Systems of Equations and Comparative Statics

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Economics 300

Systems of equations

- Model many participants
- Model many markets
- Model many related variables

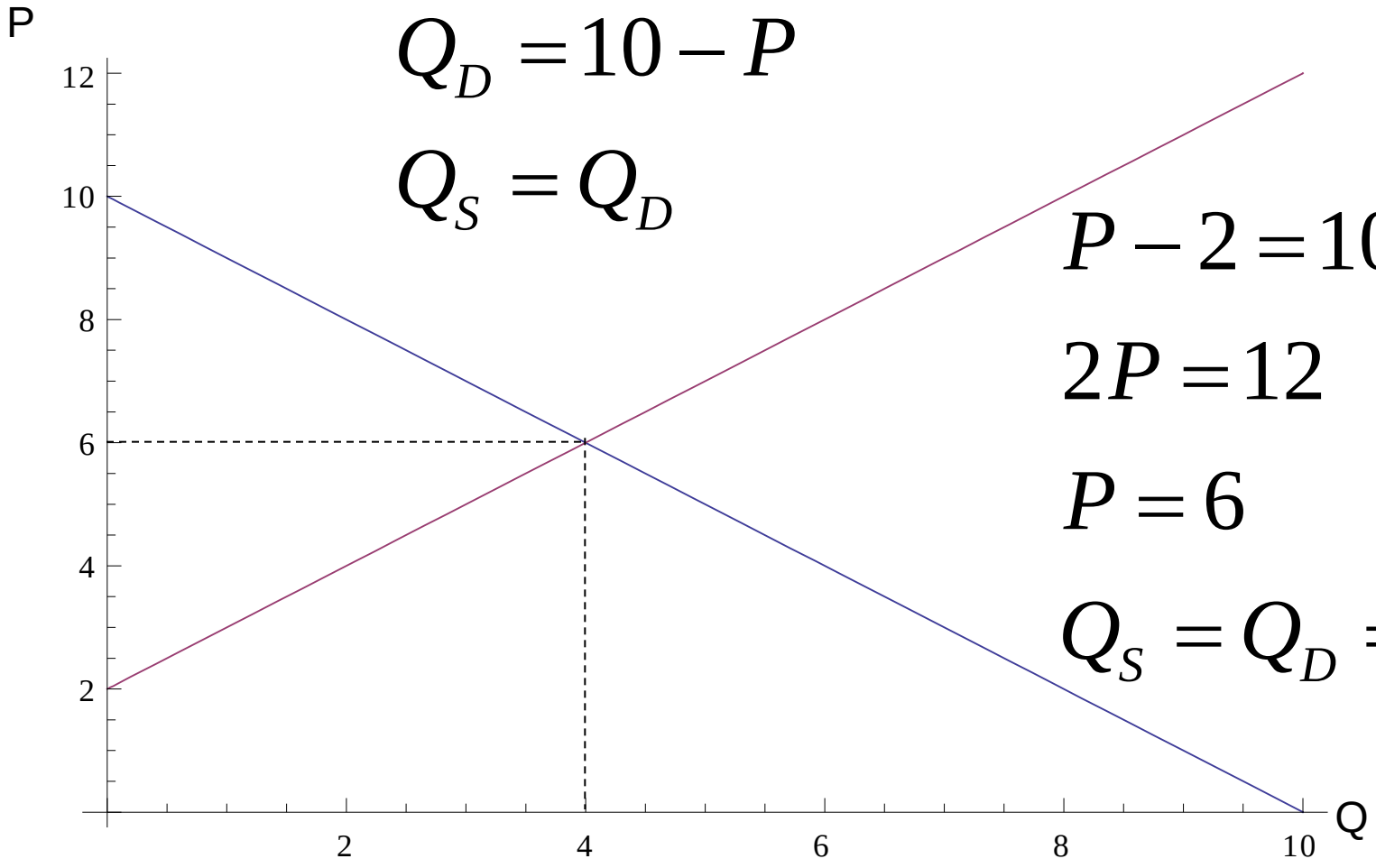
- Solving systems of equations
- Comparative statics
  - Find impact of change in exogenous variable on endogenous variables

