{n} p_i U(x_i) = p_1 U(x_1) + p_2 U(x_2) + \ldots + p_n U(x_n)
\]

- The individual will choose the lottery with the highest expected utility
- Utility is invariant to linear transformation \( V(x) = a + bU(x) \) for \( b > 0 \) is an equivalent utility function
Classification

$U'(X) < 0$, strictly concave $U(X) \Rightarrow$ Risk averse

$U'(X) = 0$, linear $U(X) \Rightarrow$ Risk neutral

$U'(X) > 0$, strictly convex $U(X) \Rightarrow$ Risk lover
Examples of commonly used Utility functions for risk averse individuals

\[ U(x) = \ln(x) \]

\[ U(x) = \sqrt{x} \]

\[ U(x) = x^a \quad \text{where} \quad 0 < a < 1 \]

\[ U(x) = 1 - e^{-ax} \quad \text{where} \quad a > 0 \]
Example: Alex is considering a job, which is based on commission & pays $3000 with 50% probability & $9000 with 50% probability. $3000 is worth 65 units of utility to Alex, and $9000 is worth 95 units of utility. The utility of the job’s earnings is the average of 65 & 95, or 80 units of utility.

We can see from the TU curve that a job paying $6000 with certainty would be worth more to Alex (85 units of utility).

A job that paid $5000 with certainty would be worth the same level of utility to Alex as the risky job.
Measuring Risk Aversion

• The most commonly used risk aversion measure was developed by Pratt

\[ r(X) = \frac{-U''(X)}{U'(X)} \]

• For risk averse individuals, \( U''(X) < 0 \)
  • \( r(X) \) will be positive for risk averse individuals
  • \( r(X) \) = coefficient of absolute risk aversion
• \( r(X) \) is same for any equivalent \( U \) (i.e., \( a+bU \))
Risk Aversion

• If utility is logarithmic in consumption
  \[ U(X) = \ln(X) \]
  where \( X > 0 \)

• Pratt’s risk aversion measure is
  \[
  r(X) = - \frac{U''(X)}{U'(X)} = - \frac{-X^{-2}}{X^{-1}} = \frac{1}{X}
  \]

• Risk aversion decreases as wealth increases
ln(x) becomes “more linear”
Risk Aversion

• If utility is exponential
  \[ U(X) = -e^{-aX} = -\exp(-aX) \]
  where \( a \) is a positive constant

• Pratt’s risk aversion measure is

  \[ r(X) = -\frac{U''(X)}{U'(X)} = -\frac{-a^2e^{-aX}}{ae^{-aX}} = a \]

• Risk aversion is constant as wealth increases
  CARA = constant absolute risk aversion
\[ 1 - e^{-2x} \]

\[ 1 - e^{-x} \]
Willingness to Pay for Insurance

• Consider a person with a current wealth of $100,000 who faces a 25% chance of losing his automobile worth $20,000

• Suppose also that the utility function is

\[ U(X) = \ln(x) \]
Willingness to Pay for Insurance

• The person’s expected utility will be

\[ E(U) = 0.75U(100,000) + 0.25U(80,000) \]
\[ E(U) = 0.75 \ln(100,000) + 0.25 \ln(80,000) \]
\[ E(U) = 11.45714 \]
Willingness to Pay for Insurance

• The individual will be willing to pay more than $5,000 to avoid the gamble. How much will he pay?

\[ E(U) = U(100,000 - y) = \ln(100,000 - y) = 11.45714 \]

\[
100,000 - y = e^{11.45714}
\]

\[
y = 5,426
\]

• The maximum premium he is willing to pay is $5,426 $426 more than “actuarially fair” insurance of $5,000.
Summary

• Expected value is an adequate criterion to choose among lotteries if the individual is risk neutral.
• However, it is not adequate if the individual dislikes risk (risk averse).
• If someone prefers to receive $B rather than playing a lottery in which expected value is $B then we say that the individual is risk averse.
• If $U(x)$ is the utility function then we always assume that $U'(x)>0$.
• If an individual is risk averse then $U''(x)<0$, that is, the marginal utility is decreasing with money ($U'(x)$ is decreasing).
• If an individual is risk averse then his utility function, $U(x)$, is concave.
• We have studied a standard measure of risk aversion and insurance.