Supply and demand

$$Q_S = P - 2$$

$$Q_D = 10 - P$$

$$Q_S = Q_D$$



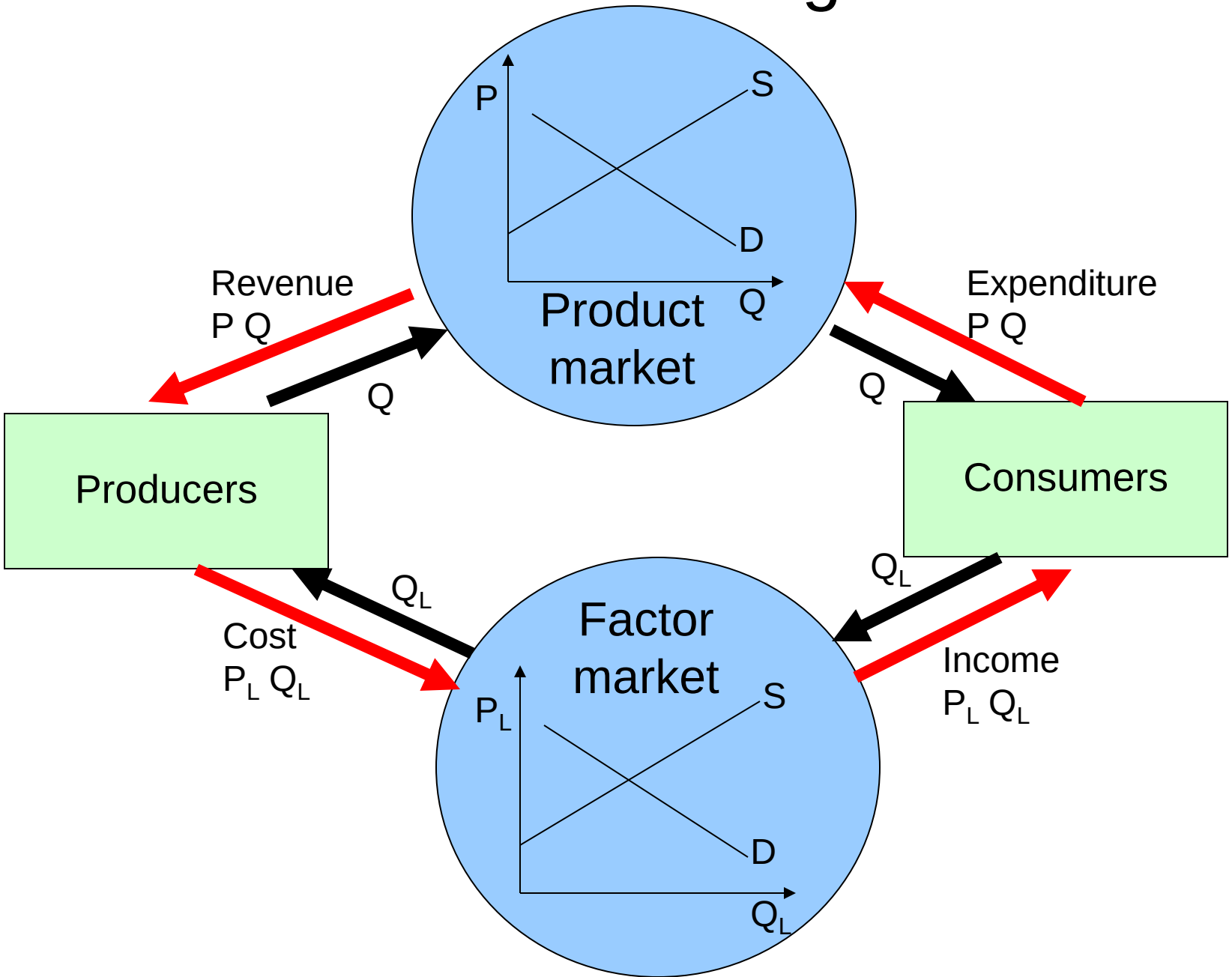
$$P - 2 = 10 - P$$

$$2P = 12$$

$$P = 6$$

$$Q_S = Q_D = 4$$

Circular flow diagram



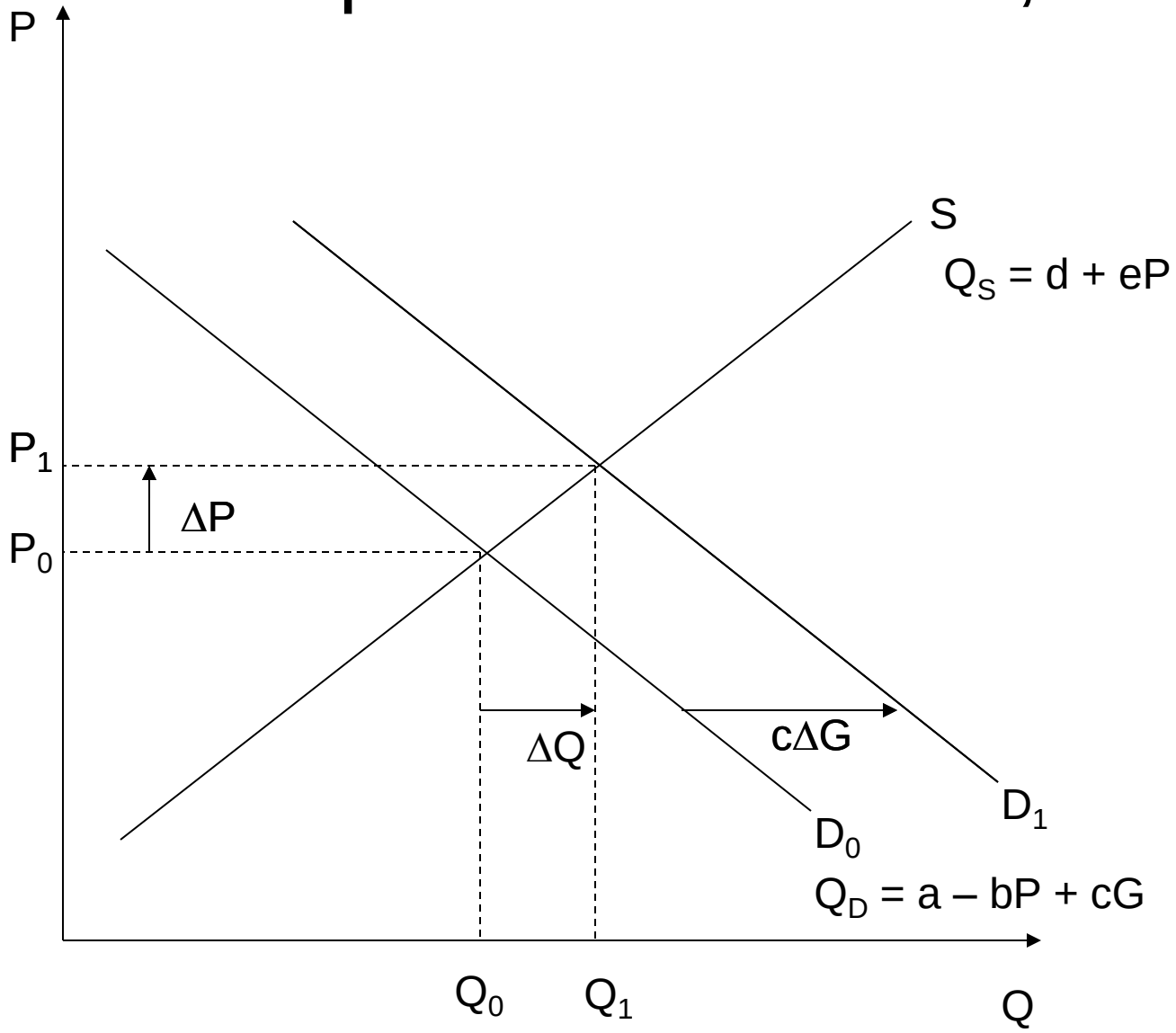
Two types of models

- Partial equilibrium
  - Individual supply and demand markets (L and Q)
  - Equilibrium price and quantity of good in isolation
    - Other goods are exogenous
- General equilibrium
  - Collection of all equations in Circular Flow Diagram
  - Equilibrium price and quantity of all goods together (all variables are endogenous)
- Both have many simultaneous equations

Partial equilibrium: supply and demand

- Demand equation $Q_D = a - bP + cG$
  - G = price of substitute good
  - a, b, c > 0 are parameters
- Supply equation $Q_S = d + eP$
- Balance equation $Q_D = Q_S = Q$
- Substitute into balance equation
  $d + eP = a - bP + cG$
- Solve for P
  $(b + e)P = a - d + cG \Rightarrow P = (a - d + cG)/(b+e)$
- Plug P into Q_S or Q_D
  $Q = d + e(a - d + cG)/(b+e)$

Comparative statics: Increase in price of substitute, ΔG



Comparative statics

- How does change in exogenous variable G impact endogenous variables P and Q?
- $P = (a – d + cG)/(b+e)$
- $Q = d + e(a – d + cG)/(b+e)$
- $\Delta G = G_1 – G_0$; $\Delta P = P_1 – P_0$; $\Delta Q = Q_1 – Q_0$
- $P_1 = (a – d + cG_1)/(b+e)$
  $P_0 = (a – d + cG_0)/(b+e)$
- $\Delta P = \Delta G\ c/(b + e)$
- $Q_1 = d + e(a – d + cG_1)/(b+e)$
  $Q_0 = d + e(a – d + cG_0)/(b+e)$
- $\Delta Q = \Delta G\ ec/(b + e)$

Exercise: Solve for x, y, z

- $x = 6z + 3h – 4a + 10$
- $y = 4z – h + 6$
- $x = y$
- Substitute 1st and 2nd into 3rd
  $6z + 3h – 4a + 10 = 4z – h + 6$
- Solve for z
  $2z = 4a – 4h – 4 \Rightarrow z = 2a – 2h – 2$
- Substitute z into 1st and 2nd
  $x = 12a – 12h – 12 + 3h – 4a + 10 = 8a – 9h – 2$
  $y = 8a – 8h – 8 – h + 6 = 8a – 9h – 2$

Exercise: Impact of $\Delta h = 3$

- $x = 8a - 9h - 2$
- $y = 8a - 9h - 2$
- $z = 2a - 2h - 2$
- $\Delta x = -9\Delta h = -27$
- $\Delta y = -9\Delta h = -27$
- $\Delta z = -2\Delta h = -6$